Study of Structure of a Rubble-Pile Asteroid using Numerical Simulation of Multi-Particle Model

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Abstract

The asteroid exploration spacecraft "HAYABUSA" took high resolution pictures of asteroid "ITOKAWA". ITOKAWA is a rubble-pile asteroid, whose shape and regolith size distribution are unique. These features are caused by composition of the Brazil-Nut effect and centrifugal force by spin. The Brazil-Nut effect is generally known as the phenomenon that causes larger particles to rise to the top of the shaken granular mixtures. In this study, the behaviors of various size particles in the space are analyzed by numerical simulation using Multi-Particle model, which is model of a rubble-pile asteroid. The results are compared with the surface of ITOKAWA, and the inferred internal structure of ITOKAWA is discussed.

多粒子数値解析を用いたラブルパイル小惑星の構造に対する考察

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摘要

小惑星探査機「はやぶさ」はイトカワの高解像度画像取得をはじめ，様々な観測を行った．イトカワは，がれきの積み重なった小惑星（ラブルパイル小惑星）であり，形状，地表面の土砂の大きさの分布の偏りなど，通常の惑星には見られない特徴が確認された．これらの特徴の主要因として，大きさの異なる粒子を混合した容器に振動を加えるとより体積の大きな粒子が浮かび上がってくる現象であるブラジルナッツ効果と，自転による遠心力の効果の複合的な影響が考えられる．そこで本研究では，多粒子系モデルを用いた数値解析により，宇宙空間においてラブルパイル小惑星を模擬した，異なる大きさの粒子を含む粒子群に振動を加え，どのような挙動を示すか解析し，実際のイトカワと比較し，考察する．

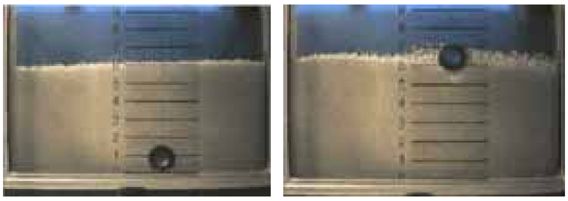
1. Introduction

The results of scientific observations and high resolution pictures of asteroid "ITOKAWA" are obtained by the asteroid exploration of "HAYABUSA". They show that ITOKAWA is a rubble-pile asteroid, whose shape and regolith size distribution are unique [1]. A rubble-pile asteroid consists of many broken pieces piled up by universal gravitation and it has no core. Its unique shape and regolith size distribution are caused by composition of the Brazil-Nut effect and centrifugal force by spin. The Brazil-Nut effect is known as the phenomenon causing the larger particles to be raised up to the granular mixture surface by shaking. Even if the larger particle has higher density than the smaller particles, it can rise up to the top. An asteroid vibrates by collision with the other falling asteroid. It is thought that the Brazil-Nuts effect takes place on a rubble-pile asteroid. In this study, the behaviors of various size particles with several vibrations on the ground or in the space are analyzed by numerical simulation. Multi-particle model is adopted for the numerical simulation. Through the numerical analysis of particles' behaviors on the ground, the deep knowledge about Brazil-Nuts effect is acquired. On the other hand, simulation of particles in the space corresponds to a rubble-pile asteroid. In the space, the particles are influenced by universal gravity, centrifugal force and impact force by collision. However, in this paper, the effect of only universal gravity and impact force are considered. The results are compared with the surface of "ITOKAWA", and the inferred internal structure and behavior of particles of "ITOKAWA" are discussed.

2. About Brazil-Nuts effect

The Brazil-Nuts effect is introduced. As shown in figure 1, Brazil-Nuts effect is the phenomenon which the larger particle is raised up to the top of the smaller particles with shaking. If the larger particle has higher density than the smaller ones, it can rise up to the top in proper conditions.

When particles are shaken, voids are created. As smaller particles have higher fluidity, they are more likely to fall in and fill the available space made by shaking. Then, the larger particle is kept on the smaller ones by arch effect. Additionally, it is thought that there is also a convective effect [3, 4]. Brazil-Nuts effect causes various results when the particles' dense ratio, amplitude of vibration or shape of particle is changed.



(a)Initial position (b)Position after shaking

Fig.1. Pictures of Brazil-Nut Effect on the ground [2].

3. Brazil-Nuts effect on the ground

3-1. Experiment

Brazil-Nut effect on the ground is confirmed by experiment. Two different size particles in the container (60x60x140[mm^3]) made from PET ware shaken. The particles are equal in density and one's diameter was twice another. Each diameter of them was 12.5[mm] and 25.0[mm]. The container was shaken vertically. The period and amplitude of shaking ware 0.33[s] and 50[mm]. The larger particles rise up to the surface of the small ones in about ten seconds. This phenomenon is Brazil-Nuts effect. In this experiment, the particles don't touch the superior wall.

3-2. Numerical simulation

In this section, Brazil-Nut effect is confirmed by simulation on the ground.

3-2-1. Simulation model

The simulation model is introduced. The simulation model was a three-dimensional Multi-Particle model and it was calculated by fourth order Runge-Kutta method. When the particle was in contact with the other particles or the walls of container, mass-spring-damper (MSD) was applied (see figure 2). Friction was considered in this model. The natural length of spring was equal to the sum of the two particles' radii which are in contact. Frictional force acted only for translational motion. Suppose the inertia moment of the particles was infinite, rotational motion of the particles was neglected here. The frictional force of static friction was considered. Normal reaction was repulsion force by spring. Gravity acted in the vertical direction.

3-2-2. The equation of motion

Define the vertical direction as z axis direction and horizontal direction as x, y axis direction. **x** in the following equations is the position of the target particle of calculation. The equation of motion for this model is given by,

not in contact,

$$m\frac{d^2\mathbf{x}}{dt^2} = -\begin{pmatrix}0\\0\\g\end{pmatrix} \qquad (1)$$

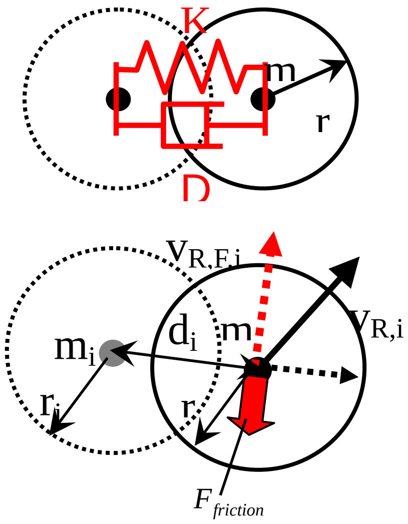


Fig.2. Schematic diagram of mass-spring-damper (MSD) model considering friction.

in contact,

$$m\frac{d^2\mathbf{x}}{dt^2} = -\begin{pmatrix}0\\0\\g\end{pmatrix} - \sum K(r + r_i - d_i)\frac{\mathbf{x}-\mathbf{x}_i}{d_i}$$
$$-\sum D\frac{d(\mathbf{x}-\mathbf{x}_i)}{dt}$$
$$-\sum \min\left[\left(1+\frac{0.001}{|\mathbf{v}_{R,F,i}|}\right)\mu, 1.0\right] K(r + r_i - d_i)\frac{\mathbf{v}_{R,F,i}}{|\mathbf{v}_{R,F,i}|} \qquad (2)$$

where

$$\mathbf{d}_i = \mathbf{x} - \mathbf{x}_i,\ d_i = |\mathbf{d}_i| \qquad (3)$$

$$\mathbf{v}_{R,i} = \mathbf{v} - \mathbf{v}_i,\ v_{R,i} = |\mathbf{v}_{R,i}| \qquad (4)$$

$$\mathbf{v}_{R,F,i} = \mathbf{v}_{R,i} - \frac{\langle \mathbf{d}_i, \mathbf{v}_{R,i}\rangle}{d_i}\frac{\mathbf{d}_i}{d_i},\ v_{R,F,i} = |\mathbf{v}_{R,F,i}| \qquad (5)$$

m is the mass of the target particle, K is spring constant, D is damping coefficient and μ is friction coefficient. The second term or later of the equation of motion (2) is added several times, which the number of the times is equal to the number of the particles in contact. Suffix i indicates each particle which is in touch.

3-2-3. Computational conditions

The conditions of simulation are indicated. It was supposed that two different size particles ware put in the container on the ground. The particles' density was 2.4[g/cm3] and the smaller particle's diameter was fixed (r_{small}=12.5[mm]). The container was shaken vertically and sinusoidally. The size of the container and number of the particles are shown in table 1. The equation of wall oscillation is given as follows.

$$\mathbf{x}_{wall} = \mathbf{A}\ \sin\left(\frac{2\pi}{\mathbf{T}}t\right) \qquad (6)$$

$$\mathbf{A} = \begin{pmatrix} 2 \\ 2 \\ A \end{pmatrix} [mm], \mathbf{T} = \begin{pmatrix} 0.1 \\ 0.2 \\ T \end{pmatrix} [s] \tag{7}$$

where **A** is amplitude of shaking, **T** is period of shaking. Time step, Δt, in this calculation was 0.0001[s].

Table.1. Computational conditions

Container size[mm]	Number of particles	
	12.5mm	25mm
75x75x∞	216	2

3-2-4. Results

The result of simulation is shown. The parameters which can be changed are amplitude of shaking, A, period of shaking, T, and ratio of the diameter of larger particle to that of the smaller particle, $R=r_{large}/r_{small}$. The following cases ware analyzed.

- Amplitude of shaking, A, is changed
- Period of shaking, T, is changed
- Ratio of the diameter of larger particle to that of the smaller particle, R, is changed

First, the cases which the amplitude of shaking is changed are discussed. Figure 3 shows the result under the condition of A=50[mm], T=0.33[s] and the ratio of each size particle's diameter, $R=r_{large}/r_{small}=2.0$. T and R are fixed here. Figure 4 shows the result under the condition of A=35[mm]. Figure 5 shows the result under the condition of A=25[mm]. Figure 6 shows images of the initial position and the position after shaking (A=50[mm], T=0.33[s] and $R=r_{large}/r_{small}=2.0$).

The larger particles rise from the initial position which is the bottom of container to the twice as high as the average height of the smaller ones as shown in figures 3 and 4. Figure 3 shows the result with the same condition as the above experiment (vertically shaking, A_1=50[mm], T_1=0.33[s] and R=2.0). The time which it takes for the larger particles to rise up to the top is 5～10 seconds and it corresponds to that of the experiment. This shows that the larger particle rises up to the top of the smaller ones as well as the experiment. It is confirmed that this model is valid for simulation of Brazil-Nuts effect.

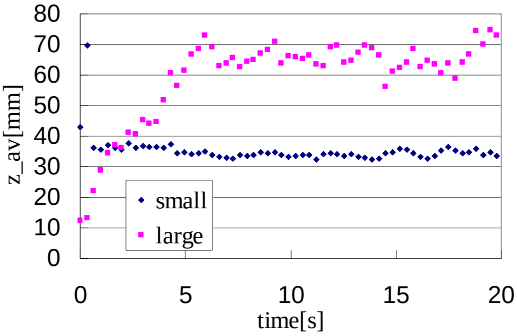


Fig.3. The average height of each size particles, z_{av}, as a function of time (A=50[mm], T=0.33[s] and R=2.0). Maximum wall acceleration, $A(2\pi/T)^2$=18126[mm/s^2].

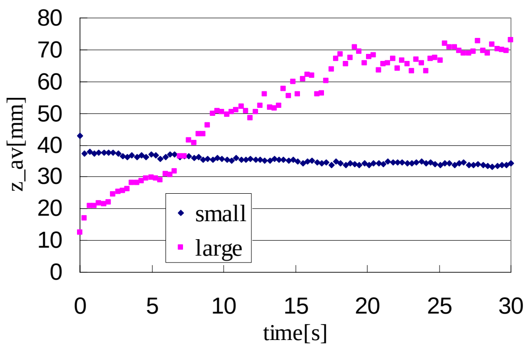


Fig.4. The average height of each size particles, z_{av}, as a function of time (A=35[mm], T=0.33[s] and R=2.0). $A(2\pi/T)^2$=12688[mm/s^2].

The z_{av} upward trend in figure 4 is more discontinuous than that in figure 3. The curve is upward in some period of time (for example, time=6～9[s]) and it is flat in other period of time (time=9～12[s]). Figure 5 shows that the larger particle doesn't rise up. These differences of continuity of upward trend depend on shaking amplitude of each case.

For Brazil-Nuts effect, making the relative displacement between a larger particle and smaller ones by shaking and keeping up the position by arch effect are needed [3]. If the relative displacement by shaking doesn't exceed the threshold of the relative displacement which is required for arch effect, the larger particle is not held up the position when it rises up instantaneously. Therefore, the results with the smaller amplitude condition (Figure 4, 5) indicate the more discontinuous up trend or no up trend.

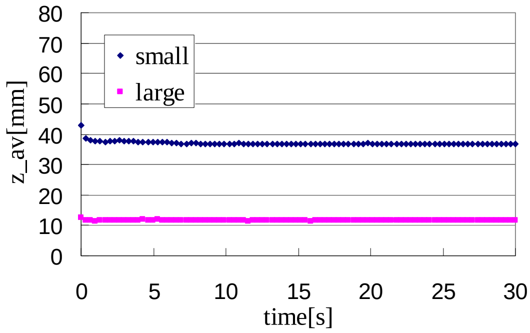


Fig.5. The average height of each size particles, z_{av}, as a function of time (A=25[mm], T=0.33[s] and R=2.0). $A(2\pi/T)^2$=9062[mm/s^2].

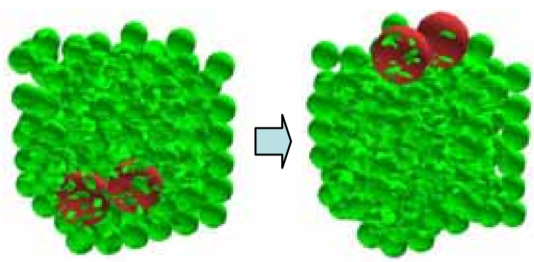


Fig.6. Images of the initial position and the position after shaking (A=50[mm], T=0.33[s] and R=2.0), which show the Brazil nuts effect.

Secondly, the cases which the period of shaking is changed are discussed. Figure 7 shows the result under the condition of A=35[mm], T=0.40[s] and R=2.0. From the comparisons between figures 4 and 5, and figures 4 and 7, it is found that when maximum acceleration of wall, $A\ (2\pi/T)^2$, is exceed the gravitational acceleration (g=9800[mm/s^2]), Brazil-Nuts effect takes place. This means $A\ (2\pi/T)^2>g$ is one of the necessary conditions for Brazil-Nuts effect. It is because if the wall moves more slowly than the free-falling particle, no void is created and no relative displacement is produced.

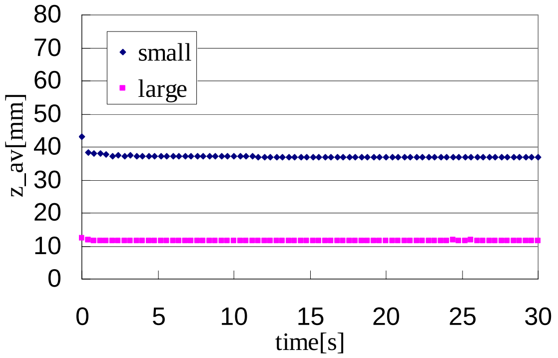


Fig.7. The average height of each size particles, z_{av}, as a function of time (A=35[mm], T=0.40 [s] and R=2.0). $A(2\pi/T)^2$=8636[mm/s^2].

Finally, the results when R is changed are indicated. Figure 8 shows the result under the condition of A_1=35[mm], T_1=0.33[s] and R=3.0. The large particles' upward trend is continuous. It is because the arch effect takes place continuously.

There is the critical ratio of diameters, R_s, which is the border between the large particles' continuously rise and discontinuously rise [3]. The two-dimensional local geometry model for getting R_c is shown as figure 9. In figure 9, the only area between W_1 and W_2 is considered. W_1 and W_2 are the virtual walls made by the particles outside them. Δh_p equal to the length of one cycle of the geometric structure. h equal to the height from the position where the W_1 and W_2 are crossed. The left diagram in figure 9 is the initial stable condition by arch effect. As the larger particle is lifted gradually, the next stable condition is found as shown in the right diagram in figure 9. Arch effect takes place when the particles 'A' are caught between the larger particle and the wall. The height of the larger particle's gravity center on the initial condition is h_0 and that on the next condition is h_1.

The initial condition shows that the larger particle is in contact with the wall. Thus, h_0 is given by,

$$h_0 = 2r_{large} = 2Rr_{small} \tag{8}$$

The next condition shows that the smaller particles 'A' are caught between the larger particle and the wall at the height nearly equal to the gravity center of the larger particle. Therefore, h_1 is given by,

$$h_1 = \sqrt{3}\left(r_{large} + \delta\right) \approx \sqrt{3}(R+2)r_{small} \tag{9}$$

where δ is the length between the larger particle and the wall in the horizontal direction. $\delta \approx 2r_{small}$ is assumed here. $\Delta h (= h_1 - h_0)$ corresponds to the threshold of the relative displacement which is required for arch effect. The proportion of the length of stable condition by arch effect to the length of one cycle of geometric structure is obtained as follow.

$$S = 1 - \frac{h_1 - h_0}{\Delta h_p} = \frac{2-\sqrt{3}}{2\sqrt{3}}R \approx 0.077R \tag{10}$$

When S=1,

$$R_c \equiv R \approx 12.9 \tag{11}$$

If S=1, arch effect takes place continuously. This means $R_c (\approx 12.9)$ is the critical ratio of diameters which is the border between the large particles' continuously rise and discontinuously rise.

Similarly, R_s in three-dimensional model is equal to 2.78 ($R_c^{3D} \approx 2.78$). As the ratio of diameters, R, in figure 8 is larger than R_c^{3D}, the larger particle rise up continuously.

In this section, the following knowledge is obtained.

- Multi-particle model is valid for simulation of Brazil-Nuts effect
- $A\ (2\pi/T)^2>g$ is one of the necessary conditions for Brazil-Nuts effect
- When $R>R_c$, upward trend is continuous

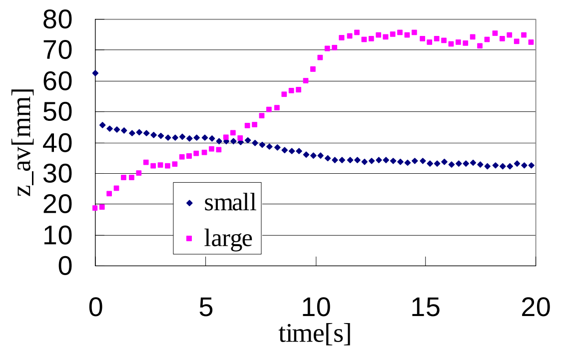


Fig.8. The average height of each size particles, z_{av}, as a function of time (A=35[mm], T=0.33[s] and R=3.0).

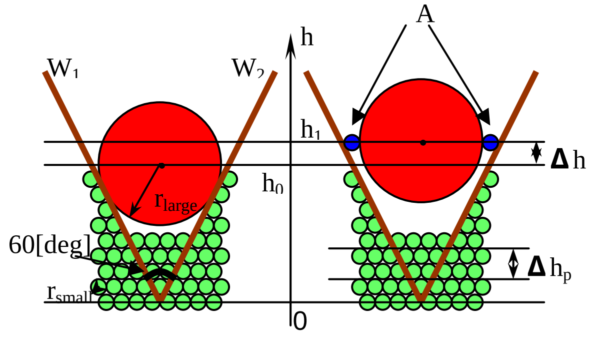


Fig.9. Concept of arch effect. The two-dimensional model of arch effect for calculating R_s. The left diagram is the initial stable condition by arch effect and the right one is the next condition where arch effect takes place.

4. Particles' behavior in the space

4-1. Simulation model

The simulation model of the particles in the space was introduced. The simulation model was a two-dimensional Multi-Particle model. When the particle was in touch with the other particles, mass-spring-damper (MSD) model was applied. Friction was considered. Gravity didn't act. Particles are gathered by one another's universal gravitation. The effect of centrifugal force by spin was neglected here. The equation of motion for this model is given by,

not in contact,

$$m\frac{d^2\mathbf{x}}{dt^2} = -\sum_{all} G\frac{mm_i}{d_i^2}\frac{\mathbf{d}_i}{d_i} \quad (12)$$

in contact,

$$m\frac{d^2\mathbf{x}}{dt^2} = -\sum_{all} G\frac{mm_i}{d_i^2}\frac{\mathbf{d}_i}{d_i} - \sum K(r+r_i-d_i)\frac{\mathbf{x}-\mathbf{x}_i}{d_i}$$
$$-\sum D\frac{d(\mathbf{x}-\mathbf{x}_i)}{dt}$$
$$-\sum \min\left[\left(1+\frac{0.001}{|\mathbf{v}_{R,F,i}|}\right)\mu, 1.0\right] K(r+r_i-d_i)\frac{\mathbf{v}_{R,F,i}}{|\mathbf{v}_{R,F,i}|} \quad (13)$$

where

$$\mathbf{d}_i = \mathbf{x} - \mathbf{x}_i, d_i = |\mathbf{d}_i| \quad (14)$$

$$\mathbf{v}_{R,i} = \mathbf{v} - \mathbf{v}_i, v_{R,i} = |\mathbf{v}_{R,i}| \quad (15)$$

$$\mathbf{v}_{R,F,i} = \mathbf{v}_{R,i} - \frac{\langle \mathbf{d}_i, \mathbf{v}_{R,i}\rangle}{d_i}\frac{\mathbf{d}_i}{d_i}, v_{R,F,i} = |\mathbf{v}_{R,F,i}| \quad (16)$$

G is universal gravitational constant.

4-2. Computational conditions

The simulation conditions are indicated in this section. The kind of particles' size was two. The particles' density was equal to asteroid "ITOKAWA", which is 3.2[g/cm3]. This value was calculated on the assumption which the void ratio of "ITOKAWA" is 40%. The smaller particles' diameter was 27.2[m] and that of the larger one was three times as long as the smaller, 81.6[m]. In order to simulate the collision with the other falling asteroid, velocity change in fixed direction (-x or -y) was given to the particle which was the most distant in fixed direction (x or y) from the center of gravity of all particles at a regular interval (see figure 10). The initial position of the larger particle was displaced in -x direction from the center of gravity. Δt in this calculation was 0.5[s].

4-2. Results

In this section, the result of simulation is discussed. Figure 11 shows the result of the simulation assumed in the space. Figure 12 shows images of the initial position and the position after shaking.

In figure 11, the curve of the case labeled 'from y' shows upward trend, but the other ('from x') doesn't. It is thought that the curve of 'from y' is the portion cut out just from the period of time when the particle rises up (corresponding to time=6～9[s] in figure 4) and that of 'from x' is the portion cut out from the period of time when the particle doesn't rise up (corresponding to time=9～12[s] in figure 4). In the other cases, no case shows down trend and some cases which momentum is given from x direction show up trend. Thus, it can not be confirmed that the larger particle's upward trend depends on the direction from which momentum is given. Behind this background, as the particles' volume and mass in space are much larger than that on the ground and the vibration scale is relatively small, the larger particle's up trend becomes more discontinuous. Therefore, time scale of the particles' behavior in the space is quite large.

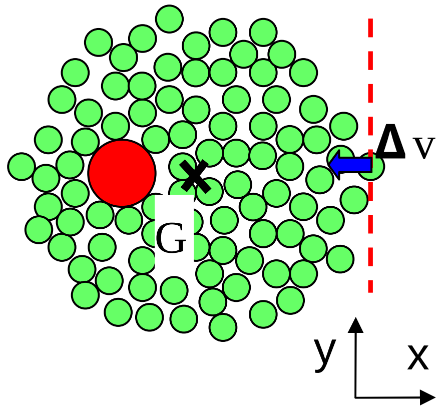


Fig.10. Concept of the model of collision with the other falling asteroid impact. The red broken line shows the most distant position of particles from the gravity center in x direction. The particle on this line is given velocity change (Δv) in -x direction.

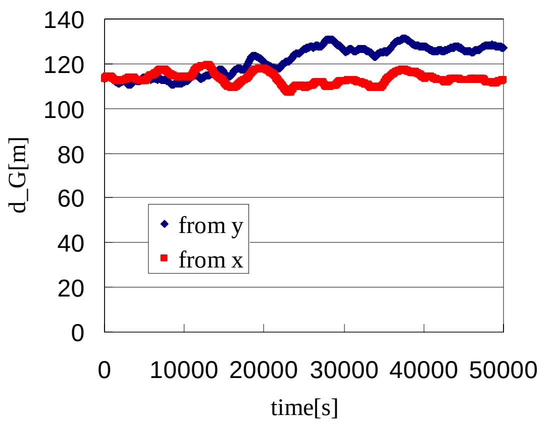


Fig.11. Distances between the larger particle and the center of gravity of the particles as a function of time. The initial position of the larger particle is displaced in -x direction from the center of gravity and momentum is given from x or y direction, which cases are labeled as 'from x' and 'from y', respectively.

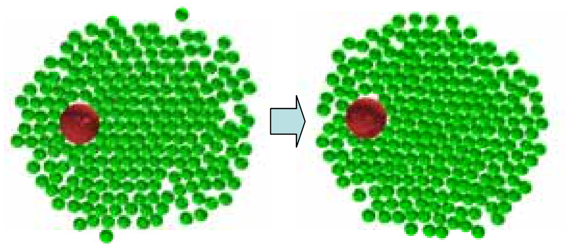


Fig.12. Images of the initial position and the position after being given momentum in the space.

As noted section 3-2-4, the relative displacement between a larger particle and the others by shaking and arch effect are needed for Brazil-Nuts effect. When the initial position of the large particle is set close to the gravity center, upward trend is not seen. Particles near the gravity center move little distance. As the relative displacement between particles is quite small, arch effect rarely takes place near the gravity center. On the other hand, if the larger particle is near the outside surface, arch effect may become easily to take place and the large particle may rises up smoothly. There is, however, no trend like that. It is because, we think, the number density of particles is smaller near the surface.

Furthermore, the value of acceleration of gravity is minimum value (zero) at the gravity center and maximum value at the surface. This is disadvantage for Brazil-Nuts effect, too.

5. Conclusions

Brazil-Nut effect on the ground is confirmed by experiment and numerical simulation. Multi-Particle model is suitable for simulation of Brazil-Nuts effect.

Knowledge about mechanisms of Brazil-Nuts effect on the ground is obtained from these results. The obtained knowledge is summarized below.

- The condition which the maximum acceleration of wall, $A(2\pi/T)^2$, is larger than the gravitational acceleration, g, $(A(2\pi/T)^2 > g)$ is one of the necessary conditions for Brazil-Nuts effect
- the larger particle's upward trend is continuous when the diameter ratio between the larger particle to the smaller one is larger than critical ratio of diameters $(R > R_c)$

When the particles in the space are shaken, the larger particle shows the trend to rise up to the surface. However, as time scale of the particles' behavior in the space is larger than that on the ground, we can see only a portion of Brazil-Nuts effect. The larger particle's upward trend depends on the vibration scale, the initial position of the larger particle, the number density of particles.

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