Estimation of the number of asteroids in the Main belt for a space probe with a clockwise orbit to approach

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Abstract

Asteroids in the Main Belt are said to have evidences of how the Solar System was formed, so they are really important for science. But it is not easy to investigate them by using telescopes, because they are too small to be examined sufficiently. One way to solve this problem is to send a space probe. the more asteroids are examined, the more discoveries can be gotten. In this present study, orbital elements of the space probe were optimized to meet as many asterods as possible in two cases. In one case, asteroids whose semi-major axes are close to 2.35AU (1AU equals 1.5 × 10^8 km) were targeted (case1). In the other case, all asteroids in the Main Belt were concluded in the calculation where our original function was introduced (case2). What is more, a quantitative way to throw the space probe into a clockwise orbit was gotten.

1 Introduction

Between Mars and Jupiter there are more than two hundred thousand asteroids circulating the Sun. As the distribution looks a belt, 'the Main belt' is commonly used.

It is said that they have floated since the Solar System was formed, so if scientific investigations of them are done, new information about the Solar System can be gotten.

Almost all of the asteroids fly around the Sun counterclockwise like all the planets except Pluto. It is obvious that if a space probe flies clockwise in the Main belt, it meets more asteroids than counterclockwise. Since exploration periods are finite, the fact that lots of asteroids can be investigated is important.

The direction the space probe flies to must be changed reversely, because the earth and asteroids fly to the same direction. All the way to realize this is swing-by, using Jupiter.

2 Simulation

In this study, as mentioned above, two cases were examined. In both cases the governing equation and conditions used in this study were same.

The governing equation is

\[ \frac{d^2 \mathbf{r}}{dt^2} = -\frac{\mu_{\text{sun}}}{r^3} \]

where \( \mathbf{r} \) is a position vector, \( t \) is time, \( \mu_{\text{sun}} \) is gravity constant (\( \mu_{\text{sun}} = 1.327 \times 10^{11} \text{km}^2/\text{s}^2 \)) and \( r \) equals | \( \mathbf{r} \) |.

The conditions are

- Ecliptic coordinates system was used.
- The exploration term is half of the period of the space probe.
- The space probe is already in the clockwise orbit when the calculation starts.
- Mean anomaly (\( M \)) was zero.

![Fig.1 Histogram about semi-major axis](image-url)
to 2.35AU were included in the calculation. Fig.1 shows why
the number 2.35AU was used. To make Fig.1, the data of two
hundred and fifty thousand asteroids were used[1]. The
peak stands where longitude equals 2.35AU, which means if
the semi-major axis of the space probe is 2.35AU, it can meet
more asteroids.
Six parameters are necessary to know how planets, aster-
oids or space probes move around a heavenly body, which are
semi-major axis($a$), eccentricity($e$), inclination($i$), ascending
node($\Omega$), peri-apsis($\omega$), and mean anomaly($M$).
About asteroids whose $a$ is close to 2.35AU, to determine
$i$ and $\Omega$, the unit normal vector of the orbital plane of each
asteroids was calculated. Distribution of $ecos\omega$ and $esin\omega$
was examined to know $e$ and $\omega$.
In the second case, two hundred and fifty thousand aster-
oids were targeted. In order to make the space probe meet
as many as possible, the optimized orbit of it was calculated
as follows:

\[ J(K, V, t) = \sum K_{xyz,i} \cdot dV \cdot dt \quad (2) \]

where $K_{xyz,i}$ is the number of asteroids in a cube, $dV$ is the
relative velocity between the space probe and an asteroid, and
d$\text{t}$ is the time it takes for the space probe to pass through
the cube. The symbols $a$, $e$, and $i$ were selected to maximize
$J(K, V, T)$, because highest $J(K, V, T)$ is desirable.

3 Results
From Fig.3, it can be said that the distribution of $\Omega$ is
equal, on the other hand, that of $i$ isn’t, because $\Omega$ is dis-
tributed in the ray-direction and $i$ in the circuit-direction.
Two rings are seen in Fig.3 and larger one where $i$ ranges
from 2.6 to 3.2 degree was targeted in this study. $\Omega$ was set
zero.
In Fig.4, $e$ is distributed in the ray-direction and $\omega$ in the
circuit direction. The distribution of $\omega$ is equal, so $\omega$ was
set zero. $e$ changes from 0.10 to 0.25. From these data, the
optimized orbit elements of the space probe are following:
$a = 2.35\text{AU}, e = 0.10 \sim 0.25, i = 2.6 \sim 3.2\text{degree}, \Omega, \omega = 0\text{degree}$
Fig. 5  The number of asteroids within a certain distance from the space probe (case1)

Fig. 6  Distribution of absolute magnitude of asteroids (case1)

From Fig. 6, it can be said that absolute magnitude of most of the asteroids ranges from 14 to 18. Many conditions influence it, but asteroids with absolute magnitude of 18 are as large as 1km in diameter.

Like Fig. 5, the number of asteroids within a certain distance from the space probe increases in proportion to the square of the distance.

Fig. 7  The number of asteroids within a certain distance from the space probe (case2)

Fig. 8  Distribution of absolute magnitude of asteroids (case2)

Fig. 9  One way to throw the space probe into a clockwise orbit by using Jupiter
In Fig. 9, the space probe is launched on 9/25/’13 and reaches Jupiter on 1/3/’15. Using Jupiter, it carries out swing-by. It passes by the earth on 9/13/’17. The trace of the space probe after that is conceptional.

4 Conclusion

- It is possible that a large number of asteroids are investigated by the space probe with a clockwise orbit.
- A realistic way to throw the space probe into a clockwise orbit is gotten.

Reference