Minimum Fuel Trajectory to an Asteroid on Elliptic Orbit

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Abstract
The purpose of this paper is to provide the information for selection of a target asteroid from many asteroids. The results of numerical calculation show the feature of trajectories to an asteroid on elliptic orbit with solar electric propulsion. It is assumed that the target orbit exists in the same plane of the earth’s orbit and the maximum available thrust is inversely proportional to the square of distance from the sun. The results show how the minimum fuel is affected by the departure time. This paper shows the results of cases that orbits of asteroid are outside of the earth’s orbit. It is concluded that orbits with higher eccentricity are less fuel and less affection of departure time.

INRODUCTION
Exploration to asteroids in the solar system is very interesting project in space sciences. The reason is that asteroids have different features from planets or satellites. Most of asteroids orbit the sun in asteroid belt, which lies between the orbits of Mars and Jupiter. Many asteroids, however, exist near the earth’s orbit, too. Itokawa, which is a target of Japanese asteroid explorer ‘Hayabusa’, is one of such asteroids. A large amount of calculation load is required to determine a target for mission planners. Their experiments and skills were necessary in the selection of target.

Solar electric propulsion(SEP) is one of key technologies for the future interplanetary exploration. Hayabusa is also accelerated by SEP system in transfer trajectories. Specific impulse of SEP is larger than one of chemical propulsion system, so it is possible to carry heavier payload. The maximum available thrust, however, is much smaller and depends on the distance from the sun. Thus impulse assumption, which is often used in preliminary trajectory design for spacecraft accelerated by chemical propulsion system, cannot be applied to design trajectories of explorers with SEP system. The numerical calculation derived using optimal control theory is necessary in trajectory design.

The authors have been studying on the trajectory optimization problems for spacecraft with SEP system[1][2]. They have shown that the objects in previous studies were transfer trajectories from one orbit to other orbit. The terminal condition is given that the explorer arrives the target asteroid. If the departure time can be selected arbitrarily, the optimal trajectory is the same as the trajectory shown in the previous papers. Thus main purpose of this paper is to show how the trajectory and fuel consumption change by the departure time change.

楕円軌道上小惑星への最小燃料軌道

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本稿は、多数ある小惑星の中から、目標惑星を選択する時に使用する情報を提供することを目的としている。電気推進等の低推力を用いた楕円軌道上の小惑星への最小燃料軌道に関して数値計算をもとにその特徴を示す。対象とする小惑星軌道は地球軌道と同一平面内の二次元問題とし、推力は太陽からの距離の二乗に反比例するモデルを使用する。出発時刻の変更が消費燃料の変化にどのように影響するかを中心にデータを示す。地球軌道の外側に小惑星軌道が存在する場合の結果を示す。離心率が高いため、消費燃料が少なく、出発時間の影響も少ないことが示される。
STATEMENT OF PROBLEM

The problems discussed in this paper are to minimize the fuel consumption of explorer from the earth in a circle orbit to an asteroid in an elliptic orbit using SEP system. To simplify the problem, the following assumptions are used in this paper. The trajectory of explorer and the orbits of the earth and the target asteroid are in the ecliptic plane. The orbits exist in center force field in which the sun locates at the center. The explorer is under the influence of only the gravitational acceleration of the sun. The equations of motion for the explorer are described in a polar coordinate.

\[
\begin{align*}
\frac{dr}{dt} &= u \quad (1) \\
\frac{d\theta}{dt} &= \frac{v}{r} \quad (2) \\
\frac{du}{dt} &= \left(T/m\right)\sin \phi + \frac{v^2}{r} - \mu/r^2 \quad (3) \\
\frac{dv}{dt} &= \left(T/m\right)\cos \phi - uv/r \quad (4) \\
\frac{dm}{dt} &= -T/c \quad (5)
\end{align*}
\]

where \( c \) represents the exhaust velocity of thruster.

\[
c = I_{SP} g_E \quad (6)
\]

where \( I_{SP} \) is specific impulse and \( g_E \) is the gravitational acceleration on the earth. It is assumed that the specific impulse is constant over the entire mission. The state variables are defined in Figure 1. The radius of the earth orbit is \( r_0 \). The input variables are the amplitude of thrust \( T \) and the angle of thrust \( \phi \), which are defined in Figure 1(b). The equations of motion of asteroid are the same as Eqs.(1)-(4) with \( T=0 \). The state variables of asteroid are presented with subscript ‘\( a \)’, for example \( r_a \).

The available power of SEP is provided by only power from the sun. The power is in inverse-square proportion to the distance between the explorer and the sun, \( r \). The maximum exhaust power of electric propulsion is in proportion to the available power. Constant exhaust velocity is assumed, thus the maximum thrust is also in inverse-square proportion to \( r \). The amplitude of thrust is given as follows.

\[
T = \varepsilon T_E \left(\frac{r_0}{r}\right)^2 \quad (7)
\]

where \( 0 \leq \varepsilon \leq 1 \)

where \( T_E \) represents the maximum thrust on the earth’s orbit. \( \varepsilon \) is a normalized thrust, which is defined as a ration of the actual thrust and the available thrust. Thus the upper and lower limits are always given as constant values for normalized thrust. The limitations of thrust are described using inequality constraints shown in Eq.(7).

The initial conditions of problem are that the explorer locates at \( \theta(0)=0 \) of the earth’s orbit and the asteroid locates at \( \theta_a(0)=\theta_{ini} \). The velocity of explorer is the same as the orbital velocity of the earth. The initial conditions for explorer are given as follows.

\[
\begin{align*}
\left(T \right)^a &= 0, \theta(0) = 0, u(0) = \frac{\sqrt{\mu}}{r_0} \quad (8) \\
v(0) &= 0, m(0) = m_0
\end{align*}
\]

where the subscript of ‘\( a \)’ indicates the initial value. \( \theta \) is defined as angle of polar coordinate in which the reference axis is a line connecting the sun and a perihelion of the orbit of target asteroid. Eccentricity and semi-major axis of the asteroid’s orbit are given as \( e \) and \( a \), respectively. The initial condition for asteroid is given as follows.

![Diagram](image)

(a) Definition of state variables in solar polar coordinate

(b) Components of velocity and input variables

Figure 1  Definition of state and input variables
The initial position of asteroid, \( f_{\text{ini}} \), is given.

The terminal condition of problem is that positions and velocities of explorer are as same as ones of asteroid.

Minimum fuel trajectories are discussed in this paper. Thus the criterion to be minimized is given as inverse value of the final mass.

\[
J = \left[ \frac{1}{m} \right] f_t \quad \text{(10)}
\]

In order to obtain high precession results, the transformed equations are used in calculation. The transformation is described in Refs.[1-3].

**OPTIMAL TRAJECTORIES**

Optimal trajectories obtained through numerical calculation are shown in this section. The following constant values are used in all examples. Specific impulse \( I_{sp} \) is 3000 s, the initial maximum available acceleration \( T_{max}/m_0 \) is 1 mm/s\(^2\), and semi-major axis of asteroid’s orbit \( a \) is 1.523. Eccentricity of asteroid’s orbit and initial position of asteroid are given for each calculation cases.

Example of optimal trajectories is plotted in Figure 2. Inner dashed line is the earth’s orbit and outer one is asteroid’s orbit. The trajectory is solid line and arrows indicate the direction of thrust. Eccentricity of asteroid’s orbit is 0.2 and the initial position of asteroid is 90 deg. At first, the explorer is accelerated a little inward. After a long coasting, the explorer is accelerated again. Finally, the explorer arrives at the asteroid with mass of 0.823\( m_0 \).

Figure 3 shows the optimal trajectory in the case of \( f_{\text{ini}}=30 \) deg. The explorer is accelerated outward at first. The transfer trajectory reaches outside of asteroid’s orbit before arriving asteroid in order to wait for the asteroid. The direction of final acceleration is also outward. Angle between the acceleration and the orbit relates to loss of energy. Thus tangential acceleration is almost optimum for minimum fuel trajectory. The final mass in Figure 3 is 80.8% in the initial mass \( m_0 \).

Figure 4 shows the optimal trajectory in the case of \( f_{\text{ini}}=150 \) deg. The departure time is later than other figures. Explorer is required to revolute faster, thus the trajectory is inside of the earth’s orbit. The direction of first acceleration is inward with a little deceleration. It is quite different from the other results. The explorer captures the asteroid at \( \theta = 370.0 \) deg with mass of 0.737\( m_0 \). The flight period is 382 days shorter than 431 days of Figure 2.
Differences of asteroid’s orbits are shown in Figures 5 and 6. The optimal trajectory in the case of $e=0.1$ is plotted in Figure 5. The asteroid’s orbit is far from the earth’s orbit where the explorer departs from. On the other hand, Figure 6 shows the result in the case of $e=0.3$. Perihelion of asteroid’s orbit is near the earth’s orbit. By comparison with Figures 2, 5 and 6, the direction of first acceleration changes from inward to tangential direction while the eccentricity increases from 0.1 to 0.3. Perpendicular element of acceleration changes the axis of elliptic orbit. It changes crossing point of two orbits. In the case of $e=0.3$, asteroid flies nearby the initial point of explorer. Thus it is not necessary for explorer to change the crossing point largely. The first acceleration is required to changes flight period, so the direction is tangential to the orbit. Thus energy loss is small. The final masses are $0.792m_0$, $0.823m_0$ and $0.844m_0$ for $e=0.1$, 0.2 and 0.3, respectively.

Figure 5 Optimal trajectory : $e=0.1 f_{ini}=90[deg]$

Figure 6 Optimal trajectory : $e=0.3 f_{ini}=90[deg]$

Figures 7 shows the final masses of optimal trajectories. Semi-major axes of asteroid’s orbit are same for all cases. Orbital energy is only a function of semi-major axis. Thus each final energy of explorer are same value. Figure shows the fuel consumption due to eccentricity or departure time. The following conclusions are obtained from this figure. (a)Less fuel is consumed for larger eccentricity. (b)Fuel consumption for larger eccentricity depends on less initial position of asteroid. Larger eccentricity in Figure 7 means that asteroid’s orbit is close to the earth’s orbit. As shown in Figure 6, explorer is accelerated in a direction of tangent to the initial orbit at first. Semi-major axis of transfer orbit depends on flight period to meet the asteroid near the initial position. Final acceleration is also in the tangential direction to satisfy the amplitude of velocity. Both accelerations are in the tangential direction, thus energy loss is small.

**CONCLUSION**

Minimum fuel trajectories to asteroid are calculated in this paper. Numerical results show better orbital condition of target asteroid from a viewpoint of fuel consumption and departure time. Less fuel consumption and less effect of departure time are obtained for asteroid whose orbit is close to the earth’s orbit.

**REFERENCES**

