Analysis of Dynamic Interactions between Satellite and Magnetic Bearing Wheel (MBW) with Inclined Magnetic Poles

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ABSTRACT

A magnetic bearing wheel (MBW) with inclined magnetic poles has been developed. This paper deals with the dynamic interactions between a satellite and the MBW. In the MBW-satellite system, the MBW rotor is coupled with the satellite by magnetic bearing forces and torques, and there exists some possibility that the magnetic bearing controller makes the satellite nutation unstable. In this paper, the equations of motion of the MBW-satellite system are formulated by Kane's method. Based on the stability analysis of this system, it is shown that the cross-feedback control of the rotor gimbal angle and integrator in an ordinary magnetic bearing controller are the instability factors responsible for satellite nutation. However, this nutation is stabilized with a cross-feedback control of the satellite angular rate estimated by a minimal-order observer from the magnetic bearing control torques. The effects of the disturbance feedback controller used in our study on the system stability are also considered, and the results show that the system stability is not affected by the disturbance feedback controller.

傾斜磁極磁気軸受ホイールと宇宙機とのダイナミクス干渉解析

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摘要

本論文は，傾斜磁極磁気軸受ホイールと宇宙機とのダイナミクス干渉解析について論じたものである．磁気軸受ホイールを搭載した宇宙機システムは，磁気軸受制御力およびトルクを介して，磁気軸受ホイール・ロータと宇宙機とが結合したシステムと見なすことができ，このとき磁気軸受制御系が宇宙機姿勢運動を不安定化する可能性がある．本論文では，傾斜磁極磁気軸受ホイール・宇宙機系運動方程式に基づいた姿勢安定性解析により，従来の磁気軸受制御系では，ロータ・ジンバル角度のクロスフィードバック制御と積分器の双方に起因して，宇宙機ニューテーションが不安定化することを示す．次にこのような問題に対して，最小次元オブザーバによる宇宙機角速度の推定と，これをクロスフィードバックする構成を追加した新たな磁気軸受制御系を適用することで，宇宙機ニューテーションを安定化できることを示し，さらに本構成に対して擾乱フィードバック制御系を追加した場合にも，系の安定性が保たれることを確認する．

1. INTRODUCTION

Recently, satellite lifetimes have increased, and the pointing requirements have become more stringent for observation missions. By applying a magnetic bearing wheel (MBW) as the attitude control actuator of a satellite, instead of an ordinary ball bearing wheel, it is expected that the lifetime of the bearing will increase and the disturbance of the wheel will decrease ; therefore, we have developed a MBW with inclined magnetic poles ; this enables the 5-DOF magnetic bearing to be composed of six electromagnets and six displacement sensors[1]~[4].

In order to equip a satellite with the MBW, the equations of motion of the MBW are accurately obtained through various examination results and then based on these equations, the equations of motion of the MBW-satellite system are formulated and the motion property of the system is considered.

With regard to the former equations of motion of the MBW and by modeling the asymmetric magnetic bearing stiffness caused by the difference in the electromagnet property as well as disturbance factors of the magnetic bearing, we derived the equations of motion that accurately describe the actual motion property[5].

On the other hand, with regard to the motion property of the MBW-satellite system, it is known that the cross-feedback control of the MBW rotor gimbal angle in the magnetic bearing controller makes the satellite motion unstable. Inoue and Ninomiya[6] proposed a method in which the integral of the satellite angular rate estimated from the MBW rotor gimbal angle is fed back to the magnetic bearing controller for the stabilization of both the MBW and the satellite motion. However, the equations of motion derived by them were not entirely accurate; therefore, a detailed analysis is required for practical application. Moreover, if the disturbance feedback controller proposed in our report[3] is applied, the motion property of the system must be considered.

This paper deals with the dynamic interactions between the satellite and the MBW. First, based on the equations of motion of the MBW[5], the equations of motion of the MBW-satellite system in which the MBW rotor is coupled with the satellite by the magnetic bearing forces and torques are formulated. Second, a stability analysis based on these equations is considered.

2. EQUATIONS OF MOTION OF THE MBW-SATELLITE SYSTEM

2 • 1 Reference Frames and Symbols The reference frames and symbols in the following discussions are defined as shown in Figure 1 and as follows.

Σ_W : MBW reference frame
(X_W, Y_W, Z_W = principal axes of inertia of the MBW)
Σ_B : satellite reference frame
($X_B, Y_B, Z_B \neq$ principal axes of inertia of the satellite)
subscript on the upper-left side : reference frame to describe a variable
M_B, $\boldsymbol{I}_B$: satellite mass, tensor of inertia
M_W, $\boldsymbol{I}_W$: MBW rotor mass, tensor of inertia
$\mu = M_B M_W / (M_B + M_W)$: conversion mass
$\boldsymbol{v}_B$: velocity of the center of mass of the satellite
$\boldsymbol{\omega}_B = [\omega_{Bx} \quad \omega_{By} \quad \omega_{Bz}]^T$: satellite angular rate
$\boldsymbol{v}_W$: velocity of the center of mass of the MBW rotor
$\boldsymbol{\omega}_W$: MBW rotor angular rate
$\boldsymbol{v}_c = [v_{cx} \quad v_{cy} \quad v_{cz}]^T$: velocity of the center of mass of the MBW-satellite system
ω_{zw} : angular rate of the MBW rotor spin
$\boldsymbol{d} = [x_w \quad y_w \quad z_w \quad \theta_{xw} \quad \theta_{yw}]^T$:5-DOF MBW rotor displacement
$\boldsymbol{f} = [f_x \quad f_y \quad f_z]^T$: magnetic bearing forces

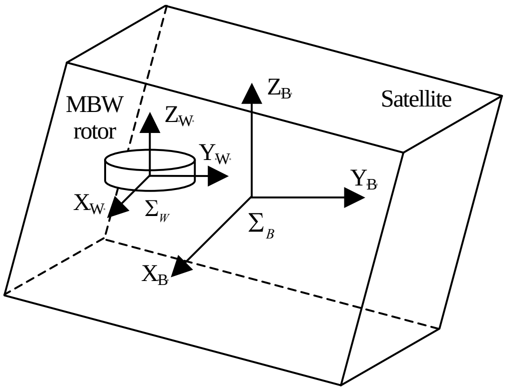


Fig. 1 Definition of reference frames

$\boldsymbol{\tau} = [\tau_x \quad \tau_y]^T$: magnetic bearing torques
$\boldsymbol{r}_{WB0} = [x_{w0} \quad y_{w0} \quad z_{w0}]^T$: position of the center of mass of the MBW rotor relative to that of the satellite under the condition $\boldsymbol{d} = \boldsymbol{0}$

2 • 2 Formulation of Equations of motion In this study, the equations of motion of the MBW-satellite system are formulated by Kane's method[7][8]. For convenience of formulation, the following assumptions are applied in the following discussions.

- The MBW and the satellite are rigid bodies, and the MBW rotor is coupled with the satellite by magnetic bearing forces and torques.
- The gravity and gravity-gradient torque are negligible.

Degrees of Freedom and Reference Frame The degrees of freedom of the MBW-satellite system are the 6-DOF of the system and 5-DOF of the relative motion. Here, these degrees of freedom are defined as

- 6-DOF of the system : Translation of the center of mass of the MBW-satellite system (3-DOF), rotation of the satellite (3-DOF)
- 5-DOF of the relative motion : Translation (3-DOF) and rotation(2-DOF) of the MBW rotor relative to the satellite

In particular, if the 6-DOF of the system are selected as stated above, the translational motion can be separated from the rotational motion in the equations of motion.

In addition, to delete the satellite attitude angle from the equations of motion, the reference frame for describing the equations of motion is determined with regard to the satellite reference frame Σ_B.

Formulation by Kane's Method The generalized speeds $u_j\,(j = 1 \sim 11)$ and generalized forces $Q_j\,(j = 1 \sim 11)$ of the MBW-satellite system are given as

$$\text{Generalized Speeds } u_j : [\,{}^B v_{cx} \ {}^B v_{cy} \ {}^B v_{cz} \ {}^B\omega_{Bx} \ {}^B\omega_{By} \ {}^B\omega_{Bz} \ {}^B\dot{x}_w \ {}^B\dot{y}_w \ {}^B\dot{z}_w \ {}^B\dot{\theta}_{xw} \ {}^B\dot{\theta}_{yw}\,] \quad (1)$$

$$\text{Generalized Forces } Q_j : [\,0 \ 0 \ 0 \ 0 \ 0 \ 0 \ {}^B f_x \ {}^B f_y \ {}^B f_z \ {}^B\tau_x \ {}^B\tau_y\,] \quad (2)$$

The equations of motion of the MBW-satellite system can be obtained by Kane's method as follows:

$$Q_j + ({}^B\tilde{\boldsymbol{f}}_B \cdot {}^B\boldsymbol{v}^j_{CB} + {}^B\tilde{\boldsymbol{n}}_{CB} \cdot {}^B\boldsymbol{\omega}^j_B) + ({}^B\tilde{\boldsymbol{f}}_W \cdot {}^B\boldsymbol{v}^j_{CW} + {}^B\tilde{\boldsymbol{n}}_{CW} \cdot {}^B\boldsymbol{\omega}^j_W) = 0 \quad (j = 1 \sim 11) \quad (3)$$

where

${}^B\tilde{\boldsymbol{f}}_B$, ${}^B\tilde{\boldsymbol{n}}_{CB}$: inertia force, inertia torque around the center of mass of the satellite
${}^B\tilde{\boldsymbol{f}}_W$, ${}^B\tilde{\boldsymbol{n}}_{CW}$: inertia force, inertia torque around the center of mass

of the MBW rotor

$^{B}\boldsymbol{v}_{CB}^{j} = \partial\, {}^{B}\boldsymbol{v}_{B} \big/ \partial u_{j}$, $^{B}\boldsymbol{\omega}_{B}^{j} = \partial\, {}^{B}\boldsymbol{\omega}_{B} \big/ \partial u_{j}$ ：partial velocity of $^{B}\boldsymbol{v}_{B}$, partial angular velocity of $^{B}\boldsymbol{\omega}_{B}$ with respect to the generalized speed u_{j}

$^{B}\boldsymbol{v}_{CW}^{j} = \partial\, {}^{B}\boldsymbol{v}_{W} \big/ \partial u_{j}$, $^{B}\boldsymbol{\omega}_{W}^{j} = \partial\, {}^{B}\boldsymbol{\omega}_{W} \big/ \partial u_{j}$ ： partial velocity of $^{B}\boldsymbol{v}_{W}$, partial angular velocity of $^{B}\boldsymbol{\omega}_{W}$ with respect to the generalized speed u_{j}

Moreover, the tensors of inertia are defined as

$$^{B}\boldsymbol{I}_{B} = [J_{Bij}] \quad (i, j = \{1,2,3\} = \{x, y, z\}) \tag{4}$$

$$^{W}\boldsymbol{I}_{W} = diag[J_{W}, J_{W}, J_{PW}] \tag{5}$$

by substituting the components of inertia forces, inertia torques, partial velocities, and partial angular velocities into equation (3); by neglecting the 2nd-order terms or more, the linearized equations of motion of the MBW-satellite system are given as follows.

$$\{J_{Bxx} + J_{W} + \mu(y_{w0}^{2} + z_{w0}^{2})\}\dot{\omega}_{Bx} + \{J_{Bxy} - \mu x_{w0} y_{w0}\}\dot{\omega}_{By} + \{J_{Bzx} - \mu z_{w0} x_{w0}\}\dot{\omega}_{Bz} - \mu z_{w0}\ddot{y}_{w} + \mu y_{w0}\ddot{z}_{w} + J_{W}\ddot{\theta}_{xw} + J_{PW}\omega_{zw}(\omega_{By} + \dot{\theta}_{yw}) = 0 \quad \cdots(6\text{-}1)$$

$$\{J_{Bxy} - \mu x_{w0} y_{w0}\}\dot{\omega}_{Bx} + \{J_{Byy} + J_{W} + \mu(z_{w0}^{2} + x_{w0}^{2})\}\dot{\omega}_{By} + \{J_{Byz} - \mu y_{w0} z_{w0}\}\dot{\omega}_{Bz} + \mu z_{w0}\ddot{x}_{w} - \mu x_{w0}\ddot{z}_{w} + J_{W}\ddot{\theta}_{yw} - J_{PW}\omega_{zw}(\omega_{Bx} + \dot{\theta}_{xw}) = 0 \quad \cdots(6\text{-}2)$$

$$\{J_{Bzx} - \mu z_{w0} x_{w0}\}\dot{\omega}_{Bx} + \{J_{Byz} - \mu y_{w0} z_{w0}\}\dot{\omega}_{By} + \{J_{Bzz} + J_{PW} + \mu(x_{w0}^{2} + y_{w0}^{2})\}\dot{\omega}_{Bz} - \mu y_{w0}\ddot{x}_{w} + \mu x_{w0}\ddot{y}_{w} = 0 \tag{6-3}$$

$$\mu(\ddot{x}_{w} + \dot{\omega}_{By} z_{w0} - \dot{\omega}_{Bz} y_{w0}) = f_{x} \tag{6-4}$$

$$\mu(\ddot{y}_{w} + \dot{\omega}_{Bz} x_{w0} - \dot{\omega}_{Bx} z_{w0}) = f_{y} \tag{6-5}$$

$$\mu(\ddot{z}_{w} + \dot{\omega}_{Bx} y_{w0} - \dot{\omega}_{By} x_{w0}) = f_{z} \tag{6-6}$$

$$J_{W}(\dot{\omega}_{Bx} + \ddot{\theta}_{xw}) + J_{PW}\omega_{zw}(\omega_{By} + \dot{\theta}_{yw}) = \tau_{x} \tag{6-7}$$

$$J_{W}(\dot{\omega}_{By} + \ddot{\theta}_{yw}) - J_{PW}\omega_{zw}(\omega_{Bx} + \dot{\theta}_{xw}) = \tau_{y} \tag{6-8}$$

Here, for brevity of expression, the subscript at the upper-left side is deleted. Note that equation (6) and the following discussions are described within the satellite reference frame Σ_{B} .

Magnetic Bearing Forces and Torques The magnetic bearing forces and torques, which are shown at the right-hand side of equations (6-4)~(6-8), depend on the MBW controller. When only the magnetic bearing controller is applied as the MBW controller, the magnetic bearing forces and torques are given as[5]

$$f_{x} = -K_{X}K_{DR}[D(\lambda)\dot{x}_{w} + K_{PR}K(\lambda)x_{w}] + f_{x\_dist} \tag{7-1}$$

$$f_{y} = -K_{Y}K_{DR}[D(\lambda)\dot{y}_{w} + K_{PR}K(\lambda)y_{w}] + f_{y\_dist} \tag{7-2}$$

$$f_{z} = -K_{Z}K_{DZ}[D(\lambda)\dot{z}_{w} + K_{PZ}K(\lambda)z_{w}] + f_{z\_dist} \tag{7-3}$$

$$\tau_{x} = -K_{Y}\left\{\begin{aligned} &K_{D\theta}[D(\lambda)\dot{\theta}_{xw} + K_{P\theta}K(\lambda)\theta_{xw}] \\ &- K_{r}\omega_{zw}D(\lambda)\dot{\theta}_{yw} + K_{a}\omega_{zw}W_{c}(\lambda)\theta_{yw} \end{aligned}\right\} + \tau_{x\_dist} \tag{7-4}$$

$$\tau_{y} = -K_{X}\left\{\begin{aligned} &K_{D\theta}[D(\lambda)\dot{\theta}_{yw} + K_{P\theta}K(\lambda)\theta_{yw}] \\ &+ K_{r}\omega_{zw}D(\lambda)\dot{\theta}_{xw} - K_{a}\omega_{zw}W_{c}(\lambda)\theta_{xw} \end{aligned}\right\} + \tau_{y\_dist} \tag{7-5}$$

where

$f_{*\_dist}, \tau_{*\_dist}$ ：excitation force, torque caused by the disturbance factors

K_{X}, K_{Y}, K_{Z} ：magnetic bearing stiffness considering the difference in the electromagnet property

$K_{DR}, K_{PR} / K_{DZ}, K_{PZ} / K_{D\theta}, K_{P\theta}$ ：control gain of the radial translation/axial translation/radial rotation

K_{r}, K_{a} ：cross-feedback gain for the stabilization of the gyroscopic motion

$D(\lambda), K(\lambda), W_{c}(\lambda)$ ：frequency transfer function of the components of the magnetic bearing controller

（refer to appendix for these internal structures）

Equations (6) and (7) are the equations of motion of the MBW-satellite system with only the magnetic bearing controller.

On the other hand, when the disturbance feedback controller is added to the magnetic bearing controller, the magnetic bearing forces and torques are given as[5]

$$f_{x} = f_{x\_mag} + K_{X}M_{W}K_{DR}K_{PR} \times \sum_{u} K(\lambda)[C_{Ru1}(\lambda)\ddot{x}_{w} - C_{Ru2}(\lambda)\ddot{y}_{w}] \tag{8-1}$$

$$f_{y} = f_{y\_mag} + K_{Y}M_{W}K_{DR}K_{PR} \times \sum_{u} K(\lambda)[C_{Ru2}(\lambda)\ddot{x}_{w} + C_{Ru1}(\lambda)\ddot{y}_{w}] \tag{8-2}$$

$$f_{z} = f_{z\_mag} + K_{Z}M_{W}K_{DZ}K_{PZ} \sum_{u} K(\lambda)C_{Zu}(\lambda)\ddot{z}_{w} \tag{8-3}$$

$$\tau_{x} = \tau_{x\_mag} + K_{Y}K_{D\theta}K_{P\theta} \times \sum_{u} K(\lambda)\left\{\begin{aligned} &J_{W}[-C_{\Theta u1}(\lambda)\ddot{\theta}_{xw} + C_{\Theta u2}(\lambda)\ddot{\theta}_{yw}] \\ &-J_{PW}\omega_{zw}[C_{\Theta u2}(\lambda)\dot{\theta}_{xw} + C_{\Theta u1}(\lambda)\dot{\theta}_{yw}] \end{aligned}\right\} \tag{8-4}$$

$$\tau_{y} = \tau_{y\_mag} + K_{X}K_{D\theta}K_{P\theta} \times \sum_{u} K(\lambda)\left\{\begin{aligned} &J_{W}[-C_{\Theta u2}(\lambda)\ddot{\theta}_{xw} - C_{\Theta u1}(\lambda)\ddot{\theta}_{yw}] \\ &-J_{PW}\omega_{zw}[-C_{\Theta u1}(\lambda)\dot{\theta}_{xw} + C_{\Theta u2}(\lambda)\dot{\theta}_{yw}] \end{aligned}\right\} \tag{8-5}$$

where

f_{*_mag}, τ_{*_mag} ：magnetic bearing force, torque with only the magnetic bearing controller （equation (7)）

$C_{Rui}(\lambda), C_{Zu}(\lambda), C_{\Theta ui}(\lambda)$ ：frequency transfer function of components of the disturbance feedback controller

（refer to appendix for these internal structures）

Equations (6) and (8) are the equations of motion of the MBW-satellite system with both the magnetic bearing controller and disturbance feedback controller.

3. STABILITY ANALYSIS OF THE MBW-SATELLITE SYSTEM

Based on the equations of motion of the MBW-satellite system formulated in chapter 2, the stability of the MBW-satellite system is considered for two cases: (1) with only the magnetic bearing controller and (2) with both the magnetic bearing controller and disturbance feedback controller.

3・1 Without The Disturbance Feedback Controller With regard to the equations of motion (6) and (7), the physical and control parameters of the MBW are shown in Table 1. On the other hand, the physical parameters of the satellite are assumed as shown in Table 2.

Table 1 Physical and control parameters of the MBW

M_W	7.58 kg	K_X	3.413×10^{-1}
J_W	4.24×10^{-2} kgm^2	K_Y	3.238×10^{-1}
J_{PW}	7.70×10^{-2} kgm^2	K_Z	3.325×10^{-1}
K_{PR}	42.0 rad/s	ω_i	1.257 rad/s
K_{PZ}	38.0 rad/s	ω_d	1.257×10^{3} rad/s
$K_{P\theta}$	38.0 rad/s	ω_c	502.7 rad/s
K_{DR}	2.50×10^{3} Ns/m	K_{IR}	1.0×10^{-2} m/N
K_{DZ}	2.75×10^{3} Ns/m	K_{IZ}	1.0×10^{-2} m/N
$K_{D\theta}$	14.4 Nms/rad	$K_{I\theta}$	5.0 rad/Nm
K_r	4.78×10^{-2} Nms^2/rad^2	ω_{lpf}	12.57 rad/s
K_a	3.82 Nms/rad^2	ω_{dis}	6.283×10^{-4} rad/s

Table 2 Physical parameters of the satellite

M_B	1000 kg	J_{Byz}	–20 kgm^2
J_{Bxx}	1200 kgm^2	J_{Bzx}	50 kgm^2
J_{Byy}	900 kgm^2	x_{w0}	–0.20 m
J_{Bzz}	1000 kgm^2	y_{w0}	0.30 m
J_{Bxy}	30 kgm^2	z_{w0}	–1.00 m

With regard to equations (6) and (7), by substituting the solution given as

$$\boldsymbol{x} = [\hat{\omega}_{Bx} \quad \hat{\omega}_{By} \quad \hat{\omega}_{Bz} \quad \hat{x}_w \quad \hat{y}_w \quad \hat{z}_w \quad \hat{\theta}_{xw} \quad \hat{\theta}_{yw}]^T e^{\lambda t} \qquad (9)$$

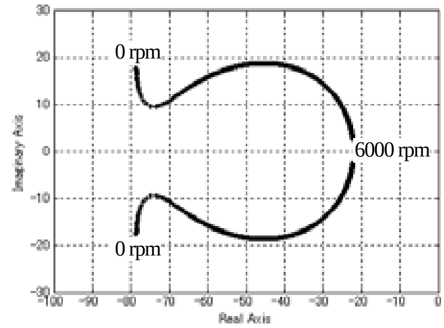
into the homogeneous equation in which the excitation force and torque caused by the disturbance factors are neglected, the following equation is derived.

$$\boldsymbol{E}(\lambda)\boldsymbol{x} = \boldsymbol{0} \qquad (10)$$

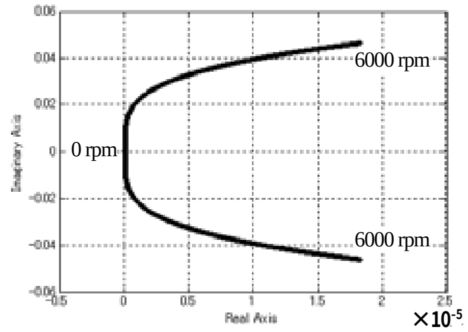
where $\boldsymbol{E}(\lambda) \in \boldsymbol{R}^{8\times8}$ is the coefficient matrix of the solution obtained using $\boldsymbol{x}$, and the characteristic equation of the MBW-satellite system is given as

$$\det \boldsymbol{E}(\lambda) = 0 \qquad (11)$$

In general, the integrator of the magnetic bearing controller makes the MBW precession unstable; however, the cross-feedback control of the MBW rotor gimbal angle can stabilize this precession[9]. On the other hand, the satellite nutation is affected by the cross-feedback control of the MBW rotor gimbal angle[6]. The root loci of these



(a) MBW precession



(b) Satellite nutation

Fig. 2 Root loci of the MBW-satellite system with an ordinary magnetic bearing control

motions based on equation (11) are shown in Figure 2. Here, all the motions of the MBW-satellite system, except the motions shown in Figure 2, are stable; therefore, the ordinary magnetic bearing controller (equation (7)) makes only the satellite nutation unstable.

In order to clarify the instability factors responsible for satellite nutation, approximate characteristic roots of the MBW-satellite system are derived. Approximations to the equations of motion (6-1,2),(6-7,8), and (7-4,5) of the MBW rotor gimbal angle θ_{xw}, θ_{yw} and the satellite angular rate ω_{Bx}, ω_{By} are applied as

- $x_w, y_w, z_w, \omega_{Bz}, \mu$; the product of inertia of the satellite are negligible.
- the principal moment of inertia of the MBW rotor J_W is negligible.
- in the magnetic bearing controller, the derivative control, cross-feedback control gain K_r for the stabilization of the MBW nutation, and the phase delay of the controller are negligible ($W_c(\lambda), D(\lambda) = 1$).
- $J_B \approx J_{Bxx} \approx J_{Byy}, K_{XY} \approx K_X \approx K_Y$

For an approximate characteristic equation of motion of the satellite nutation, equation (11) is simplified as

$$\det \begin{bmatrix} \lambda - j\cdot h/J_B & (h/J_B)^2 & 0 \\ 1 & \lambda + \dfrac{K_C}{h} + j\dfrac{h^2 + J_B K_P}{J_B h} & j\dfrac{K_P \omega_i}{h} \\ 0 & -1 & \lambda \end{bmatrix} = 0$$

$\cdots$(12)

where $h = J_{PW}\omega_{zw}$ gives the MBW rotor angular momentum; $K_P = K_{XY}K_{D\theta}K_{P\theta}$, the proportional control gain; $K_C = K_{XY} \times K_a\omega_{zw}$, the cross-feedback gain of the gimbal angle; and $\omega = \omega_{Bx} + j\cdot\omega_{By}, \theta = \theta_{xw} + j\cdot\theta_{yw}$, the complex representation. In addition, for convenience of derivation of an approximate characteristic root by the perturbation method, the state variable of the system is given as

$$\boldsymbol{x}' = \left[\omega - j\cdot(h/J_B)\theta \quad \theta \quad \int\theta dt\right]^T \tag{13}$$

The infinitesimal term of equation (12) is $(h/J_B)^2$; therefore, by 1st-order perturbation of this term, the approximate characteristic root of the satellite nutation λ_S is given as

$$\lambda_S = j\frac{h}{J_B} + \frac{h^4\left\{J_B(J_BK_P\omega_i + K_Ch) - j\cdot h(2h^2 + J_BK_P)\right\}}{J_B\left\{J_B^2(J_BK_P\omega_i + K_Ch)^2 + h^2(2h^2 + J_BK_P)^2\right\}} \tag{14}$$

The approximate root locus of the satellite nutation is shown in Figure 3; compared to the root locus in Figure 2(b), approximate characteristic root λ_S can completely describe the satellite nutation.

Because equation (12) is the complex representation, an imaginary number of the characteristic root based on equation (14) and Figure 3 is a positive number. This shows that the direction of the satellite motion and that of the MBW rotor angular momentum are the same.

Moreover, equation (14) shows that both the cross-feedback control K_C of the MBW rotor gimbal angle and the integrator ω_i in the ordinary magnetic bearing controller are the instability factors responsible for the satellite nutation.

For the stabilization of satellite nutation, the cross-feedback control of the satellite angular rate is assumed to be effective. With this cross-feedback control, the magnetic bearing control torque τ' is given as

$$\tau' = \tau - j\cdot K_S\omega \tag{15}$$

where K_S denotes the cross-feedback gain of the satellite angular rate and $\tau = \tau_x + j\cdot\tau_y$, the ordinary magnetic bearing control torque. The characteristic equation corresponding to equation (12) is given as

$$\det\begin{bmatrix} \lambda - j\cdot h/J_B & (h/J_B)^2 & 0 \\ \dfrac{h-K_S}{h} & \lambda + \dfrac{K_C}{h} + j\dfrac{h(h-K_S)+J_BK_P}{J_Bh} & j\dfrac{K_P\omega_i}{h} \\ 0 & -1 & \lambda \end{bmatrix} = 0 \quad \cdots(16)$$

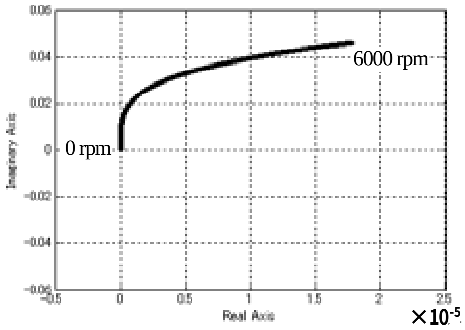


Fig. 3 Approximate root locus of satellite nutation with an ordinary magnetic bearing control

By 1st-order perturbation of the $(h/J_B)^2$ term of equation (16), an approximate characteristic root λ_S with cross-feedback control of the satellite angular rate is given as

$$\lambda_S = j\frac{h}{J_B} + \frac{h^3(h-K_S)\left\{J_B(J_BK_P\omega_i + K_Ch) - j\cdot h[h(2h-K_S)+J_BK_P]\right\}}{J_B\left\{J_B^2(J_BK_P\omega_i + K_Ch)^2 + h^2[h(2h-K_S)+J_BK_P]^2\right\}} \quad \cdots(17)$$

The cross-feedback gain K_S is larger than the MBW rotor angular momentum h; hence, the satellite nutation can be stabilized.

However, in general, the magnetic bearing controller cannot observe the satellite angular rate. Therefore, in practice, the observer estimates the satellite angular rate, and the cross-feedback controller of the estimated angular rate is applied.

The state equations of the approximated model corresponding to equation (12) are expressed as

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \omega\end{bmatrix} = \begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \omega\end{bmatrix} + \begin{bmatrix}j/h\\ -1/J_B\end{bmatrix}\tau \qquad \theta = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \omega\end{bmatrix} \tag{18}$$

Based on equation (18), a minimal-order observer of the satellite angular rate is given as[10]

$$\frac{dz}{dt} = Lz + L^2\theta - \left(\frac{1}{J_B} + j\frac{L}{h}\right)\tau \qquad \hat{\omega} = z + L\theta \tag{19}$$

where z denotes the state variable of the observer; L, the observer gain; and $\hat{\omega}$, the estimated satellite angular rate.

Because the natural angular frequency of the satellite is almost equal to h/J_B in equation (14), the observer gain can be given as

$$L = -\alpha\frac{h}{J_B} \quad (\alpha > 1) \tag{20}$$

By substituting equation (20) into equation (19), the estimated satellite angular rate is given as

$$\hat{\Omega} = \frac{-\alpha hs}{J_Bs + \alpha h}\Theta + \frac{-1 + j\cdot\alpha}{J_Bs + \alpha h}\mathrm{T} \tag{21}$$

where the capital letters denote Laplace transforms. However, the orders of the right-side terms in equation (21) are estimated as

$$\alpha hs\Theta \sim 10^{-1}\Theta \ll (-1+j\cdot\alpha)\mathrm{T} \sim 10^{2}\Theta \qquad (22)$$

Therefore, the first term on the right-hand side can be neglected, and the satellite angular rate can be estimated by magnetic bearing torques, as follows.

$$\hat{\Omega} \approx \frac{-1+j\cdot\alpha}{J_B s+\alpha h}\mathrm{T} \qquad (23)$$

Equation (23) shows that the satellite angular rate can be estimated by low-frequency components of the magnetic bearing torques transformed in the complex plane.

Based on the above discussion, a new magnetic bearing controller (motion of radial rotation) for the stabilization of the satellite nutation is proposed, as shown in Figure 4. In this controller, a broken line block is added to the ordinary magnetic bearing controller[5]; based on Figure 4, the magnetic torques are expressed as

$$\tau_x = F(\lambda)\tau_{x\_mag} - G(\lambda)(K_Y/K_X)\tau_{y\_mag} \qquad (24\text{-}1)$$

$$\tau_y = G(\lambda)(K_X/K_Y)\tau_{x\_mag} + F(\lambda)\tau_{y\_mag} \qquad (24\text{-}2)$$

where

$$F(\lambda) = \frac{(J_B\lambda+\alpha h)\left[J_B\lambda-\alpha(K_S-h)\right]}{\left[J_B\lambda-\alpha(K_S-h)\right]^2+K_S^2} \qquad (25\text{-}1)$$

$$G(\lambda) = \frac{K_S(J_B\lambda+\alpha h)}{\left[J_B\lambda-\alpha(K_S-h)\right]^2+K_S^2} \qquad (25\text{-}2)$$

In this case, equations (6),(7-1)～(7-3), and (24) are the equations of motion of the MBW-satellite system.

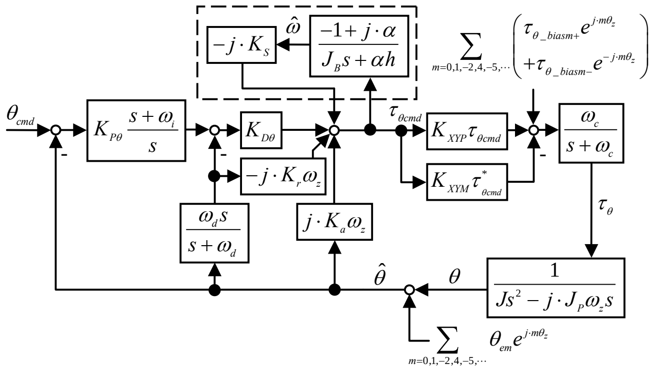


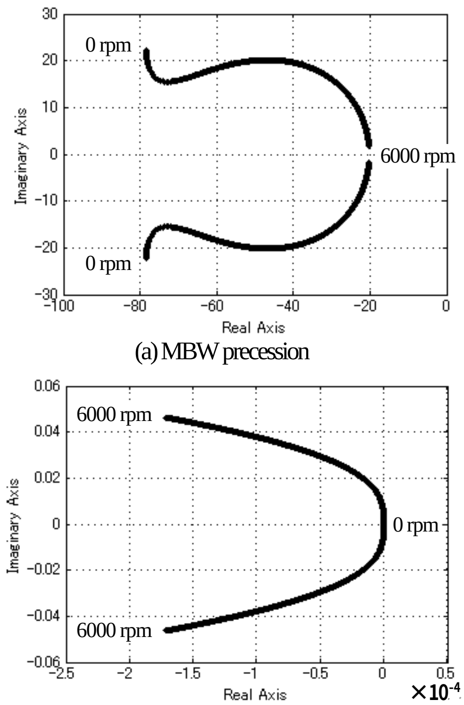
Fig. 4 Block diagram of the proposed magnetic bearing control

With regard to equations (6), (7-1)～(7-3), and (24), the numerical analysis of the root loci of the MBW precession and the satellite nutation are shown in Figure 5.

Here,

$$\begin{cases} \alpha = 3 \\ K_S = 500\,Nms/rad \\ J_B = (J_{Bxx}+J_{Byy})/2 \end{cases} \qquad (26)$$

Figure 5 shows that both the MBW motion and the satellite motion become stable by the proposed magnetic bearing controller.



(a) MBW precession

(b) Satellite nutation

Fig. 5 Root loci of the MBW-satellite system with the proposed magnetic bearing control

3・2 With The Disturbance Feedback Controller The stability of the system in which the disturbance feedback controller is added to the proposed magnetic bearing controller (Figure 4) is considered.

In this case, the magnetic bearing torques are given as equation (27); equations (6),(8-1)～(8-3), and (27) are the equations of motion of the MBW-satellite system.

$$\begin{aligned}\tau_x = {} & F(\lambda)\tau_{x\_mag} - G(\lambda)(K_Y/K_X)\tau_{y\_mag} + K_Y K_{D\theta}K_{P\theta}\times \\ & \sum_{u} K(\lambda)\begin{Bmatrix} J_W\left[\left(-F(\lambda)C_{\Theta u1}(\lambda)+G(\lambda)C_{\Theta u2}(\lambda)\right)\ddot{\theta}_{xw}+\left(G(\lambda)C_{\Theta u1}(\lambda)+F(\lambda)C_{\Theta u2}(\lambda)\right)\ddot{\theta}_{yw}\right] \\ -J_{PW}\omega_{zw}\left[-\left(-F(\lambda)C_{\Theta u1}(\lambda)+G(\lambda)C_{\Theta u2}(\lambda)\right)\dot{\theta}_{yw}+\left(G(\lambda)C_{\Theta u1}(\lambda)+F(\lambda)C_{\Theta u2}(\lambda)\right)\dot{\theta}_{xw}\right]\end{Bmatrix}\end{aligned} \qquad (27\text{-}1)$$

$$\begin{aligned}\tau_y = {} & G(\lambda)(K_X/K_Y)\tau_{x\_mag} + F(\lambda)\tau_{y\_mag} + K_X K_{D\theta}K_{P\theta}\times \\ & \sum_{u} K(\lambda)\begin{Bmatrix} J_W\left[\left(-F(\lambda)C_{\Theta u1}(\lambda)+G(\lambda)C_{\Theta u2}(\lambda)\right)\ddot{\theta}_{yw}-\left(G(\lambda)C_{\Theta u1}(\lambda)+F(\lambda)C_{\Theta u2}(\lambda)\right)\ddot{\theta}_{xw}\right] \\ -J_{PW}\omega_{zw}\left[\left(-F(\lambda)C_{\Theta u1}(\lambda)+G(\lambda)C_{\Theta u2}(\lambda)\right)\dot{\theta}_{xw}+\left(G(\lambda)C_{\Theta u1}(\lambda)+F(\lambda)C_{\Theta u2}(\lambda)\right)\dot{\theta}_{yw}\right]\end{Bmatrix}\end{aligned} \qquad (27\text{-}2)$$

Based on these equations of motion, the numerical analysis of the root loci of the MBW precession, satellite nutation, and $\pm 1N$ disturbance feedback controller are shown in Figure 6 with the same parameters in Figure 5.

A comparison between Figure 5 and Figure 6 shows that the motion properties of the MBW rotor precession and the satellite nutation are not affected by the disturbance feedback controller, and they are found to be stable. It is assumed that the third term on the right-hand side of equation (27), which is caused by the disturbance feedback controller, affects only the uN component（$\pm 1N$ component in Figure 6）.

On the other hand, the disturbance feedback controller, which is added to the proposed magnetic bearing controller, remains stable; therefore, the MBW-satellite system is stable with the disturbance feedback controller.

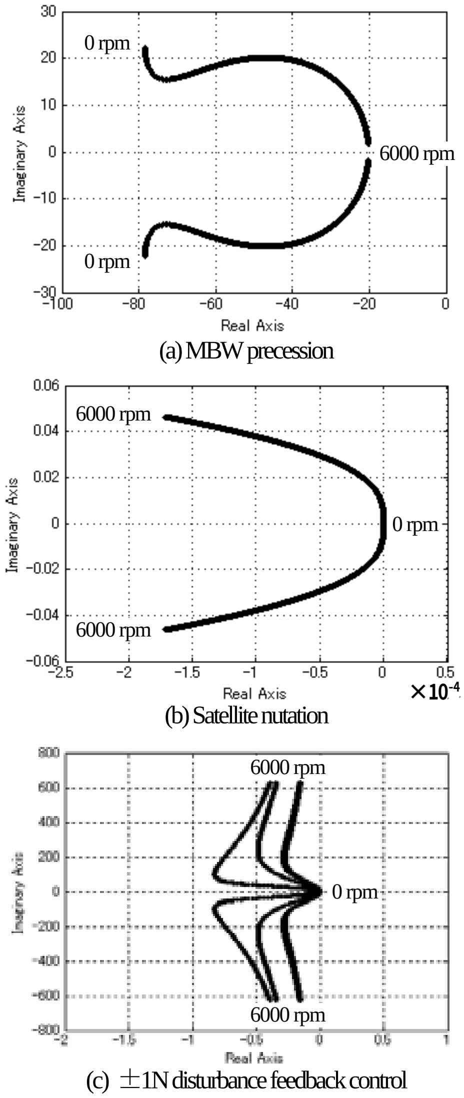


Fig. 6 Root loci of the MBW-satellite system with the proposed magnetic bearing control and disturbance feedback control

4. CONCLUSION

The dynamic interactions between the satellite and the MBW with inclined magnetic poles are considered. Based on the stability analysis of the MBW-satellite system, it is shown that both the cross-feedback control of the rotor gimgal angle and integrator in an ordinary magnetic bearing controller make the satellite nutation unstable.

In order to solve this problem, a new magnetic bearing controller is proposed. In this controller, the cross-feedback control of the satellite angular rate estimated by the minimal-order observer from the magnetic bearing control torques is added to the ordinary magnetic bearing controller. The proposed magnetic bearing controller can stabilize both the MBW and the satellite motion, including the satellite nutation; further, even if the disturbance feedback controller is added to the proposed magnetic bearing controller, the MBW-satellite system remains stable.

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APPENDIX INTERNAL STRUCTURE OF THE FREQUENCY TRANSFER FUNCTION

A. Magnetic Bearing Controller Assuming that the polar form of the variable in the stationary state is $a \cdot e^{\lambda t}$, the internal structures of the frequency transfer functions of the magnetic bearing controller are given as

$$W_c(\lambda) = \omega_c / (\lambda + \omega_c)$$
$$D(\lambda) = W_c(\lambda) \cdot \omega_d / (\lambda + \omega_d)$$
$$K(\lambda) = W_c(\lambda) \cdot (\lambda + \omega_i) / \lambda$$

where

ω_c ：break frequency of the 1st-order system affecting the electromagnetic force
ω_d ：break frequency of the defective differentiation
ω_i ：break frequency of the integrator

B. uN Disturbance Feedback Controller The internal structures of frequency transfer functions of the uN disturbance feedback controller are given as

$$C_{Ru}(\lambda) = \frac{K_{IR}\omega_{dis}\omega_{lpf} e^{j\cdot\alpha_u}}{(\lambda - j\cdot u\omega_{zw} + \omega_{dis})(\lambda - j\cdot u\omega_{zw} + \omega_{lpf})}$$
$$C_{Zu}(\lambda) = \frac{K_{IZ}\omega_{dis}\omega_{lpf} e^{j\cdot\beta_u}}{(\lambda - j\cdot u\omega_{zw} + \omega_{dis})(\lambda - j\cdot u\omega_{zw} + \omega_{lpf})}$$
$$C_{\Theta u}(\lambda) = \frac{-K_{I\theta}\omega_{dis}\omega_{lpf} e^{j\cdot\gamma_u}}{(\lambda - j\cdot u\omega_{zw} + \omega_{dis})(\lambda - j\cdot u\omega_{zw} + \omega_{lpf})}$$
$$C_{Ru1} = \frac{C_{Ru}(\lambda) + C_{Ru}(\lambda^*)^*}{2}, \quad C_{Ru2} = \frac{C_{Ru}(\lambda) - C_{Ru}(\lambda^*)^*}{2j}$$
$$C_{\Theta u1} = -\frac{C_{\Theta u}(\lambda) + C_{\Theta u}(\lambda^*)^*}{2}, \quad C_{\Theta u2} = -\frac{C_{\Theta u}(\lambda) - C_{\Theta u}(\lambda^*)^*}{2j}$$

where

subscript on the upper-right-side of the equations * ：conjugate complex
ω_{lpf} ：break frequency of the low-pass filter
ω_{dis} ：break frequency of the defective integrator
$K_{IR}, K_{IZ}, K_{I\theta}$ ：gain of the defective integrator
$\alpha_u, \beta_u, \gamma_u$ ：phase lead of the translation/rotation command