Design and Control of Approach Trajectory for a space robot flying near a target satellite
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Based on the Hill’s equation, orbital dynamics of a space robot (chaser) flying around a trouble satellite (target) on a circular orbit around the earth is considered. Approach trajectories by impulsive thrust and control methods on relative angle were proposed by authors. This paper proposes a new control method based on the strategy named GAG (Gaining Angle of Gaze) of ball catching task using a mobile robot. The effectiveness of proposed method is shown by simulation under practical conditions.

1 Introduction

In order to provide services for an Earth-orbiting satellite, it is required to be able to use a fully autonomous servicing space robot that approaches and captures a satellite. In practice, NASA examined to capture the troubled SFU (Space Flyer Unit) in a Space Shuttle mission STS-72 in 1996[1]. Before capturing the SFU by the manual robot manipulator, the manned space shuttle approached the SFU. NASA developed ETS-VII (Engineering Test Satellite-VII) so as to examine automatic rendezvous docking by the unmanned space robot in 1998[2]. ETS-VII consisted of the target and the chaser, and the target was cooperative with the chaser capturing it. In these missions, the approaching task was executed by continuous control of the thrusters that need a large amount of fuel consumption. However, future space service missions are driving the need for small, low cost satellites whose motion relative to the target satisfies the following conditions.

- It is necessary to perform accurate approach in order to avoid a collision.
- The amount of fuel consumption is small.
- The trajectory can be maintained under disturbances by simple thruster control.

Before capturing the target, the chaser needs to approach it. Such approach trajectory should be designed so as to satisfy the three conditions. Approach trajectory that satisfies the above-mentioned three conditions is realized by two or more impulsive velocity changes was proposed[3]. Based on discrete modeling of error system, discrete LQR was employed to stabilize the trajectory subject to disturbances. In order to estimate the whole state variables of the chaser, the Extended Kalman Filter (EKF) is applied. The effectiveness of proposed method was shown by an integrated simulation including LQR, EKF, and VIC (Velocity Increment Cutoff) under practical conditions. As a result, it was clarified that these methods were dependent on the accuracy of an estimated initial state.

In this paper, orbital dynamics of a space robot (chaser) flying around a satellite (target) in trouble on a circular orbit around the earth is considered in order to approach the target as one of the recovery or repair mission by robotic systems. The method of approaching the target with sufficient accuracy is examined, even if there is an estimated initial error. We pay attention a new motion strategy named GAG (Gaining Angle of Gaze) of ball catching task using a mobile robot[4][5] and the strategy is applied to the space robot approaching the target.

In the following sections, relative motion on circular orbit, flyaround trajectory before approaching the target, strategy of the approach trajectory, Finally, an effectiveness of the proposed methods is tested through a simulation under practical conditions.
2 Relative Motion on Circular Orbit

Let us consider an orthogonal coordinate frame \( X, Y, \) and \( Z \) which moves together with the target on the circular orbit of the earth as shown in Fig. 1. Its origin coincides with the target position, \( Y \) axis is directed to the center of the earth, \( Z \) axis is orthogonal to the orbital plane, and \( X \) axis is defined according to the right-hand rule. This coordinate frame is rotating on a circular orbit of the target with the orbital angular velocity \( \omega \) around \( Z \) axis.

Because the relative distance between the chaser and the target can be assumed to be sufficiently small in comparison with a radius of the target’s orbit, the linearized differential equations of relative motion is obtained, which is called the Hill’s equation. Introducing the normalized time \( \tau = \omega t \) and its derivatives as \( \frac{d}{d\tau} = \left( \frac{d}{dt} \right) \) and \( \frac{d^2}{d\tau^2} = \left( \frac{d^2}{dt^2} \right) \), the equation is written as follows.

\[
\begin{align*}
\dot{x} &= 2\dot{y} + \alpha_x \\
\ddot{y} &= -2\dot{x} + 3y + \alpha_y \\
\dot{z} &= -z + \alpha_z
\end{align*}
\]

where \( \alpha_i = A_i/\omega^2 \) (\( i = x, y \)). \( A_i \) is the acceleration applied by thrust. The C-W solution in case of free motion\( (\alpha_x = \alpha_y = 0) \) can be written in the following forms.

\[
\begin{align*}
x(\tau) &= x_0 + 2\dot{y}_0 (1 - \cos \tau) + (4\dot{x}_0 - 6\dot{y}_0) \sin \tau + (6y_0 - 3\dot{x}_0)\tau \\
y(\tau) &= 4y_0 - 2\dot{x}_0 + (2\dot{x}_0 - 3y_0) \cos \tau + \dot{y}_0 \sin \tau \\
z(\tau) &= \dot{z}_0 \sin(\tau) + z_0 \cos(\tau)
\end{align*}
\]

where \( x_0 = x(0) \), \( \dot{x}_0 = \dot{x}(0) \), \( y_0 = y(0) \), \( \dot{y}_0 = \dot{y}(0) \), \( z_0 = z(0) \), and \( \dot{z}_0 = \dot{z}(0) \).

3 Flyaround Trajectory

Flyaround trajectories was proposed to observe the target before the chaser approaches the target[6]. Flyaround trajectories by impulsive velocity and free motion were designed. In case of free motion, the chaser moves in a three dimensional circular trajectory written by

\[
\begin{bmatrix}
x(\tau) \\
y(\tau) \\
z(\tau)
\end{bmatrix} = \begin{bmatrix}
2y_0 \sin \tau \\
y_0 \cos \tau \\
z_0 \sin \tau
\end{bmatrix}
\]

Though this trajectory which crossing the orbital plane at an angle of 60 degrees does not need any fuel consumption, the flyaround period \( \tau_f \) is fixed as \( \tau_f = 2\pi \), the orbital period of the target. Fig. 2 shows the flyaround trajectory in the case of the distance between the target and the chaser being 100 m. In this paper, the chaser begins to approach the target from initial state like the free motion in Fig. 2.

4 Approach Trajectory

The design of the approach trajectory was proposed[3]. It is the method of realizing the trajectory by two or more impulsive velocity changes. On the assumption that the on-board equipment on the chaser is poor and the target is not cooperative to the chaser, methods of realizing approach trajectory are developed. The chaser can observe only LOS (Line Of Sight) angle and VIC (Velocity Increment Cutoff) is adopted as the simplest method of driving thrusters. Based on discrete modeling of error system, discrete LQR is introduced to stabilize the trajectory subjected to disturbance. In order to estimate the whole state variables of the chaser, the Extended Kalman Filter (EKF) that observes only the LOS angle is designed. The chaser can approach the target with sufficient accuracy by the proposed methods if there is no estimated initial state error, that is, the accuracy of the final point is influenced by the estimated initial state error.

Let us consider the method that the chaser can approach correctly the target even if there is the es-
estimated initial error. We pay attention to a new motion strategy named GAG (Gaining Angle of Gaze) of ball catching task using a mobile robot[4][5] as same method. It is assumed that the required information for GAG is the ball’s elevation angle and azimuth angle measured by a monocular vision system fixed on the robot. The task objective is to approach a point just below the ball. The detail of GAG strategy is described in Appendix A.

When controlling a approach trajectory using a thruster system on the chaser, the controller needs available information. If optical sensors are employed in Fig.3, the relative angle between the satellites is more easily available than the relative distance. Therefore, it is assumed that the chaser can observe LOS (Line Of Sight angle) angles in Fig.4. Moreover, thrusters are mounted on the chaser in X, Y and Z directions and are assumed to be controlled independently and continuously without restriction.

GAG strategy is applied to the space robot (chaser) approaching the target. It is assumed that the chaser can move only $X_c - Y_c$ plane, that is, the thruster mounted on the chaser in X and Y direction is only used and Z direction is free motion.

Angle $\alpha$ in Fig. 5 and angle $\beta$ in Fig. 6 are described by

$$\alpha = |\theta_2|$$

$$\beta = \theta_1 - \theta_{1_{initial}}$$

We propose that the strategy to approach the target generates the desired velocity $v_r$ directed to the target on $X_c - Y_c$ plane in Fig.5 and the desired velocity $v_w$ in Fig.6 of the form

$$v_r = k_r \sin 2\alpha$$

$$v_w = k_w \sin \beta \sin 2\alpha$$

where, $k_r$ and $k_w$ are positive constants. Eq.(7)(8) can be regarded as a nonlinear feedback control law with respect to $\alpha$ and $\beta$ satisfying the conditions

$$0 \leq \alpha < \frac{\pi}{2}, \quad 0 \leq |\beta| \leq \pi$$

Introducing deviations from the relative velocity $v_{real}$ of the form

$$a_r = k_1(v_r - v_{real} \cos \theta_4)$$

$$a_w = k_2(v_w - v_{real} \sin \theta_4)$$

$$\theta_4 = \arctan(\dot{y}, \dot{x}) - \arctan(-y, -x)$$
5 Simulation

The proposed approach strategy in Sec 4 is tested in simulation. The parameters of the simulation are shown in Table I. The initial state is shown in Table II and is derived from flyaround trajectory in Sec 3. The final point is \([x, y, z] = [0, 0, 0]\). Approach trajectory simulation in the case of condition(a) and

\[
 \begin{align*}
 k_r &= k_w = k_1 = k_2 = 0.2 \\
 k_r &= k_w = k_1 = k_2 = 0.4 \\
 k_r &= k_w = k_1 = k_2 = 0.2 \\
 \end{align*}
\]

is shown in Fig. 7—Fig. 11. Figures are respectively X-Y trajectory for fixed time \((30\text{s})\) interval, angle \(\alpha\), relative velocity, acceleration by thrust, and X-Y-Z trajectory. Approach trajectory simulation in the case of condition(b) and

\[
 \begin{align*}
 k_r &= k_w = 0.4 \\
 k_r &= k_w = 0.4 \\
 k_r &= k_w = 0.4 \\
 \end{align*}
\]

is similarly shown in Fig. 12—Fig. 16. It is shown in Fig. 7 and Fig. 12 that the chaser can approach the target while reducing the relative velocity. Angle \(\alpha\) is converging on \(\frac{\pi}{2}\) [rad] in Fig. 8 and Fig. 13. Fig. 9 and Fig. 14 shows that relative velocity in the direction of X and Y axis is controlled well and converges \(0\) [m/s]. It is clarified that large acceleration by thrust is required in early stages of the simulation by Fig. 10 and Fig. 15. Three dimensional trajectory is shown in Fig. 11 and Fig. 16 and the chaser can reach the target. However, the motion in the Z axis is the free motion by hill’s equation. It is necessary to consider the approach based on the distance or the angle of only this direction in future. In the viewpoint of fuel consumption, the proposed strategy should be applied to the approach trajectory in the case of the distant between the target and the chaser being short, although accurate approach is performed to avoid collision.

6 Conclusion

In this paper, we propose the control method of approach trajectory. The proposed method is based
chaser can approach the target by the proposed method on LOS Angles in the simulation. It is necessary to consider the motion in the direction of Z axis to approach safely on a three dimensional trajectory. It can be assumed that distance can be measured and the motion in the direction of Z axis can be controlled on the distance, if the distance between the target and the chaser is very short.

References


A GAG (Gaining elevation Angle of Gaze)

A strategy for projectile interception called GAG (Gaining elevation Angle of Gaze) that enables the fielder to catch a fly ball in any situation with the monocular vision. The required information for GAG is the projectile’s elevation angle and azimuth angle measured by monocular vision of the fielder. The validity of GAG can be shown mathematically by considering a mobile robot as substitute for the fielder. Especially, GAG for a wheel type mobile robot with non–holonomic constraint generates dexterous pursuing motion notwithstanding its simplicity. On certain occasions, the robot starts getting forward to pursue the ball after adjusting itself by rotation backward. The concept of GAG is explained in the following three cases.

Let us consider a powered wheel steering robot with non–holonomic constraint and assume that the information available for the robot is the ball’s elevation angle $\alpha$ and azimuth angle $\gamma$ measured by a monocular vision system fixed on the robot. The task objective in this case is to approach a point just below the ball.

A circle centered on the vertical projection of the ball to the ground in Fig. 17 is introduced. EC (Equian-<n>gular circle) is a circle centered on the vertical projection of the ball to the ground on which $\alpha$ is un-<n>changing. If the robot on EC moves inward as depicted with a dotted line, $\alpha$ increases. Therefore, as the robot repeats this sort of movement, it gets closer and closer to the center of EC. This is the most ba-<n>sic idea of GAG. However, the center of EC is not measurable with the monocular vision system. GAG is a strategy that leads the robot so as to keep the tangent of $\alpha$ increasing only by using $\alpha$ and $\gamma$.

The movable direction of the mobile robot with non–holonomic constraint depends on its posture. To move this kind of robot toward the inside of EC, GAG generates the translational velocity $v$ and the angular velocity $\omega$ of the form

$$ v = k_1 \cos \gamma \sin 2\alpha \quad (14) $$

$$ \omega = k_2 \sin \gamma \sin 2\alpha \quad (15) $$

where, $k_1$ and $k_2$ positive constants. Before showing the validity of GAG, we will give another interpretation of these equations. From (14)(15), we can see that the curvature of the path is proportional to $\tan \gamma$. In addition, the robot comes to rest just below the ball because $v$ and $\omega$ are proportional to $\sin 2\alpha$. We can also regard (14)(15) as a nonlinear feedback control law with respect to $\alpha$ and $\gamma$ satisfying the conditions

$$ 0 \leq \alpha < \frac{\pi}{2}, \quad 0 \leq |\gamma| \leq \pi \quad (16) $$

The path of the robot subject to this control law varies according to the initial posture, namely $\gamma$. If $0 < \gamma \leq \frac{\pi}{2}$, the robot moves toward the inside of EC and gets closer and closer to the center of EC, though the robot does not know where the center is. If $\frac{\pi}{2} < \gamma \leq \pi$, first the robot rotates toward the center of EC while getting backward, and then starts getting forward after the posture becomes less than $\frac{\pi}{2}$, that is, $0 < \gamma \leq \frac{\pi}{2}$. In case that $\gamma$ has a minus sign, the robot moves in the same manner except rotating inversely.