A Study of Spacecraft Thermal Control using Thermoelectric Conversion Device
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Abstract
This paper discusses spacecraft thermal control using a thermoelectric conversion device, which is one of thermal control devices. The thermoelectric conversion device can change thermal conductivity and achieve thermal distribution inside spacecraft. In this paper, stability analysis of the thermal control is conducted, and results of numerical simulations and experiments are considered.

1. Introduction
For the purpose of cooling components in spacecraft, a heat exhauster, for example a heat pipe radiator, is conventionally used. In this paper, a thermoelectric conversion device is applied for thermal control. The device is expected to be able to control temperature of components actively and stably. When the thermoelectric conversion device is applied to a system, it becomes a bilinear system which is a nonholonomic system and difficult to control stably. The purposes of this research are shown in the following.
(1) We show that the thermoelectric conversion device can change temperature distribution.
(2) We show that the bilinear system can be stable by proper control.
(3) We show a transient response is improved by using the thermoelectric conversion device.

2. Thermal Control of Spacecraft
2.1 Principle of Thermoelectric Conversion Device
In this research, a peltier device is applied as one of the thermoelectric conversion devices. The peltier device is one of thermocouples which consist of a P-type semiconductor and a N-type semiconductor as shown in fig.2.1.

![Fig.2.1 Principle of Thermoelectric Conversion Device](image)

When a current of electricity is flown as shown in fig.2.1, the interface of the metal plate 1 absorbs heat, and the interface of the metal plate 2 exhausts heat. This phenomenon is Peltier effect. Thermoelectricity of both branches is indicated by $\alpha_{pn}$. Internal resistances of both branches are $r_p$ and $r_n$. Thermal conduction is $K$. Electrical resistivity is $\rho$. A coefficient of thermal conductivity is $\kappa$. A cross section of the branch is $S$. Length of the branch is $l$. An internal resistance and heat conduction of the system are shown in the following.

$$r_{pn} = r_p + r_n = \left(\frac{\rho_p}{S_p} + \frac{\rho_n}{S_n}\right)l$$  \hspace{1cm} (0.1)

$$K_{pn} = K_p + K_n = \kappa_p S_p + \kappa_n S_n$$  \hspace{1cm} (0.2)

Then, total absorption of heat is derived as follows.
\[ Q_{\text{total}} = \alpha \rho I_{m} - K_{m} (T_{H} - T_{C}) - \frac{1}{2} I_{m}^{2} r_{m} \]  
\[ \text{(0.3)} \]

### 2.2 Discretization of Equation of Heat Conduction

Discretization of one dimensional equation of heat conduction as shown in eq.(2.4) is considered as a model of this research.

\[ \frac{\partial}{\partial x} \left( -k \frac{\partial u(x,t)}{\partial x} \right) - Q + \rho c \frac{\partial u(x,t)}{\partial t} = 0 \]  
\[ \text{(0.4)} \]

Where, \( u(x,t) \) is temperature of position \( x \) at time \( t \). \( \rho \) is density. \( c \) is specific heat. \( Q \) is a heating value.

Here, temperature \( u(x,t) \) is rewritten as shown in the following.

\[ u(x,t) = N(x,\theta) \]  
\[ \text{(0.5)} \]

Where, \( N \) is a shape function of \( x \). \( T \) is temperature of an element, which is a function of time \( t \). A partial differential equation of \( u(x,t) \) with respect to \( x \) is derived as follows.

\[ \frac{\partial \hat{u}}{\partial x} - \frac{\partial N}{\partial x}, T(t) = B, T \]  
\[ \text{(0.6)} \]

When the Galerkin’s method is applied for one element and a weighted residual is zero, the following equation is derived.

\[ C^{(\varepsilon)} \hat{T} + K^{(\varepsilon)} T = p^{(\varepsilon)} \]  
\[ \text{(0.7)} \]

Where, element matrix \( C^{(\varepsilon)} \), \( K^{(\varepsilon)} \), \( p^{(\varepsilon)} \) are shown in the following.

\[ C^{(\varepsilon)} = \int_{\Omega^{e}} \rho c N^{T} N dD \]  
\[ \text{(0.8)} \]

\[ K^{(\varepsilon)} = \int_{\Omega^{e}} k B^{T} B dD \]  
\[ \text{(0.9)} \]

\[ p^{(\varepsilon)} = \int_{\Omega^{e}} Q N^{T} dD \]  
\[ \text{(0.10)} \]

Total matrix consisted of the element matrix is shown as follows.

\[ C T + K T = P \]  
\[ \text{(0.11)} \]

This equation means discretization of heat conduction of eq.(2.4).

### 2.3 Thermal Model

A thermal model of this research is shown in fig.2.2. when \( \theta_{1A} = \theta_{0}, \theta_{1B} = \theta_{1}, \theta_{2A} = \theta_{0}, \theta_{2B} = \theta_{2}, C_{0} \sqcup \infty, C_{1} = C_{2} = C, c_{1} = c_{2} \) are considered, the system is a bilinear system expressed as shown in the following.

\[ C x = A x + B u + P (u \otimes x) + q \]  
\[ \text{(0.12)} \]

Where, 

\[ C = \text{diagonal}(C_{0}, C_{1}, C_{2}) \]

\[ x^T = (\theta_{0}^T, \theta_{1}^T, \theta_{2}^T), u = (i_{i}) \]

\[ A = K \begin{pmatrix} -1 & 1 & 0 \\ 2 & -2 & 1 \\ 0 & 1 & -1 \end{pmatrix} \]

\[ B = \frac{1}{2} \begin{pmatrix} v_{1} & 0 \\ v_{1} & v_{2} \\ 0 & v_{2} \end{pmatrix} \]

\[ P = \alpha_{p} \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{pmatrix} \]

\[ q^T = (Q_{0} Q_{1} Q_{2}) \]

\[ \text{Fig.2.2 Thermal Model} \]
\[ C_0 \theta_0^* + C_1 \theta_1^* + C_2 \theta_2^* = \int (v_i i_1^* + v_2 i_2^*) dt + \text{const.} \]  \hspace{1cm} (0.16)

In the case that target temperature is constant, the first term is always positive.

\[ Q_0 < -Q_1 - Q_2 \]  \hspace{1cm} (0.17)

This equation means radiation.

In the case that a constant current is flown in the pertier device, eq.(2.12) is rewritten as shown in the following.

\[ C \delta x = A x + Bu + P \begin{pmatrix} i_1^* & 0 \\ 0 & i_2^* \end{pmatrix} x + q \]  \hspace{1cm} (0.18)

\[ = A x + Bu + q \]  \hspace{1cm} (0.19)

Therefore we achieve constant temperature which satisfies the following equation.

\[ \dot{A} x + Bu + q = 0 \]  \hspace{1cm} (0.20)

This equation shows that the thermoelectric conversion device can change temperature distribution. Therefore the purpose of this research (1) is achieved. When the second term of eq.(3.7), that is an additional heat generation, is ignored, the equation means heat radiation is purposely variable.

In the eq.(2.12) equilibrium temperature is expressed by \[ x^* \]. With eq.(3.1) and \[ x = x^* + \delta x \], we can derive the following equation when no constant current is flown in the pertier device.

\[ C \delta x = A \delta x + Bu + P \begin{pmatrix} x_1^* & 0 \\ 0 & x_2^* \end{pmatrix} u \]  \hspace{1cm} (0.21)

When a constant current is flown in the device, a same type equation as eq.(3.8) with appropriate an equilibrium condition is derived.

### 3.2.1 Lyapunov Stability Analysis

Here, the following control law is applied.

\[ u = G \delta x \]  \hspace{1cm} (0.22)

Then,

\[ \mu \delta x = \delta x^T \left( \left[ A^{1/2} \right]^T Q + Q \left[ A^{1/2} \right] \right) \delta x \]
\[ + P \delta x \left[ (p - \delta x)^T Q (p - \delta x) \right] \]  \hspace{1cm} (0.23)

Here, if we design a gain to assurance \[ A_{1/2} \].

There is positive definite matrix \[ A_{1/2} \].

In the eq.(2.12) equilibrium temperature is expressed by \[ x^* \]. With eq.(3.1) and \[ x = x^* + \delta x \], we can derive the following equation when no constant current is flown in the pertier device.

\[ C \delta x = A \delta x + Bu + P \begin{pmatrix} x_1^* & 0 \\ 0 & x_2^* \end{pmatrix} u \]  \hspace{1cm} (0.24)

When,

\[ V = \delta x^T Q \delta x \]
\[ Q > 0 \]  \hspace{1cm} (0.25)

The following condition expression is derived.

\[ \mu \delta x = -[\delta x^T (\sigma (Y) + \sigma (M) - \gamma (S) Q S) \sigma (G^T G) \delta x] (M \leq 0) \]
\[ \quad -[\delta x^T (\sigma (Y) - \sigma (M) - \gamma (S) Q S) \sigma (G^T G) \delta x] \]  \hspace{1cm} (0.26)

If the inside of grouping symbol of eq.(3.14) is positive, the system is asymptotically stable.

### 3.2.2 Popov’s Hyperstability Theorem

In this section, hyperstability of the model is discussed. The system is expressed as follows.

\[ C \delta x = A \delta x + Bu + P \begin{pmatrix} x_1^* & 0 \\ 0 & x_2^* \end{pmatrix} u \]  \hspace{1cm} (0.27)

A control input is \( y = 1 \delta x \). When \( i_1 = 4 i \), for all \( T > t_0 \), the following equation is satisfied.

\[ \int_{t_0}^{t} y^T w \delta t \geq -\gamma_0^2 \]  \hspace{1cm} (0.28)

where \( \gamma_0^2 \) = positive const.

This equation expresses that a compensator \( H \) satisfies Popov’s integral inequality. In this model, a transfer function is shown in the following.

\[ G(s) = 1(Cs - A)^{-1} \]  \hspace{1cm} (0.29)

\( A \) is symmetrical matrix, and its all eigen values are real number. For this thermal model, it is confirmed that all eigen values are zero or negative. Therefore we can
extend the problem to a heat transfer problem. The pole of the transfer function for open loop is derived from the following equation.
\[
det(Cs - A) = 0
\] (0.30)
The pole of the characteristic equation for open loop is not positive.
\[
x^T \{ G(j\omega) + G(-j\omega) \} x = \bar{y}^T (2A) \bar{y} \geq 0
\] (0.31)
\[
y \in \mathbb{R} \cup (Cj\omega + A)^{-1} x
\] (0.32)
Therefore, \( G(s) \) is positive real. From the Popov's hyperstability theorem, the bilinear model is asymptotically stable by a nonlinear compensator \( H \).
We show that the bilinear system can be stable by a proper control. (Purpose of this research (2))

### 3.3 Design of Control Gain

In section 3.2, the controller is indicated to be hyperstability. When a design for an unstable system is considered, an adjustment value \( \beta \) is introduced and the following equation is applied.
\[
\tilde{A} \rightarrow \tilde{A} + 1/2 \beta I
\] (0.33)

An optimal regulator problem is applied to decide a control gain.
\[
J = \int \left( \frac{1}{2} \delta x^T \bar{y} \delta x + \frac{1}{2} u^T Ru \right) dt
\] (0.34)

For minimizing the \( J \), the following Riccati equation is solved.
\[
A^T \bar{Q} + QA - QBR^T B^T \bar{Q} + Y = 0
\] (0.35)
And, \( G \) is expressed as
\[
G = -R^T B^T \bar{Q}
\] (0.36)
If we set \( R = 1/2B^T B \), electrical power is set in the evaluation function. Therefore the following equation is solved.
\[
J = \int \left( \frac{1}{2} \delta x^T \delta x + \frac{1}{2} u^T B^T Bu \right) dt
\] (0.37)

### 4. Numerical Simulation

#### 4.1 Equilibrium Temperature and Transient Response (No Control)

A temperature change and a transient response with a constant current are described in Fig.4.1, 4.2, and 4.3. From fig.4.1 and 4.2, it is cleared that the thermoelectric conversion device can change temperature distribution. Fig.4.3 shows that the bilinear system can be stable by proper control.
4.2 Equilibrium Temperature and Transient Response (With Control)

Target temperature is $\theta_1 = 290[\text{K}]$, $\theta_2 = 298[\text{K}]$. A current applied to the peltier device is controlled. A temperature change and a transient response are shown in fig.4.4. The current is shown in fig.4.5. The transient response is shown in fig.4.6 and 4.7. Fig.4.4 and 4.5 show that the thermoelectric conversion device can change temperature distribution. Fig.4.6 and 4.7 show that the transient response is improved by using the thermoelectric conversion device.

5. Experiment

In this experiment, heater 1 is set 1.3[V], 2.0[A] and heater 2 is set 1.9[V], 3.0[A]. These heaters are supposed to generate a constant heat value. A constant current and a constant voltage to each peltier device are changed, then temperature is measured. Comparing results, experiment 0 with two peltier devices is off is indicated in table 5.1. In Ex. 1-7, $i[A]$, $v[V]$ are applied to each peltier device. Temperature of a water tank and each peltier device are $\theta_0$, $\theta_1$, $\theta_2$[K]. In Ex.1, 6, 7, $\theta_2$ is lower than Ex.0. In Ex.2-5, $\theta_1$, $\theta_2$ are lower than Ex.0. These results shows peltier devices cool components. Therefore, we can actually confirm that the
thermoelectric conversion device can change temperature distribution.

Fig. 5.1 Experiment Model

Fig. 5.2 Experimental Setup

Table 5.1 Equilibrium Temperature for Each Experiment

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6. Conclusion
Firstly, we show that system using a thermoelectric conversion device becomes a bilinear system which is a nonholonomic system and difficult to control stably. Secondly, we can show the following items.
(1) We show that the thermoelectric conversion device can change temperature distribution.
(2) We show that the bilinear system can be stable by proper control.
(3) We show transient response is improved by using the thermoelectric conversion device.

References