DYNAMICS ANALYSIS OF SOLAR SAIL MEMBRANE USING IMPROVED MULTI-PARTICLE MODEL

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Abstract
Japan Aerospace Exploration Agency (JAXA) is currently studying on the “Solar Sail” propulsion for future deep space explorations. One of the key technologies to realize the solar sail is how light and how compact we can make the photon acceptance surface. JAXA has conducted extensive studies on utilizing centrifugal force to deploy the photon acceptance surface. The final objective is to realize the 7.5µm-thickness and 50m diameter polyimide membrane, combined with thin flexible solar cells, as the photon acceptance surface that will be needed around the Jupiter orbit. In the August 9, 2004, JAXA has launched the S-310 sounding rocket, which tested two different shapes of membranes during the zero-gravity flight. The first type of the membrane looks like a “clover-leaf”, and another is like a “fan”. These two membranes, both of them have 10m diameter, were unfolded sequentially during the zero-gravity flight under the free spin condition, and their behavior was observed by onboard cameras. This paper focuses on the “clover-leaf” solar sail, which was fully deployed successfully, and compares its result with a numerical simulation model called improved multi-particle model, to validate its applicability for the analysis of actual 50m size solar sail dynamics.

1 Introduction
Japan Aerospace Exploration Agency (JAXA) is currently studying on the “Solar Sail” propulsion for future deep space explorations. One of the key technologies to realize the solar sail is how light and how compact we can make the photon acceptance surface. JAXA has conducted extensive studies on utilizing centrifugal force to deploy the photon acceptance surface, which we assume the most reliable and weight-effective solution to deploy the sail. Our baseline mission plan uses the 7.5µm-thickness and 50m diameter polyimide membrane, combined with thin flexible solar cells, as the photon acceptance surface that will be needed around the Jupiter orbit. The dynamics study of the membrane is conducted by the computational and experimental approaches, and as a milestone of this dynamics study, JAXA has launched the S-310 sounding rocket in the August 9, 2004, which tested two different shapes of membranes during the zero-gravity flight. S-310 is an ISAS (Institute of Space and Astronautical Science)/JAXA’s sounding rocket that flies up to 200km and supplies zero-gravity environment for about 3 minutes. The first type of the membrane looks like a “clover-leaf”, and another is like a “fan”. These two membranes, both of them have 10m diameter, were unfolded sequentially during the zero-gravity flight under the free spin condition, and their behavior was observed by onboard cameras. The major objective of the experiment is as follows;

(1) Combined with the past experiments using vacuum chamber (membrane diameter: φ1.5m), high altitude large balloon (φ4m), and the spinning table (φ2.5m)²³⁴, the scale dependency on the membrane dynamics is obtained to tune the simulation parameters, so that the reliable extrapolation to the 50m diameter sail should be-
3.1 Normalization Parameter of Dynamics

(2) To obtain the damping ratio of the membrane structure under the actual zero-gravity environment.

(3) To find anything that can only be seen in the zero-gravity environment.

The post-flight analysis has been done mainly using the “clover-leaf” result, which was fully deployed successfully. In this paper, the S-310 flight result is quickly reviewed, and the dynamics analysis of the clover-leaf sail and the parameter-tuning of the model based on the flight result is shown.

2 S-310-34 Flight Result

The flight sequence of S-310-34 is shown in Fig.1. The motor case is not jettisoned to hold as much angular momentum as possible for stable sail deployment, and also to observe the sail behavior from the tail of the motor by cameras. S-310-34 was launched at 17:15JST, on August 9, 2004 from Uchinoura Space Center. After the successful deployment of yo-yo despinner, nose cone and the cameras’ windshields, the first sail, the clover-sail, was deployed at X+100s. After the successful deployment of the clover-sail, the fan-sail was triggered to deploy at X+190s, which was stuck halfway.

The clover sail, shown in Fig.2, is considered so that all the folding lines should form large angles with the direction of the centrifugal force. The deployment sequence is divided into two stages (Fig.3). The first stage is similar to a simple yo-yo despinner dynamics, while the second stage is the dynamics of a thin flexible structure itself. The material of the sail is the 7.5μm-thickness polyimide, on which the 100Å aluminum is deposited on both sides to avoid static electricity.

The maximum diameter of the sail is 9.75m, and the weight is 0.72kg. To stabilize the deployment dynamics, eight 0.02kg weights and four 0.07kg weights were attached on the tip of the clover-sail.

The deploying clover-sail was successfully captured by the tail camera(Fig.4) and side-view camera.

3 Sail Dynamics Analysis

3.1 Normalization Parameter of Dynamics

Let us consider the outer-plane dynamics of the membrane. Here the membrane is assumed to be circular and its dynamics is axisymmetric for simplicity. The dynamics equation (as a thin plate) is expressed as:

$$ D\Delta\Delta w - q_z - T_r \frac{\partial^2 w}{\partial r^2} - T_\theta \left( \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) - 2S_\theta \left( \frac{1}{r} \frac{\partial^2 w}{\partial r \partial \theta} + \frac{1}{r^2} \frac{\partial w}{\partial \theta} \right) + q_r \frac{\partial w}{\partial r} + q_\theta \frac{1}{r} \frac{\partial w}{\partial \theta} = 0 \quad (1) $$

where $q_r$, $q_\theta$, $q_z$ are the body forces along radial, circumferential and outer-plane directions, respectively. $w$ is the outer-plane displacement, $T_r$, $T_\theta$, $S_\theta$ are the radial and circumferential tensile stresses and shearing stress, respectively. $D$ is the representative bending rigidity. Using elasticity $E$, thickness $h$, Poisson’s ratio $\nu$, $D$ is written as follows:

$$ D = \frac{Eh^4}{8(1-\nu^2)} \quad (2) $$

It is known that the rotating circular plate receives the following stresses:

$$ \sigma_r = \frac{(3+\nu)\rho\Omega^2 R^2}{8} \left( 1 - \frac{r^2}{R^2} \right) \quad (3) $$

$$ \sigma_\theta = \frac{(3+\nu)\rho\Omega^2 R^2}{8} \left( 1 - \frac{1+3\nu}{3} \frac{r^2}{R^2} \right) \quad (4) $$

$$ \gamma_{r\theta} = 0 \quad (5) $$

where $\rho$ is the density, $R$ the radius, $\Omega$ the spin rate of the membrane respectively. Using (3)-(5), the following substitutions become available:

$$ q_z = -\rho\dot{w} \quad (6) $$

$$ T_r = h\sigma_r + \frac{m}{2\pi R} \quad (7) $$

$$ T_\theta = h\sigma_\theta \quad (8) $$

$$ q_\theta = q_r = S_\theta = \frac{\partial}{\partial \theta} = 0 \quad (9) $$

where $-\rho\dot{w}$ is an inertial force, $m$ is a tip mass attached on the edge of the membrane. Substituting
Figure 4: Deploying Clover-Sail of S-310-34

(6)-(9) into (1), we get;

$$D \Delta \Delta w - \mu \ddot{w} - \frac{(3 + \nu)}{8} \mu \Omega^2 \left[ \left( 1 - \frac{r^2}{R^2} \right) R^2 \frac{\partial^2 w}{\partial r^2} \right] + \left( 1 - \frac{1 + 3\nu}{3 + \nu} \frac{r^2}{R^2} \right) \frac{R^2}{r} \frac{\partial w}{\partial r} - \frac{m}{2\pi \Omega^2} \frac{\partial^2 \dot{w}}{\partial r^2} = 0 \quad (10)$$

where $\mu = \rho h$ is the surface density. The first term of (10) is the bending, the second is the inertial force, the third is the centrifugal force affected on the membrane, and the last term is the centrifugal force on the tip mass.

Now let $\tilde{r}, \tilde{w}$ denote the normalized lengths defined as $r = R\tilde{r}, w = R\tilde{w}$. Then (10) is normalized as;

$$\ddot{\tilde{w}} = \frac{D}{\mu R^3} \Delta \Delta \tilde{w} - \frac{(3 + \nu)}{8} \Omega^2 \left[ \left( 1 - \tilde{r}^2 \right) R^2 \frac{\partial^2 \tilde{w}}{\partial \tilde{r}^2} \right] + \left( 1 - \frac{1 + 3\nu}{3 + \nu} \tilde{r}^2 \right) \frac{R^2}{\tilde{r}} \frac{\partial \tilde{w}}{\partial \tilde{r}} - \frac{m}{2\pi \Omega^2} \frac{\partial^2 \dot{\tilde{w}}}{\partial \tilde{r}^2} \quad (11)$$

Then let us define $\tau_C, \tau_D, \tau_B$ as the time constants of the membrane (continuous mass) dynamics, weights (discrete mass) dynamics and bending effect, respectively. They are derived from (11) as follows;

$$\tau_C = \frac{1}{\Omega} \quad (12)$$
$$\tau_D = \sqrt{\frac{2\pi \mu R}{m \Omega}} \quad (13)$$
$$\tau_B = \sqrt{\frac{\mu}{D} R^2} \quad (14)$$

It can be seen from (12) that when the membrane mass is dominant (no bending, no tip mass), the dynamics is only governed by the spin rate. Also from (13), when the tip mass effect is not negligible, the dynamics does not only depend on the spin rate, but also on the size of the membrane. From (14), the transient behavior is related to $R^2$, but not influenced by the spin rate when the bending is dominant.

3.2 Multi-Particle Model Simulation

To numerically analyze the dynamics of the membrane, we have developed a “Multi-Particle Model(MPM)” MPM is suited for fast analysis of transient or dynamic behavior of the thin flexible structure. MPM has several versions suited for simulation objectives, but basically it is modeled as masses connected by springs and dashpots.

Fig.5 shows examples of MPM simulations, with three different elasticity($E$) parameters. The interval between each snapshot is 0.1sec in this figure. This simulates the second stage deployment of the clover membrane, adopting a simple rectangular membrane model, by omitting the 8 sector parts from the clover sail.

Fig.6 shows the time history of the deployed area rate, kinetic energy, spin rate of the edge of the membrane, projected distance from the center to the tip of the membrane, respectively. From Fig.5 and 6, it can be said that the dynamics does not depend
much on $E$, as is indicated in Eq.(12).

Fig.7 shows the spin rate and membrane size dependency on the dynamics. In these graphs, the vertical axes represent the time that the deployed radius reaches 50%, which is indicated by $T_{0.5}$. The continuous lines on the graph represent the fitting curves for 50% deployment using Eq.(12). The numerical results well agree with these curves. In the low spin rate or the small membrane size regions, the numerical results deviate from theoretical curves, because in these regions, the effect of the tip mass is not negligible, and the dynamics is governed more by Eq.(13).

Fig.7 also plots the S-310-34 flight result. The 50% second stage deployment in the actual S-310-34 flight result took less than 1 sec, as is observed from Fig.4(2nd stage deployment started at X+128s) and the other telemetry data. This flight result is plotted on Fig.7 with error bars, and shows good agreement with the theoretical and numerical results.

4 Damping Ratio Estimation

The damping of the sail membrane is one of the most interesting issues that we want to know by zero-gravity experiments, as it is quite difficult to determine in the Earth’s gravity and atmosphere environment.

Fig.8 is the MPM simulation result of rectangular membrane deployment simulations using three different damping ratios. Though it is oscillative for low damping ratio, the simulations with two larger damping ratios ($\gamma > 0.1$) show little difference. In these cases, the damping is fast enough compared with the large deformation dynamics of the membrane, indicating that the selection of the appropriate damping parameter for MPM simulation is not so severe, as long as a membrane has above a certain damping ratio.

Fig.9 is the inplane rotating angle of three representative points after the second stage deployment, observed by the on-board camera. In the S-310-34 experiments, a one-way clutch mechanism is equipped, so that the angular momentum of the rocket should transfer to the sail, but once the spin rate of the sail exceeds the spin rate of the rocket body, the angular momentum of the sail should not transfer back to the rocket. This mechanism avoids the sail from re-winding around the rocket body even when the over-spinning occurs.

Because of this one-way clutch mechanism, the inplane motion in Fig.9 shows half a period membrane dynamics, and half a period one-way clutch dynamics, periodically. The damping ratio can be estimated by extracting two neighboring magnitudes of the membrane dynamics. Using $\theta_1$ and $\theta_2$ in Fig.9, the damping ratio can be solved from the following
The result is compared with the S-310-34 stage deployment sequence is simulated using MPM.

The damping ratio of 0.1 is valid for this polyimide sail, but it is not so severe value, as its sensitivity on the deployment dynamics is small for around this damping ratio.

5 Stress Distribution

The stress and wrinkle affected on the sail are important factors, for it directly affects the static shape of the sail, and the performance of the solar-sailing.

MPM also provides an easy way to analyze the stress and wrinkle distributions. From the formulation of the MPM, the stress acted on one triangle element is assumed uniform as follows;

\[
\sigma = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = DBQ \delta
\]

where \( \sigma \) is the stress vector defined in the local coordinate system.

<table>
<thead>
<tr>
<th>point</th>
<th>( \theta_1 ) [deg]</th>
<th>( \theta_2 ) [deg]</th>
<th>( \theta_1/\theta_2 )</th>
<th>( \gamma )</th>
<th>( \omega_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>320</td>
<td>104</td>
<td>0.325</td>
<td>0.118</td>
<td>0.162</td>
</tr>
<tr>
<td>B</td>
<td>323</td>
<td>109</td>
<td>0.337</td>
<td>0.115</td>
<td>0.162</td>
</tr>
<tr>
<td>C</td>
<td>325</td>
<td>107</td>
<td>0.329</td>
<td>0.117</td>
<td>0.162</td>
</tr>
</tbody>
</table>

Table 1: Damping Ratio of S-310-34 Sail
Figure 10: Comparison of S-310-34 Result and MPM Simulation

Figure 11: MPM Local Element Coordinates (Left) and Integrated Mesh for Clover-Sail (Right)

Figure 12: Wrinkling in S-310-34 and MPM Wrinkle Simulation

cal element coordinates (Fig.11), $\mathbf{F} = [f_1 \ f_2 \ f_3]^T$ is the forces acted along the sides of the element. $\mathbf{D}, \mathbf{B}, \mathbf{Q}$ are the stress-strain conversion matrix, strain-displacement conversion matrix, coordinate transformation matrix, respectively and defined as follows:

$$
\mathbf{D} = \frac{E}{1 - \nu^2} \begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1 - \nu}{2}
\end{bmatrix}
$$ (17)

$$
\mathbf{B} = \begin{bmatrix}
\frac{1}{x_A} & 0 & 0 \\
0 & \frac{1}{y_B} & 0 \\
-x_A & \frac{1}{y_B} & 0
\end{bmatrix}
$$ (18)

$$
\mathbf{Q} = \begin{bmatrix}
\frac{x_B - x_A}{l_1} & 0 & \frac{y_B}{l_1} \\
0 & \frac{x_B - x_A}{l_2} & \frac{y_B}{l_2} \\
1 & 0 & 0
\end{bmatrix}^{-1}
$$ (19)

From (16), we can calculate the principle stress $\sigma_{\text{max}}, \sigma_{\text{min}}$.

In the MPM simulation, wrinkle is to occur when $\sigma_{\text{min}} < 0$. In that situation, it is treated that the element should not sustain the compressive force.

The right figure of Fig.12 shows the calculated stress and wrinkle distribution by MPM. The wrinkle is expressed as black lines. The wrinkle distribution well corresponds with the S-310-34 results, as is seen from Fig.12, which in turn, implies that the stress distribution is also well simulated in MPM.

6 Conclusion

JAXA has launched the S-310 sounding rocket on August, 9, 2004, to deploy $\phi$10m membranes for
solar-sail. The sail was made of the aluminized polyimide, whose thickness is 7.5µm. The clover-sail was successfully deployed and fully extended, and the results were retrieved completely.

The camera and the other telemetry data were compared with the numerical simulation model called “multi-particle model” (MPM), and its validity was confirmed by combining a theoretical analysis, S-310-34 result, and MPM simulations.

Additionally the damping ratio, which had been difficult to evaluate in the ground-based experiments, was tuned by this S-310-34 experiments, and the wrinkling was confirmed to be well simulated.

Using these tuned model and parameters, the multi-particle model becomes possible to extrapoleatively deal with the dynamics of a φ50m-class actual solar-sail membrane.

Though S-310-34 deployed the sail from the free-spinning rocket, the actual solar-sail spacecraft planned in ISAS will have the mechanism to deploy the sail slowly and gradually, in the quasi-static condition, which is the next step of our solar-sail R&D.

References