

# A study on design method of vibration isolator for reaction wheel assembly in consideration of the torque transmissibility

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## Abstract

Recently, observation satellites are required extremely high pointing accuracy and stability to achieve various and complicated missions. However, reaction wheel assembly used as an actuator of the attitude control system can be a major source of disturbances. Therefore, isolation of disturbances is one of the most significant issues in designing a precise attitude control system. On the other hand, reaction wheel is an actuator of attitude control system. Therefore, the purpose of this study is to propose the new design method of vibration isolator in consideration of the torque transmissibility.

## トルクの伝達を考慮したリアクションホイールの振動アイソレータ設計法に関する研究

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## 摘要

衛星姿勢制御用アクチュエータとして広く利用されているリアクションホイールは、主要な擾乱源であることが知られている。そのためリアクションホイールの振動を低減することは、今後要求される姿勢安定度要求を満たすために重要な課題の一つとなっている。しかしリアクションホイールは姿勢制御用のアクチュエータであるため、制御トルクは伝達するが、擾乱トルクは低減しなければならない。そこで、姿勢制御用のトルク伝達率を考慮に入れた振動アイソレータの設計法について述べる。

## 1. Introduction

Observation satellites are required high pointing accuracy and stability to achieve various and complicated missions<sup>[1]</sup>. For example, figure 1 shows the overview of Nano-JASMINE satellite<sup>[2]</sup>. Nano-JASMINE is developed by Nakasuka Laboratory at University of Tokyo and National Astronomical Observatory of Japan. For this satellite, required stability is 740 milli-arcsecond at 8.8 sec. However, reaction wheel assembly used as an actuator of attitude control system can be a major source of disturbances<sup>[3,4]</sup>. Therefore, isolation of disturbance is one of the most significant issues in designing a precise attitude control system. On the other hand, reaction wheel is an actuator of attitude control system. Therefore, disturbance must be isolated from satellite, and control torque must be transmitted. The purpose of this study is to propose the new design method of vibration isolator in consideration of control torque transmissibility.

## 2. Numerical model of vibration isolator

A parallel link mechanism is known to achieve a high rigidity with lightweight construction because it is a truss construction.



Figure 1 Overview of Nano-JASMINE

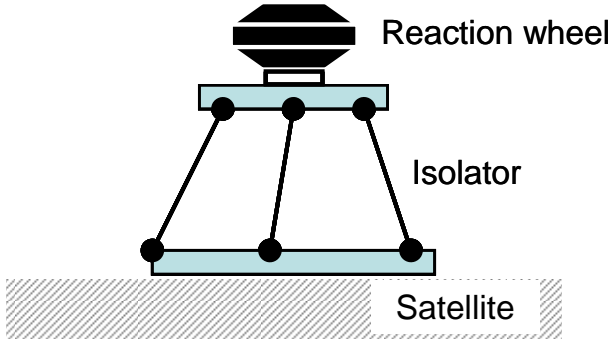


Figure 2 Concept of parallel link isolator

Therefore, parallel link mechanism is adopted to design a vibration isolator. Figure 2 shows the concept of parallel link isolator for reaction wheel assembly.

Numerical model of parallel link isolator is derived by kinematics or inverse kinematics. In kinematics, position and attitude are calculated from a length of each leg. However, because of an indeterminate structure, kinematics is difficult to solve. On the other hand inverse kinematics, a length of each leg is calculated from position and attitude of platform. A length of each leg can be calculated by Jacobian matrix easily.

Therefore in this study, inverse kinematics is adopted to analyze the isolator.

Figure 3 shows the definition of coordinates and vector. Coordinates and vectors are described as follows:

- $\{p\}$  Coordinate fixed to the C.M. of platform
- $\{b\}$  Coordinate fixed to the C.M. of base plate
- $r_i$  Vector from origin of  $\{b\}$  to the bottom of each leg
- $p_i$  Vector from origin of  $\{p\}$  to the top of each leg
- $X_0$  Vector from origin of  $\{b\}$  to origin of  $\{p\}$
- $C^{b/p}$  DCM from  $\{p\}$  to  $\{b\}$
- $e_i$  Vector from bottom to top of each leg (unit vector)

Where,  $e_i$  is written by elevation angle and azimuth angle as follow,

$$e_i = [\cos \theta_{EL,i} \cos \theta_{AZ,i} \quad \cos \theta_{EL,i} \sin \theta_{AZ,i} \quad \sin \theta_{EL,i}]^T$$

The position and attitude are described by  $w = (x, y, z, \theta_x, \theta_y, \theta_z)^T$ , and a length of each leg is described by  $q = (q_1, q_2, \dots)^T$ . Then, following relation works out.

$$q = [e_i e_i^T \quad -e_i e_i^T \tilde{p}_i] w$$

The equations of motion are written as follows:

$$ms^2 d\mathbf{r} + \sum_{i=1}^n \left\{ (cs + k) [e_i e_i^T \quad -e_i e_i^T \tilde{p}_i] \mathbf{w} \right\} = \mathbf{f}_{RW}$$

$$Is^2 d\theta + \sum_{i=1}^n \left\{ (cs + k) [\tilde{p}_i e_i e_i^T \quad -\tilde{p}_i e_i e_i^T \tilde{p}_i] \mathbf{w} \right\} = \boldsymbol{\tau}_{RW}$$

Where, m is a mass of platform, c is a viscosity, k is a spring constant, I is a inertia moment of platform and s is a Laplace operator respectively. Force and torque transmitted to the

## Position & Attitude

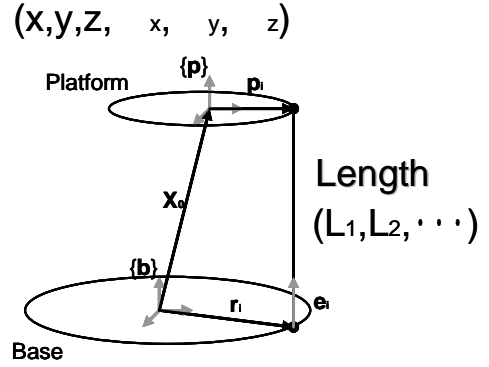


Figure 3 Definition of coordinates and vectors

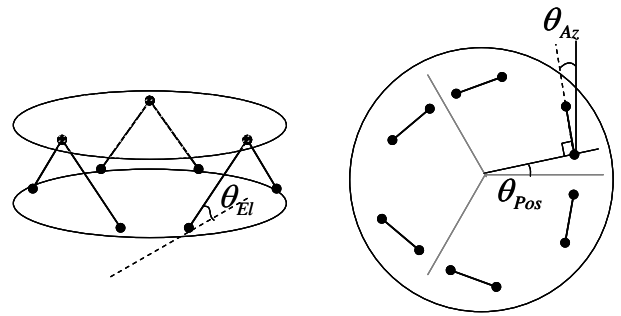


Figure 4 Definition of elevation angle and azimuth angle

satellite are calculated by

$$\mathbf{f}_{iso} = \sum_{i=1}^n \left\{ (cs + k) [e_i e_i^T \quad -e_i e_i^T \tilde{p}_i] \mathbf{w} \right\}$$

$$\boldsymbol{\tau}_{iso} = \sum_{i=1}^n r_i \times \mathbf{f}_i = \sum_{i=1}^n \left\{ (cs + k) [\tilde{r}_i e_i e_i^T \quad -\tilde{r}_i e_i e_i^T \tilde{p}_i] \mathbf{w} \right\}$$

Then, transfer function from disturbance force, torque and control torque to transmitted force and torque is written by

$$\begin{bmatrix} \mathbf{f}_{Sat} \\ \boldsymbol{\tau}_{Sat} \end{bmatrix} = G \begin{bmatrix} \mathbf{f}_{RW} \\ \boldsymbol{\tau}_{RW} \end{bmatrix}$$

## 3. Design method of vibration isolator

### 3.1. Break down from mission requirement

This section describes the optimal design condition of vibration isolator. First, mission requirement is translated into the allowed disturbance torque for satellites. Attitude variation by disturbances is assumed as follow:

$$\theta(t) = a \sin(\omega t) = a \sin(2\pi f t)$$

The amplitude of attitude variation is calculated by mission requirement as follow

$$a \leq \begin{cases} \frac{\Delta \theta}{\sin(\pi f T_s)} & (0 < f < f_s) \\ \Delta \theta & (f \geq f_s) \end{cases}$$

Finally, allowed disturbance is calculated by this amplitude and

property of controller:

$$|\tau| = I_{sat} a \sqrt{(\omega_c^2 - \omega_d^2)^2 + (2\xi_c \omega_c \omega_d)^2}$$

Where,  $\xi_c$  is a damping ratio of controller,  $\omega_c$  is a natural frequency of controller respectively.

For example, allowed disturbance for Nano-JASMINE is shown in figure 5.

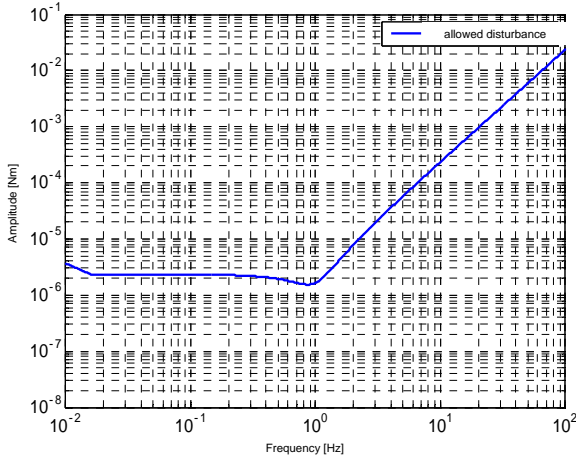


Figure 5 Allowed disturbance for Nano-JASMINE

Next, design condition for isolator is derived by allowed disturbance. Disturbance caused by reaction wheel is modeled as follow

$$\tau_{whl} = a\omega^2$$

Where a is a proportional constant and  $\omega$  is a angular velocity of reaction wheel. Figure 6 shows the allowed disturbance and disturbance caused by reaction wheel assembly. Finally, required property for vibration isolator is calculated. Figure 7 shows the design condition for vibration isolator.

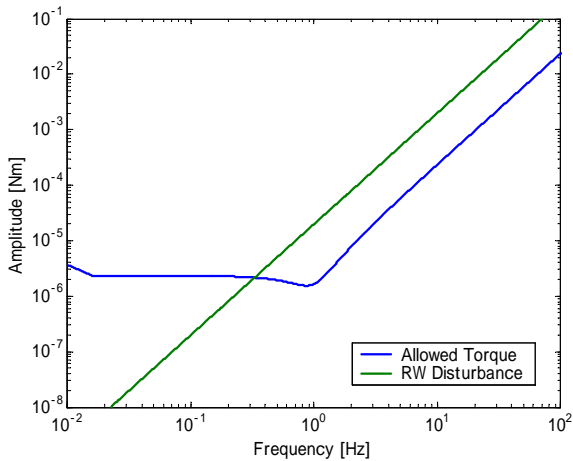


Figure 6 Allowed disturbance and reaction wheel disturbance

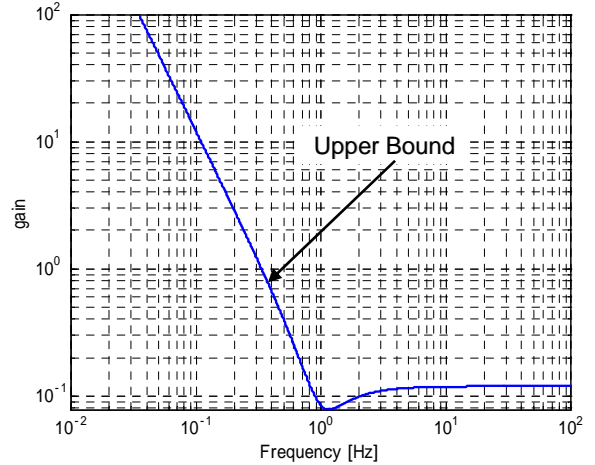


Figure 7 Design condition for vibration isolator

### 3.2. Optimization

In the previous section, required property is derived from mission requirement. In this section, parameters of isolator (spring constant, viscosity, elevation angle, azimuth angle and so on) are optimized to satisfy the design condition.

Figure 8 shows the overview of optimization loop. First, initial parameters (spring constant, viscosity and so on) are set and requirements for attitude stability, control property and robustness are derived by mission requirement. Then, controller is designed for augmented system to satisfy these requirements. Next, required property for isolator is derived and compared with initial parameters. Finally, evaluate value is calculated and this loop is iterated to find optimal parameters by sequential quadratic programming (SQP).

Figure 9 shows the result of optimization. In this figure blue line shows required isolator property for x and y axis, red line shows for z axis, and green and light blue line show the result of optimization for x,y and z axis respectively. Required property shows the upper bound for isolator. It is confirmed that result of optimization is lower than upper bound.

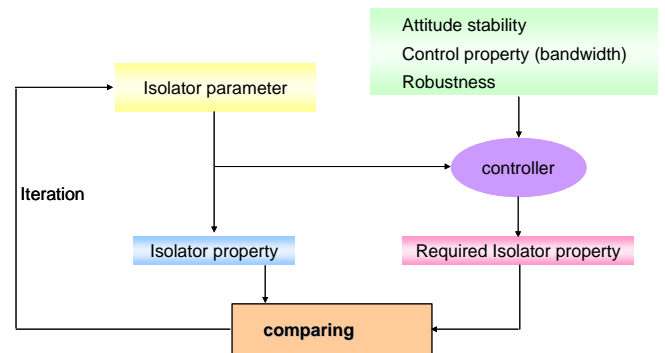


Figure 8 Overview of optimization loop

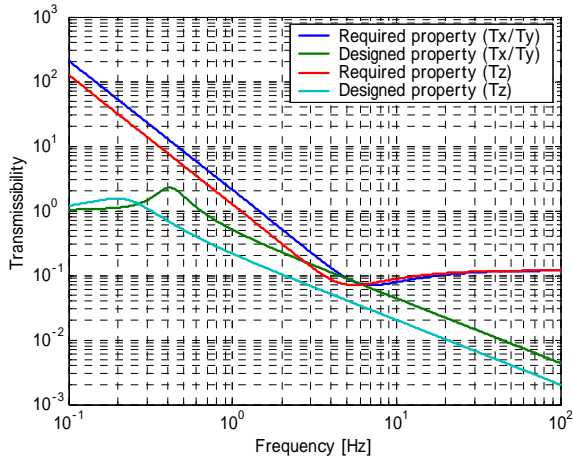


Figure 9 Result of optimization

#### 4. Validation by numerical analysis

Numerical analysis is conducted to validate proposed algorithm. Numerical model includes the dynamics of satellite, reaction wheel assembly and vibration isolator. Satellite is modeled single rigid body. In terms of isolator, numerical model written in section 2 is used. Reaction wheel assembly is modeled by using Herzian contact theorem. Then, two parameters of isolator (optimal and not optimal) are simulated.

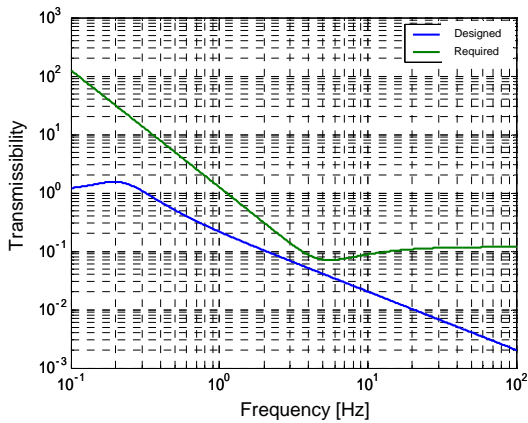


Figure 10 Optimal parameters for validation

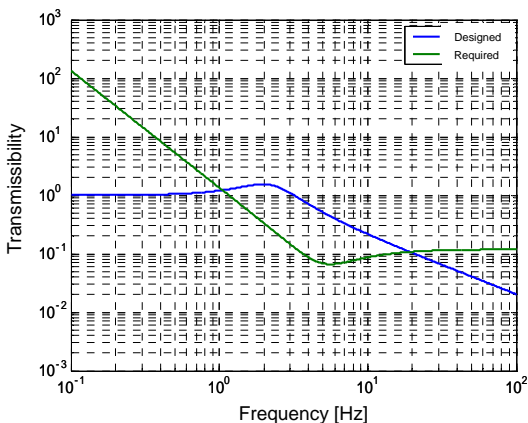


Figure 11 Not optimal parameters for validation

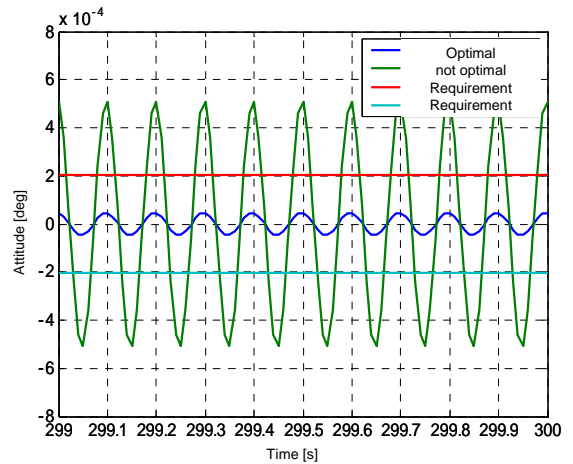


Figure 12 Result of validation

The result of numerical analysis is shown in figure 12. As a result, it is confirmed that isolator with optimal parameter satisfy the requirement. Therefore, the proposed algorithm is validated.

#### 4. Conclusion

Vibration isolator for reaction wheel assembly is needed to isolate disturbance, and to transmit control torque. Therefore, there is a trade-off in designing an isolator. In this study, new design method is proposed in consideration of torque transmissibility. In proposed method, optimal design condition of isolator is derived by mission requirement. Numerical analysis is conducted to verify proposed method. As a result, it is confirmed that optimal parameters calculated by proposed method satisfy the trade-off.

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