Attitude Control Law of Spacecraft with Control Moment Gyros
for Rapid Multitarget Pointing Mission

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Abstract: Both of high attitude maneuverability and stability are required for a spacecraft to realize a multitarget pointing mission. Control moment gyros (CMGs) are effective to obtain the high attitude maneuverability because they can generate large torques. On the other hand, the attitude stability is degraded by inner torques caused by the CMG particular dynamics such as multibody dynamics coupling, gimbal rate control error, and so on. In this paper, an angular momentum based inner torque compensation method is proposed. In this method, a CMG angular momentum control loop is added to the general gimbal servo system. As results, inner torques are compensated efficiently with low computation cost.

1. Introduction

For remote sensing spacecrafts, high attitude maneuverability is beneficial to realize large coverage. And high pointing accuracy and stability are essential to increase the image quality. For example, Centre National d’Etudes Spatiales (CNES) is developing the PLEIADES which is an earth observation satellite. Its image resolution is better than 1.0 m and attitude maneuver rate is 60 deg par 25 s[1], [2]. For such a spacecraft, a control moment gyro (CMG) is useful to obtain sufficient attitude maneuverability. But to increase the pointing accuracy and stability, particular kinematics and dynamics such as kinematics singularity and inner torque coupling must be carefully considered when an attitude control system is developed.

The kinematics singularity is the phenomena that control torque vector is restricted within a two dimensional space and three axis control become impossible when CMG gimbal angles takes certain values. Many researchers have examined this problem and many singularity avoidance laws are proposed. Nakamura and Hanafusa developed the singularity robust inverse method[3]. It was originally developed for handling the singularities in robotic manipulators, but because of the similarity of the CMG kinematics to that of the robotic manipulator, the singularity robust inverse method is effective to the CMG. Wie et al. developed the generalized singular robust inverse method[4], [5]. In this method, weighting matrix is added to the original singularity robust inverse method, and singularity avoidance ability and torque transit are well adjusted. Joseph and Paradiso
developed the directed search method[6]. In this method, gimbal profiles are found based on the feedforward gimbal trajectory. And the susceptibility to unknown disturbances is also investigated. Hoelscher and Vadali proposed the optimal control method[7]. In this method, all of the optimal attitude maneuver path and gimbal profiles are determined to avoid the singularity during the attitude maneuvering.

The inner torque coupling is also a significant problem when CMGs are used for the precision attitude control. CMG itself or dynamics coupling with attitude motion generate many kinds of inner torques besides ideal gyro moment, and they disturb the attitude dynamics. Dzielski et al. proposed the feedback linearization method[8]. In this method, a CMG equipped spacecraft is formulated and its nonlinear dynamics is compensated by a nonlinear feedback controller. Heiberg et al. proposed the disturbance accommodation method[9]. In this method, inner disturbances caused by gimbal rate control error is investigated and its real-time estimation and compensation method is verified.

In our study, an angular momentum based inner torque compensation method is developed. In this method, inner torques are not compensated directly, but compensated indirectly by a CMG angular momentum control loop, which is added to the general gimbal servo system. As results, inner torques are compensated efficiently with low computation cost. Usefulness of this method is demonstrated through numerical simulations about a rapid multitarget pointing mission.

2. Angular Momentum Control Method

Let us denote the number of CMGs as $n$. Tracking to the reference attitude for the maneuvering and target pointing is equivalent to make the main body of the spacecraft to have corresponding angular momentum. So the ideal angular momentum of the main body $N \mathbf{h}_r^A \in R^{3 \times 1}$ is given as a function of reference attitude angular velocity $N \mathbf{\omega}_r^A \in R^{3 \times 1}$ and gimbal angles $\mathbf{\sigma} \in R^{n \times 1}$ as follows.

$$N \mathbf{h}_r^A = N \mathbf{h}_r^A(\mathbf{\sigma}, N \mathbf{\omega}_r^A)$$ (1)

And this angular momentum is transferred from the CMGs to the main body. Therefore, if total angular momentum of the spacecraft is equal to 0, Eq. (1) is equivalent to the following equation.

$$N \mathbf{h}_r^D = -N \mathbf{h}_r^A(\mathbf{\sigma}, N \mathbf{\omega}_r^A)$$ (2)

where $N \mathbf{h}_r^D$ is an ideal angular momentum of whole CMGs. If total angular momentum of the spacecraft is not equal to 0, total angular momentum is added to the right term in Eq. (2). But without loss of generality, Eq. (2) is considered hereafter. Equation (2) is used as an angular momentum command to a gimbal servo system (including a steering law), where gimbal angles are controlled to realize the required CMG angular momentum. Actual CMG angular momentum $C \mathbf{h}_D \in R^{3 \times 1}$ is calculated from observed gimbal angle as follows.

$$C \mathbf{h}_r^D = C \mathbf{h}_r^D(\mathbf{\sigma})$$ (3)

Angular momentum error is calculated from Eqs. (2) and (3), and it is compensated. For example, a torque command $\mathbf{\tau}_{ff} \in R^{3 \times 1}$ to the gimbal servo system is determined to be proportional to the angular momentum error as follows.

$$\mathbf{\tau}_{ff} = K_f (N \mathbf{h}_r^D - C \mathbf{h}_r^D)$$ (4)

where $K_f \in R^{3 \times 3}$ is a constant gain. Equation (4) is regarded as a feedforward torque command to compensate inner torques. And remaining error is compensated by a feedback controller. For example, feedback torque command $\mathbf{\tau}_{fb} \in R^{3 \times 1}$ is determined by a following PD regulator[10],

$$\mathbf{\tau}_{fb} = K_p (q_{err13} + K_v (N \mathbf{\omega}_r^A - N \mathbf{\omega}_r^A))$$ (5)

where $K_p \in R^{3 \times 3}$ is a proportional gain,
$K_v \in \mathbb{R}^{3 \times 3}$ is a differential gain, and $N q_{err}^A_{13} \in \mathbb{R}^{3 \times 1}$ is a vector part of quaternion defined as follows.

$$N q_{err}^A = N q_r^A \otimes q^N$$

Whole of the control system is shown in Fig.1. In this system, inner torques are not calculated directly, but ideal angular momentum of CMGs to realize ideal attitude maneuver is calculated. And a local feedback loop for the CMG angular momentum control is added to the original gimbal servo system. As results, tracking ability both to the torque command and the angular momentum command is realized. This is effective to reduce the attitude perturbation caused by the inner torques. And this requires much lower computation cost than the method where inner torques are directly calculated. Because angular momentum is related to the attitude angular velocity and the gimbal angle with simple kinematics equations, on the other hand inner torques are formulated with complex dynamics equations.

3. Numerical Simulation

Four CMGs case ($n = 4$) is considered and their arrangement is shown in Fig. 1, where gimbal axes $e_i$ ($i = 1, ..., 4$) are normal to pyramid shape surfaces, and $\beta = 53.73$ deg for all CMGs. Simulation parameters are summarized in Table 1. Attitude control performances for multitarget pointing mission are investigated, where parameter error about the main body inertia moment is considered. This parameter is used to calculate the angular momentum command, so parameter error results in the inner torque compensation error.

All of observation targets are supposed to be fixed on the earth surface. And they are moving relative to the inertia reference frame because of the earth rotational motion. And spacecraft translational motion around the earth is supposed to the sun synchronous orbit and altitude is about 600 km. Pointing direction vectors from the spacecraft position to multitargets are depicted in Fig. 3, where 11 targets are distributed from Kagoshima to Hokkaido and time interval between each target acquisition is 30 s. Reference attitudes of the spacecraft at the target pointing moments are determined to make the yaw axis be parallel to the pointing direction, and pitch axis is parallel to the equatorial plane of the earth. And reference angular velocities at the target pointing moments are determined to make the yaw axis completely track the targets with considering the earth rotational motion and the spacecraft translational motion; that is, relative velocity of the line-of-sight to the targets is controlled to be 0 at the target pointing moments. Through these constraints, reference attitude and reference angular velocity relative to the inertia coordinate are derived[11]. Calculation results are shown in Figs. 4-6, where marked points represents target pointing moments. Attitude quaternion and angular velocity are connected with smooth curves, and angular
acceleration curves are also smooth and equal to 0 at the target pointing moments.

Now let us denote main body inertia matrix used for angular momentum control as $\hat{I}^A$. Attitude control performances when $\hat{I}^A$ includes 2% error are investigated. These results are shown in Figs. 7-13. Figure 7 shows attitude control torque command and Fig. 8 shows CMG angular momentum command. Figure 9 shows attitude control error which is the difference between the reference attitude and the actual value. And Fig. 10 shows attitude angular velocity control error which is the difference between the reference angular velocity and the actual value. And Fig. 11 shows angular momentum control error which is the difference between the angular momentum command and the actual value. In particular at the target pointing moments in Figs. 9-11, control errors are minimized. Figure 12 shows gimbal angle and Fig. 13 shows gimbal rate.

### Table 1 Simulation Parameters

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<thead>
<tr>
<th>Main Body Inertia Matrix $I^A$</th>
<th>kgm$^2$</th>
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<tbody>
<tr>
<td>$\begin{bmatrix} 1500 &amp; 80 &amp; -50 \ 80 &amp; 1200 &amp; 30 \ -50 &amp; 30 &amp; 1000 \end{bmatrix}$</td>
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<table>
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<tr>
<th>CMG Inertia Matrix $I^D_i$</th>
<th>kgm$^2$</th>
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<tbody>
<tr>
<td>$\begin{bmatrix} 0.023 &amp; 0 &amp; 0 \ 0 &amp; 0.023 &amp; 0 \ 0 &amp; 0 &amp; 0.046 \end{bmatrix}$</td>
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<tr>
<th>CMG Angular Momentum $C_{ij}^D$</th>
<th>Nms</th>
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<tbody>
<tr>
<td>$\begin{bmatrix} 0 &amp; 0 &amp; 29 \end{bmatrix}$</td>
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Attitude control performances when $\hat{I}^A$ includes 1, 5, 10, or 20 % error or no error are also investigated. These results are summarized in Figs. 14 and 15. If the inertia matrix error is within few percents, the control performances are almost limited. So if the inertia matrix error is within few percents, the proposed method works well.
4. Conclusion

Angular momentum based inner torque compensation method is proposed. In this method, the CMG angular momentum is controlled by the additional control loop to the general gimbal servo controller. This is effective to compensate the inner torques with low computation cost.

References


