Stability of Spinning Solar Sail-craft containing A Huge Membrane

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Abstract

The solar sails are the spacecraft that are propelled by sunlight. The Institute of Space and Aeronautical Science (ISAS) has studied the solar sails, which are spinning and deployed by centrifugal force. One of the technological difficulties to be realized is how to design the spinning solar sail to maintain the stability while those huge membranes unfurl being deformed and oscillating. In this paper, an attention is focused on the out-of-plane oscillation and an analysis is presented about the dynamics so that the characteristic parameters are identified as for the stability of those spinning solar sails.

1. Introduction

The solar sails are well known as a kind of propulsion means that propel taking the advantage of the photons momenta. Recently some kinds of deployment methods have been investigated, and the spin-deployed configuration has been studied at the Institute of Space and Aeronautical Science, Japan Aerospace Exploration Agency (ISAS/J AXA) [1~3]. There are revealed some differences between the solar sails system and the other spacecraft. One of the most important differences is in the structure. While the spinning and oblate spacecraft with rigid structure are generally stable, the flexible membrane structure of the solar sail-craft may make it unstable. Photon pressure in space easily causes the oscillation to the membrane. Therefore it is required to...
design the spinning solar sail-craft to keep stability while the huge membrane shows oscillatory behavior.

There have been conducted so far some experimental or numerical methods to investigate the stability of the spacecraft with flexibility. Such kinds of methods are effective to investigate the stability of the spacecraft with flexible appendage. Such as the membrane. However, the gravity disturbance applies to the experiments on the ground and the results are subject to the experiment conditions provided [2]. Generally it is difficult to conclude any rigorous stability condition only from the numerical simulations. In contrast, analytical methods can derive the stability conditions with some explicit parameters that are useful in assessing the stability for the spacecraft possessing the huge membrane structure.

The purpose of this paper is to derive the stability conditions of the solar sail-craft through an analytical method. First, we derive the Newton-Euler equations of the solar sails motion with a circular membrane structure. And next, the discussion focuses its attention on the out-of-plane oscillation associated with the membrane and concludes the stability conditions. As a result, it is found that a few key parameters govern the stability of the motion. This result helps the sail-craft design and the inertia property of those spin-deployed solar sails system.

2. The membrane dynamics

In this section, the oscillation of a circle membrane is derived from the Newton's equations using the continuum model (Fig.1). The spacecraft dynamics with first-order mode is described in the following simplified form. The dynamics of the membrane is described as

\[
\frac{\partial}{\partial r} \left( r \sigma_r \frac{\partial w}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \sigma_\theta \frac{\partial w}{\partial \theta} \right) = \mu(r)(\ddot{w} + r \cos \Omega \omega_x + r \sin \Omega \omega_y) + r \sin \theta \dot{\omega}_x - r \cos \theta \dot{\omega}_y,
\]

where

- \( w(r,\theta,t) \); Z-direction displacement of the membrane,
- \( \mu(r) \); Density of the membrane,
- \( \sigma_r, \sigma_\theta \); Radial and hoop stress,
- \( \omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \Omega \end{bmatrix} \); Angular velocity of the spacecraft

Fig.1 A simplified model of a solar sail

Assuming that \( w = \alpha(r) \beta(\theta) g(t) \),
where\[g + (\lambda \Omega)^2 g = 0,
\]
\[\beta + v^2 \beta = 0,\]
and \[\mu(r) \lambda^2 \gg \frac{m^2}{r^2}\]
results. The assumption (3) requests that the value of \(\sigma\), \(\nu\), and \(\lambda\) are sufficiently small. By excluding high-order modes, the homogeneous solution of (1) is described as
\[w(r, \theta, t) = k_r r \sin \theta \cdot g_0(t) + k_r r \cos \theta \cdot g_0'(t) + k_r r \sin \theta \cdot g_1(t) + k_r r \cos \theta \cdot g_1'(t) + \ldots \]
By considering \(mv=0, \quad m=I\), this equation is re-written as follows:
\[w(r, \theta, t) = k_r r g_0(t) + k_r r \sin \theta \cdot g_1(t) + k_r r \cos \theta \cdot g_1'(t) + \ldots \]
The first term of this equation expresses the symmetric mode to the axial direction and is negligible in this analysis. Denoting the radius of the attachment point of the membrane as \(r^*\), assuming that \(r_0 > r^*\), it is derived the following equations:
\[w(r, \theta, t) = k_r (r-r^*) \sin \theta \cdot \phi(t) - k_r (r-r^*) \cos \theta \cdot \psi(t) \]
where \(r_0\) stands for the radius of the membrane.

Substituting this solution into the original equation of motion (1) and integrating, it is obtained
\[I_2 \ddot{\phi} + I_1 \Omega^2 \dot{\phi} + I_1 \left(\Omega \dot{\omega}_x + \dot{\omega}_z\right) = 0
\]
\[I_2 \ddot{\psi} + I_1 \Omega^2 \psi + I_1 \left(-\Omega \dot{\omega}_x + \dot{\omega}_z\right) = 0,\]
where
\[I_1 = \int_0^\infty \mu(r) r^2 dr, \quad I_2 = \int_0^\infty \mu(r) (r-r^*)^2 dr.
\]
and the damping effect,
\[\ddot{\phi} + c \dot{\phi} + (1+K)\Omega^2 \dot{\phi} + (1+K)(\dot{\omega}_x + \Omega \dot{\omega}_x) = 0
\]
\[\ddot{\psi} + c \dot{\psi} + (1+K)\Omega^2 \psi + (1+K)(\dot{\omega}_z - \Omega \dot{\omega}_z) = 0,\]
results.

3. The Stability of the Solar Sail Structure

In this section, the dynamics of the total system is described through the Euler's equations using the coordinate system fixedly defined on the membrane. And the characteristic parameters of the system stability are identified.

The total angular momentum is written
\[\hat{L} = I \omega_s + \hat{\rho} \times (\dot{\rho} + \omega_m \times \dot{\rho}) \quad \text{dm}
\]
\[\left(\begin{array}{c}
I \omega_x + \Delta (\phi + \Omega_M \psi) \\
I \omega_y + \Delta (\psi - \Omega_M \phi) \\
I \Omega_s + I \Omega_M
\end{array}\right),
\]
where, about the membrane,
\[\hat{\rho} = \begin{pmatrix}
r \cos \theta \\
r \sin \theta \\
w(r, \theta, t)
\end{pmatrix}, \quad \omega_m = \begin{pmatrix}
\omega_x \\
\omega_y \\
\Omega_M
\end{pmatrix},
\]
and also about the spacecraft,
\[I = \begin{pmatrix}
I_{xy} & 0 & 0 \\
0 & I_{xy} & 0 \\
0 & 0 & I_z
\end{pmatrix}, \quad \omega_s = \begin{pmatrix}
\omega_x \\
\omega_y \\
\Omega_s
\end{pmatrix},
\]
and \[I = I_{xy} + \frac{1}{2} I_1, \quad J = I_z + I_1, \quad \Delta = \frac{1}{2} I_2.\]
Here, both \(\omega_m\) and \(\omega_s\) are described in the membrane-fixed coordinate system. The
total angular momentum is conserved and expressed as follows:

\[ I \dot{\omega}_1 + (J - I) \Omega_\omega \omega_1 + \Omega_\omega (\ddot{\phi} + \Omega_\omega^2 \phi) = 0, \]
\[ I \dot{\omega}_2 - (J - I) \Omega_\omega \omega_2 + \Omega_\omega (\ddot{\psi} + \Omega_\omega^2 \psi) = 0, \]

where \( J = I_z + \Omega_M^2 + I_1 \).

The Laplace transform of the equation (6) and (8) expressions are written as follows:

\[ BW_j + (J - I) \Omega_M W_j - 2(1+K)\Omega_\omega^2 (s^2 + i\Omega_\omega^2 + (1+K)\Omega_M^2) = 0. \]
\[ BW_j - (J - I) \Omega_M W_j - 2(1+K)\Omega_\omega^2 (s^2 + i\Omega_\omega^2 + (1+K)\Omega_M^2) = 0. \]

The necessary condition for the system to be stable is that all coefficients of the terms of the 6th-order characteristic equation are positive. It is satisfied if

\[ \frac{\Omega_M^2}{\Omega_\omega^2} I_z - I_\omega + \frac{1}{2} I_3 > 0, \]
\[ 2I_{xy} - \frac{\Omega_M^2}{\Omega_\omega^2} I_z > 0, \]

where \( K \approx 0, I_3 = 2\pi \hbar \int_0^{\infty} \mu(r) r^2 r^2 dr \) .

If the membrane structure had a rigid body, the system would be almost stable in any case. Under the condition \( \Omega M \Omega_\omega \), Eq.(10), (11) \( \Leftrightarrow I_{xy} - \frac{1}{2} I_3 < I_z < 2I_{xy} \)

Or if \( \Omega M \Omega_\omega \),

\( (10) \Leftrightarrow \frac{1}{2} I_3 > I_{xy} \)

4. Results and Discussion

Considering the first-order oscillation, it is found that the characteristic parameters \( I_{xy}, I_z, I_3 \) represent the stability of the sail system. This is correct only if \( K = 0 \). It should be confirmed that eq. (10) is consistent with the eigen values of the characteristic equations, where \( K \approx 0 \) (\( \Leftrightarrow r_0 \gg r^* \)).

The results are shown in Table-1 and Table-2. They are composed of the eigen values of the characteristic equation of (9) and they are \( I_z - I_\omega + I_3/2 \) or \( -I_{xy} + I_3/2 \).

Table-1 and 2 show these values:

<table>
<thead>
<tr>
<th>( \Omega_\omega )</th>
<th>( \Omega_M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

respectively. All of them are calculated under the parameters below:

\( \mu = 9.75 \times 10^2 (r \leq r_0/2), 9.75 \times 10^3 (r > r_0/2) \ [kg/m^2] \)

\(<\text{Results}>\)

Major results are summarized below:

1) \( \square \square \square \square = 1.0 \wedge I_z > I_\omega \).

For all \( r_0 \) and \( r^* \), the system is found stable. The criterion (10) is also positive. (Table1.1)

\( \square \square \square \square = 1.0 \wedge I_z > I_\omega \).

For \( r_0 \) such that (10), the system is stable. The larger \( r^* \) admit the smaller \( r_0 \). (Table1.2, 1.3)

\( \square \square \square \square = 1.0 \wedge I_z > I_\omega \).

Regardless of \( I_z \), the system is stable for \( r_0 \) such that (10). The larger \( r^* \) admit the smaller \( r_0 \). (Table 2.1, 2.2)

In case the criterion (10) is not satisfied,
just one pair of conjugate eigen values have positive real parts.

5. Sample Control compensation Strategy for Stabilization

When \( I_{xy} < I_z \), the system becomes unstable with relatively small value of \( r_0 \) (Table 2.1–2.3, Fig 2.1–2.2). For the purpose of stabilizing this system, by introducing the thruster or a certain suitable control torque proportional to the angular velocity of the core body, the equation (9) becomes:

\[
\begin{align*}
&kW_x - (J + K_\Omega)W_x = \frac{(1 + K)(s^2 + \Omega^2)}{s^2 + c \Omega^2} (sW_x + \Omega W_y) = - \kappa W_x \\
&kW_y - (J + K_\Omega)W_y = \frac{(1 + K)(s^2 + \Omega^2)}{s^2 + c \Omega^2} (sW_x - \Omega W_y) = - \kappa W_y
\end{align*}
\]

The characteristic equation of (15) shows that the system becomes stable for all \( r_0 \), if \( K \) is a certain large number and \( K \approx 0 \). The results are shown in Fig 2.3.

6. Conclusion

Here is derived the stability condition of the simple continuum model of the solar sail-craft carrying a huge membrane, assuming the first-order mode is dominant to the stability. It is found that the 3 parameters affect it dominantly. This condition shows that the spacecraft is stabilized, if the radius of the membrane and that of the attachment point where the membrane attaches to are enough large to a certain extent.

Next, here is presented a simple control strategy applicable to the unstable cases.

These do show a design guideline for the spinning solar sail-craft with a huge and highly flexible membrane structure.

What is going to be investigated and shown are the numerical verification of the results obtained here, using the appropriate means such as the multi-particle methods.

7. References

$\rho=100\,\text{kg/m}^3$, $c=10^{-3}$

Table 1.1  ($\omega_s=\omega_m=1.0\,\text{rad/s}$, $I_z=120\,\text{kg/m}^3$, $r*=0.5\,\text{m}$)

<table>
<thead>
<tr>
<th>$r_0$ [m]</th>
<th>I1/I2</th>
<th>\begin{align*} \text{eigenvalue} \end{align*}</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.01</td>
<td>\begin{align*} -6.3+9.0j \end{align*}</td>
</tr>
<tr>
<td>50</td>
<td>1.02</td>
<td>\begin{align*} -0.42+4.2j \end{align*}</td>
</tr>
<tr>
<td>10.5</td>
<td>1.09</td>
<td>\begin{align*} -1.7\times10^{-3}+1.2j \end{align*}</td>
</tr>
<tr>
<td>1</td>
<td>3.76</td>
<td>\begin{align*} -5.0\times10^{-4}+1.9j \end{align*}</td>
</tr>
</tbody>
</table>

Table 1.2  ($\omega_s=\omega_m=1.0\,\text{rad/s}$, $I_z=80\,\text{kg/m}^3$, $r*=0.5\,\text{m}$)

<table>
<thead>
<tr>
<th>$r_0$ [m]</th>
<th>I1/I2</th>
<th>\begin{align*} \text{eigenvalue} \end{align*}</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.5</td>
<td>1.02</td>
<td>\begin{align*} -0.43+4.3j \end{align*}</td>
</tr>
<tr>
<td>20.5</td>
<td>1.05</td>
<td>\begin{align*} -1.6\times10^{-2}+1.6j \end{align*}</td>
</tr>
<tr>
<td>11.99</td>
<td>1.08</td>
<td>\begin{align*} -3.0\times10^{-3}+1.2j \end{align*}</td>
</tr>
<tr>
<td>1.5</td>
<td>2.65</td>
<td>\begin{align*} -5.0\times10^{-4}+1.6j \end{align*}</td>
</tr>
</tbody>
</table>

Table 1.3 ({$\omega_s=0$, $\omega_m=1.0\,\text{rad/s}$}, $I_z=80\,\text{kg/m}^3$, $r*=0.01\,\text{m}$)

<table>
<thead>
<tr>
<th>$r_0$ [m]</th>
<th>I1/I2</th>
<th>\begin{align*} \text{eigenvalue} \end{align*}</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.01</td>
<td>1.00</td>
<td>\begin{align*} -12+1.2j \end{align*}</td>
</tr>
<tr>
<td>50.01</td>
<td>1.00</td>
<td>\begin{align*} -0.72+1.2j \end{align*}</td>
</tr>
<tr>
<td>45.16</td>
<td>1.00</td>
<td>\begin{align*} -5.0\times10^{-1}+1.2j \end{align*}</td>
</tr>
<tr>
<td>1.01</td>
<td>1.01</td>
<td>\begin{align*} -5.0\times10^{-4}+1.0j \end{align*}</td>
</tr>
</tbody>
</table>

Table 2.1 ({$\omega_s=0$, $\omega_m=1.0\,\text{rad/s}$}, $I_z=\text{any}\,\text{kg/m}^3$, $r*=0.5\,\text{m}$)

<table>
<thead>
<tr>
<th>$r_0$ [m]</th>
<th>I1/I2</th>
<th>\begin{align*} \text{eigenvalue} \end{align*}</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.5</td>
<td>1.02</td>
<td>\begin{align*} -4.8\times10^{-1}+4.7j \end{align*}</td>
</tr>
<tr>
<td>30.5</td>
<td>1.03</td>
<td>\begin{align*} -7.7\times10^{-2}+2.7j \end{align*}</td>
</tr>
<tr>
<td>20.7</td>
<td>1.04</td>
<td>\begin{align*} -2.3\times10^{-2}+2.0j \end{align*}</td>
</tr>
<tr>
<td>1.5</td>
<td>2.65</td>
<td>\begin{align*} -5.0\times10^{-4}+1.6j \end{align*}</td>
</tr>
</tbody>
</table>

Table 2.2 ({$\omega_s=0$, $\omega_m=1.0\,\text{rad/s}$}, $I_z=\text{any}\,\text{kg/m}^3$, $r*=0.01\,\text{m}$)

<table>
<thead>
<tr>
<th>$r_0$ [m]</th>
<th>I1/I2</th>
<th>\begin{align*} \text{eigenvalue} \end{align*}</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.01</td>
<td>1.00</td>
<td>\begin{align*} -11+2.0j \end{align*}</td>
</tr>
<tr>
<td>80.01</td>
<td>1.00</td>
<td>\begin{align*} -4.9+2.0j \end{align*}</td>
</tr>
<tr>
<td>77.23</td>
<td>1.00</td>
<td>\begin{align*} -4.3+2.0j \end{align*}</td>
</tr>
<tr>
<td>0.51</td>
<td>1.01</td>
<td>\begin{align*} -5.0\times10^{-4}+1.0j \end{align*}</td>
</tr>
</tbody>
</table>

Fig 1.1 (Table 1.1; $r=0.5\sim30\,\text{m}$)

Fig 2.1 (Table 2.1; $r=0.5\sim30\,\text{m}$)

Fig 2.2 (Table 2.1; $r=0.5\sim30\,\text{m}$)