Robust Lunar Orbit Injection by the usage of Moon Synchronous Orbit

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Abstract

Discussed in this paper is a lunar approach strategy tolerant of lunar orbit injection (LOI) failure. LOI is one of the most critical events for a lunar orbiting mission. If the injection is not performed, the spacecraft flies by the moon, and in the worst case, it escapes not only from the moon but also from the earth, which leads to the mission failure. The proposed strategy is to design the trajectory so as to provide the opportunity and condition for the mission recovery even in the case of unexpected fly-by. That is, to design the trajectory so as to re-encounter the moon even in the case that the LOI is not performed. The trajectory design procedure is introduced and an example of designed trajectory is shown.

1. Introduction

With the opening of this new century, the moon attracts the attention again as the target of the space exploration. SMART-1 launched by ESA, which is now still orbiting the moon, is the lead-off visitor to the moon of this century. Japan is developing two lunar explorers, LUNAR-A and SELENE, which are planned to be launched within a few years. China and India are planning their first mission to the moon, and the United States refocuses on the human exploration to the moon.

Lunar orbit injection (LOI) is one of the most critical events for a lunar orbiting mission. If the injection is not performed, the spacecraft flies by the moon, and in the worst case, it escapes not only from the moon but also from the earth. Fig. 1 and Fig. 2 show an example of lunar transfer sequence, drawn in earth Centered Inertial Coordinate System. $Z$ -axis is in the direction of the orbit plane vector of the moon, $X$ -axis is in the direction of the perigee of the moon, and $Y$ -axis is perpendicular to $X$ and $Z$ -axes. Three axes are fixed at the beginning of the sequence, and not updated during the sequence. As is shown in Fig. 2, if the injection is not performed, the spacecraft on the hyperbolic orbit flies by the moon and escape from the moon. Moreover, as is shown in Fig. 1, as a result of the lunar swing-by, the spacecraft is accelerated and escape from the earth.

Fig. 1 Example of lunar transfer sequence

Fig. 2 Example of lunar transfer sequence (Close up of lunar approach)
One method to avoid this kind of risk is to use the gravity capture by the moon. This method is firstly demonstrated by Japanese explorer HITEN and is planned to be used by LUNAR-A. In this method, the perturbation by the earth’s gravity is effectively utilized to decelerate the explorer on the lunar approaching trajectory. Finally, the explorer is captured by the moon and orbit around the moon naturally without any active usage of the propulsion system, which result in the complete avoidance of the above mentioned risk. However, this method is inconvenience in that, it requires long process and duration to prepare the lunar approaching condition to be captured.

Another method to avoid the risk is to design the lunar approaching trajectory considering the case that the injection is not performed. That is to say, to design the trajectory so as to provide the opportunity and condition for the mission recovery even in the case of unexpected fly-by and escape. An example of this method is a free return trajectory used in Apollo program. By the usage of this trajectory, the astronauts could have been able to come back safely to the earth even in the case of the LOI failure. However, in this case of manned mission, to return to the earth was the mission recovery.

Discussed in this paper is a lunar approach strategy based on the latter risk avoidance method. That is to say, the trajectory is designed to re-encounter the moon even in the case that the LOI is not performed. In other words, the trajectory is designed so that the explorer is injected into the moon synchronous orbit (MSO) after the fly-by in case of the injection failure.

In case that the injection failure is due to a transient anomaly (such as operation failure, configuration set up delay, etc.), re-encounter the moon provides another opportunity of LOI, which leads to the almost perfect mission recovery.

In case that the injection failure is due to a permanent anomaly (such as main engine trouble, etc.), drastic restructure of the trajectory may be required for the mission recovery. Even in this case, re-encounter the moon provides the most effective means of orbit maneuver, that is, lunar swing-by.

From this point of view, proposed in this paper is a lunar approach strategy tolerant of LOI failure (Robust LOI). The restriction and the expense to adopt this strategy are also discussed.

2. Robust LOI at the First Encounter

2.1 Injection into Moon Synchronous Orbit

The moon synchronous orbit (MSO) is a geocentric orbit whose orbital period $P$ can be expressed as

$$P = \frac{m}{n} P_m$$  \hspace{1cm} (1)

where $P_m$ is the orbital period of the moon, $m$ and $n$ are integers, and they are assumed to be irreducible. If a spacecraft is injected into an MSO at the position of the moon, it will re-encounter the moon after $n$ orbital periods (the gravity of the moon and the other perturbations are neglected here.) Even if the LOI is not performed normally, if the spacecraft is injected into an MSO after the fly-by, it will re-encounter the moon and be able to retry the LOI.

To investigate the condition to inject into an MSO, the Local Coordinate System is defined as Fig. 3. The origin of the frame is the moon, $x$ axis is in the direction of the earth, $y$ axis is perpendicular to $x$ axis and the north of the moon, and $z$ axis is perpendicular to $x$, $y$ axes. For simplification, $y$ axis is assumed to be exactly opposite to the velocity of the moon, and $z$ axis is assumed to be identical with the north pole direction of the moon.

Fig. 3 Definition of local coordinate system

Fig. 4 gives a simplified model of the lunar swing-by, projected on the $xy$ plane of the Local Coordinate System. The velocity of the moon ($v_m$) is in minus $y$ direction. The velocity of the spacecraft before the swing-by ($v_{pre}$) is also in minus $y$ direction, assuming that the spacecraft is at the apogee of the long elliptical transfer orbit. The relative velocity of the spacecraft against the moon before the swing-by ($v_{rel}$) is deflected by the swing-by to be $v_{post}$ after the swing-by. The inertial velocity after swing-by ($v_{in}$) is obtained by adding $v_m$ and $v_{post}$. The maximum deflection angle ($\theta_{max}$) is determined from the gravity constant of the moon, $v_{in}$, and minimum perilune radius. And it limits the range of $v_{out}$ and $v_{post}$ attainable by a single swing-by.

Fig. 4 Simplified model of the lunar swing-by
On the contrary, if a target $v_{\text{post}}$ is given, $v_{\text{out}}$ is obtained by subtracting $v_m$ from $v_{\text{post}}$. Assuming a free swing-by, the length of $v_{\text{out}}$ is constrained to be the same with that of $v_{\text{ein}}$ (that is, the top of $v_{\text{post}}$ is constrained to be on the circle, whose center is the top of $v_{\text{ein}}$, and whose radius is the length of $v_{\text{ein}}$). The relation between $v_{\text{out}}$ and $v_{\text{ein}}$ gives the swing-by condition to attain the target $v_{\text{post}}$. For example, the velocities to inject the spacecraft into MSOs are shown in Table 1. Here, the radius of the orbit of the moon ($R_m$) is assumed to be constant here ($\approx 3.844 \times 10^8$ m).

Drawn in Fig. 5 are the contours of $v_{\text{post}}$ to inject the spacecraft into the MSO, overwritten on the swing-by diagram of Fig. 4. $v_{\text{out}}$ to attain the MSO are obtained as the intersections of the contours and the circle of attainable $v_{\text{out}}$.

### Table 1 Velocities to attain MSO

<table>
<thead>
<tr>
<th>Period of MSO</th>
<th>Velocity at $R_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times P_m$</td>
<td>1018.3 m/s</td>
</tr>
<tr>
<td>$(1/2) \times P_m$</td>
<td>654.1 m/s</td>
</tr>
<tr>
<td>$2 \times P_m$</td>
<td>1191.9 m/s</td>
</tr>
<tr>
<td>$(2/3) \times P_m$</td>
<td>845.6 m/s</td>
</tr>
<tr>
<td>$(2/5) \times P_m$</td>
<td>404.7 m/s</td>
</tr>
</tbody>
</table>

![Fig. 5 $v_{\text{out}}$ to attain MSO](image)

#### 2.2 Mission Requirement and MSO Injection

To discuss the approach condition to attain the MSO, Ballistic Plane Coordinate System is introduced. $y_B$ axis is in the direction of $v_{\text{ein}}$, $x_B$ axis is perpendicular to $y_B$ axis and the north of the moon, and $z_B$ axis is perpendicular to $x_B$, $y_B$ axes. Under the simplified assumption introduced in the discussion above, 3 axes are identical with those of Local Coordinate System. An approaching condition is specified as a point (Target Point) on the B-plane ($x_B, z_B$ plane of the B-plane Coordinate System.) The Target Point is the intersection of B-plane and asymptote of approaching hyperbola.

Approaching conditions to attain the MSO after the swing-by (MSO condition) forms a circle (with its center at origin) under this simple assumption. In Fig. 5, they are drawn in thin lines, and labeled with their orbital periods.

Main task of the LOI is to inject the spacecraft into the mission orbit. In the following discussions, as a typical case, it is assumed that the mission requirement assigns the radius and the inclination at the perilune passage ($r_p$ and $i$.) The approaching condition to attain the mission requirement can be also discussed on the B-plane. Assuming the target radius as 1838 m (that is, altitude of 100 km), and the target inclination as 90 deg., approaching conditions to attain the individual target form thick circle and line respectively in Fig. 6. The red circles are the Target Point to attain the mission requirement. Clearly, they are not on the circles of the MSO condition, that is, the mission requirement and the MSO condition are not satisfied simultaneously. Additionally, it shows that, if we are to satisfy the MSO condition, changing target $i$ is meaningless due to the symmetry in the figure, and change (enlargement) of $r_p$ is necessary, which leads to the increase in LOI $\Delta v$.

![Fig. 6 Mission requirement and MSO condition (in simplified model)](image)

It is true that the perfect symmetry shown in Fig. 6 is caused by the adoption of the simplified assumption. However, the situation does not change drastically even in the practical settings. Fig. 7 is drawn based on the approach condition of the sequence shown in Fig. 1 and 2. The relation between $v_{\text{in}}$ and $v_{\text{ein}}$ (or $y_B$ axis) is essential, and in this case, $v_{\text{m}}$ is (227.3 m/s, -1038.7 m/s, 6.0 m/s) on B-plane Coordinate System (note that, in simplified model, $v_{\text{m}}$ is exactly in the minus $y_B$ direction.) The circles of the MSO condition slightly warp and shift, however, the composition does not change so much.

This is the general characteristics as far as adopting near optimal lunar transfer. In near optimal lunar transfer, the transfer orbit is highly elliptical orbit whose apogee radius is approximately $R_m$. The velocity of the spacecraft around the apogee, which is expressed as $v_{\text{pec}}$ in Fig. 4, is much smaller than $v_{\text{m}}$, and $v_{\text{ein}}$ is almost in the opposite direction of $v_{\text{m}}$. That is, the spacecraft always approach from the front of the moon.
3. Robust LOI at the Second Encounter

3.1 Basic Concept

As is discussed in the previous section, the MSO condition on the B-plain changes if the approaching direction changes. To change the direction of the relative velocity of the spacecraft against the moon, the efficient way is to use a swing-by. Of course, after the direction of the relative velocity is changed by the swing-by, the spacecraft must re-encounter the moon to perform actual LOI. That is, the spacecraft must be injected into MSO after the first swing-by.

To summarize the concept, at the first encounter with the moon, the spacecraft is “planned” to fly-by the moon. By the swing-by, the spacecraft is injected into MSO, at the same time, the approaching direction at the second encounter is set so that the mission requirement and the MSO condition are satisfied simultaneously. At the second encounter, if the LOI is performed normally, the spacecraft is injected into the mission orbit. And even if the LOI is not performed normally, the spacecraft is injected into an MSO after the fly-by, and it will re-encounter the moon and be able to retry the LOI.

3.2 Conditioning of the Second Encounter

At the second encounter, the mission requirement and the MSO condition must be satisfied simultaneously. The LOI which satisfied this condition is called “Robust LOI” here after. Investigated in this section is the approaching direction to attain the Robust LOI condition.

Here we use the simplified model introduced in Section 2.1. Additionally, the periods of MSO before and after the second encounter are assumed to be the same, and \(v_{e, in}\) at the second encounter and \(v_{e, out}\) of the first encounter are assumed to be the same. The latter assumption means that the orbit of the moon and MSO are both non-perturbed elliptical orbit.

Firstly discussed is the orbit plane (of the approaching hyperbola) which satisfies the MSO condition and the mission requirement as to \(r_p\) (Fig. 8.) From MSO condition, the angle \(\alpha_{\text{sync}}\) between \(v\) and \(v_{e, out}\) (and between \(v\) and \(v_{e, in}\)) is determined. Additionally, the mission requirement as to \(r_p\) determines the deflection angle \(\theta_p\) between \(v_{e, in}\) and \(v_{e, out}\). From this two angles, \(\alpha_{\text{sync}}\) and \(\theta_p\), the angle \(\gamma\) between \(v\) and orbit plane is obtained using spherical geometry,

\[
\cos \gamma = \frac{\cos \alpha_{\text{sync}}}{\cos(\theta_p/2)}
\]

If the orbit plane vector \(n_{op}\) is defined in the direction of the orbit angular momentum, there are two possible \(n_{op}\) for an given orbit plane. And the angle between \(v\) and \(n_{op}\) is \(\gamma + \pi/2\) or \(\gamma - \pi/2\).

In summary, the MSO condition on the B-plain does change if the approaching direction changes. However, in near optimal lunar transfer, the approaching direction cannot be changed so much, and the situation does not change so much.
Obtained by the procedure above is the approaching direction $V_{\text{in}}$, which satisfies the Robust LOI condition at the second encounter. And from the assumption, it is the same with $V_{\text{out}}$ of the first encounter. The approaching condition at the first encounter is determined to attain this $V_{\text{out}}$ after the swing-by.

Fig. 10 shows an example of the targeting at the first encounter. The approach condition at the first encounter is the same with that of Fig. 7. The periods of MSO before and after the second encounter are assumed to be $P_{\text{in}}$. And no perturbation are considered for the orbit of the moon and MSO. Four targeting points are placed corresponds to the four possible solutions shown in Fig.9. Of course, they are on the circle of $(1 \times P_{\text{in}})$ MSO condition.

Fig. 11 Mission requirement and MSO condition (at the second encounter)

3.3 Example Sequence

Using the result of the previous section (Target #4 in Fig. 10) as an initial estimate, a consistent sequence is constructed under multi-body model. The perilune passage condition at the second and the third encounter are adjusted by several maneuvers. Note that the optimization process is not well completed, and large maneuvers still remain.

Fig. 12 shows the trajectory of the sequence in the Earth Centered Inertial Coordinate System. $Z$ axis is in the direction of the orbit plane vector of the moon, $X$ axis is in the direction of the perigee of the moon, and $Y$ axis is perpendicular to $X$, $Z$ axis. Three axes are fixed at the beginning of the sequence, and not updated during the sequence (the orbit of the moon is perturbed during the sequence, and it is observed in Fig. 12 (a).) Fig. 12 (a) shows the projection on the $XY$ plane, that is the orbit plane of the moon, and Fig. 12 (b) shows the projection on the $XZ$ plane.

Firstly, the spacecraft transfer to the moon with almost the same trajectory with that of the Fig. 1. By the swing-by at the first encounter the moon, the spacecraft is injected into MSO. The MSO is that carefully chosen to satisfy Robust LOI condition at the second encounter. About one month after, the spacecraft re-encounter the moon. The close up of the trajectory around the second encounter is shown in Fig. 13. It is the trajectory projected on the $YZ$ plane. At the perilune passage, normally, LOI is performed and the spacecraft is injected into the mission orbit (polar circular orbit with altitude 100km.) In case that the LOI is not performed for some reasons, the spacecraft fly-by the moon. However, even in this case, the spacecraft is injected into MSO again, and about one month after, the spacecraft re-encounter the moon. The close up of the trajectory around the third encounter is shown in Fig. 14. It is the trajectory projected on the $YZ$ plane.
At the perilune passage, LOI is re-performed and the spacecraft is injected into the mission orbit.

![Diagram](image)

Fig. 12 Sequence of Robust LOI

(a) Projection on \(XY\) plane

(b) Projection on \(XZ\) plane

Fig. 13 Sequence of Robust LOI

(Close up of the second encounter)

Fig. 14 Sequence of Robust LOI

(Close up of the third encounter)

5. Conclusion

Discussed in this paper is a lunar approach strategy tolerant of LOI failure. The proposed strategy is to design the trajectory so as to provide the opportunity and condition for the mission recovery even in the case of unexpected fly-by. That is, to design the trajectory so as to re-encounter the moon even in the case that the LOI is not performed. The trajectory design procedure is introduced and an example of designed trajectory is shown.

References