Instability analysis of HAYABUSA attitude control by using one momentum wheel

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abstract

Since HAYABUSA has lost two of three wheels, its attitude is currently stabilized by one-axis bias-momentum wheel. This method should lead to simple attitude keeping, but the motion showed nutation divergence in March 2007. This paper shows that the cause of the instable motion lies in the difference between the axis of wheel and of inertia, and a way to avoid this instability is proposed.

1ホイールによる"はやぶさ"姿勢制御の不安定性解析

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概要

地球へ向けて帰還中の「はやぶさ」は現在 3 台中 2 台のホイールが故障し、いわばバイアスモーメンタム方式で 1 軸姿勢制御を行っている。この方式ではシンプルな姿勢安定が実現されるはずだが、2007 年 3 月にニューテーションが発散する傾向を示した。本稿では、この現象がホイール回転軸と探査機慣性主軸の不一致によることを解析と数値シミュレーションによって示し、防止策を提案する。

1.Introduction

The asteroid explorer HAYABUSA was launched on May,9,2003. Along the way, HAYABUSA has lost two of three wheels, its attitude is currently stabilized by only one-axis bias-momentum wheel. Based on an ideal theory, such satellites are stable for 3 axes since one axis is stabilized through exchange of angular momentum with the wheel under a PD control, and the other two axes are stabilized by the gyroscopic stiffness derived from the momentum of the wheel.

HAYABUSA had been regarded as such an ideal satellite as well. However, with a little products of inertia caused by the fuel leakage, the wheel axes are different from the principal axes, and thus the dynamics differs from ideal ones. Because of this misaligment, a nutational divergence happened to HAYABUSA in March 2007.

This paper analyzes the cause of this phenomenon and discuss the influences of such misalignment upon a bias-momentum one wheel satellite like HAYABUSA.

2.Theory

2-1 Equation of motion

To analyze the dynamics of HAYABUSA, consider a typical bias-momentum satellite with only one momentum wheel. We define an inertial reference frame $[a_0]$ and a body-fixed reference frame $[a_1]$ and a principal axes-fixed reference frame $[a_2]$. A direction cosine vector of the wheel axis which expresses a difference with the principal axis is described by $\boldsymbol{\mu}^T = (\mu_1, \mu_2, \mu_3)$ in $[a_2]$(Fig.1). We suppose the wheel is pointed at around z-axis of $[a_2]$.

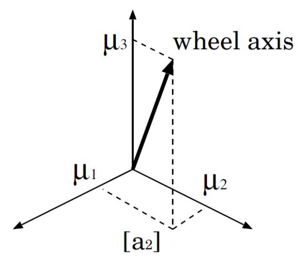

Fig.1: Tilt of the wheel
from the principal axes-fixed frame

Consider $A_{ij}, \boldsymbol{\omega}_{ij}, \boldsymbol{\phi}_{ij}$ as a direction cosine matrix, an angular velocity vector and an angular vector, respectively. They are described in $[a_i]$ relative to $[a_j]$. The whole angular momentum vector L in $[a_1]$ is written as

$$L = J_1\boldsymbol{\omega}_{10} + \boldsymbol{H}_w \tag{1}$$

where J_1 is the inertia matrix in $[a_1]$ and $\boldsymbol{H_w}$ is the angular momentum vector of the wheel. For simplicity, neglecting disturbance torque, Euler equation is written as

$$J_1\dot{\boldsymbol{\omega}}_{10} + \tilde{\boldsymbol{\omega}}_{10}(J_1\boldsymbol{\omega}_{10} + \boldsymbol{H_w}) = -\dot{\boldsymbol{H}}_w \tag{2}$$

where $\dot{\boldsymbol{H}}_w$ is the control torque and $\tilde{\boldsymbol{x}}$ is a matrix describing cross product $\boldsymbol{x}\times$.

We use *rotation vector* to express the attitude. Using $\boldsymbol{e}$ as a direction cosine of Euler's Eigenaxis and $0 \leq \theta \leq \pi$ as rotation angle about the axis, rotation vector $\boldsymbol{\phi}$ and its time derivative is defined as[1)]

$$\boldsymbol{\phi}^{\boldsymbol{T}} = (\phi_x, \phi_y, \phi_z) = \theta\boldsymbol{e}^T \tag{3}$$

$$\begin{aligned}\dot{\boldsymbol{\phi}} &= \boldsymbol{\omega} + \frac{1}{2}\tilde{\boldsymbol{\phi}}\boldsymbol{\omega} + \Theta\tilde{\boldsymbol{\phi}}^2\boldsymbol{\omega} \\ \Theta &= \frac{2\sin\frac{\theta}{2} - \theta\cos\frac{\theta}{2}}{2\theta^2\cos\frac{\theta}{2}} \\ \frac{1}{12} &\leq \Theta \leq \frac{1}{\pi^2} \quad (0 \leq \theta \leq \pi)\end{aligned} \tag{4}$$

Eqs.(2)(4) is written as

$$J_1\dot{\boldsymbol{\omega}}_{10} + \tilde{\boldsymbol{\omega}}_{10}(J_1\boldsymbol{\omega}_{10} + \boldsymbol{H}_w) = -\dot{\boldsymbol{H}}_w \tag{5}$$

$$\dot{\boldsymbol{\phi}}_{10} = \boldsymbol{\omega}_{10} + \frac{1}{2}\tilde{\boldsymbol{\phi}}_{10}\boldsymbol{\omega}_{10} + \Theta\tilde{\boldsymbol{\phi}}_{10}^2\boldsymbol{\omega}_{10} \tag{6}$$

in $[a_1]$.

In general, we can use below equations for transforms among reference frames.

$$\begin{aligned}A_{21}J_1A_{12} &= J_2 \\ A_{21}\boldsymbol{\omega}_{10} + \boldsymbol{\omega}_{21} &= \boldsymbol{\omega}_{20} \\ A_{21}\boldsymbol{\phi}_{10} + \boldsymbol{\phi}_{21} &= \boldsymbol{\phi}_{20} \\ A_{21}\tilde{\boldsymbol{x}}A_{12} &= \widetilde{A_{21}\boldsymbol{x}}\end{aligned} \tag{7}$$

Since the definitions about $[a_1]$, $[a_2]$ and $\boldsymbol{\mu}$, J_2 is a principal inertia matrix which consists of principal moments of inertia J_x, J_y, J_z and we can write

$$\begin{aligned}\boldsymbol{\omega}_{21} &= 0 \\ \dot{\boldsymbol{\phi}}_{21} &= 0\end{aligned} \tag{8}$$

$$\begin{aligned}A_{21}\boldsymbol{H}_w &= h\boldsymbol{\mu} \\ A_{21}\dot{\boldsymbol{H}}_w &= \dot{h}\boldsymbol{\mu}\end{aligned} \tag{9}$$

where h is the magnitude of $\boldsymbol{H}_w$.

By combining A_{21} and Eqs.(5)(6), and using above equations, we obtain

$$J_2\dot{\boldsymbol{\omega}}_{20} + \tilde{\boldsymbol{\omega}}_{20}(J_2\boldsymbol{\omega}_{20} + h\boldsymbol{\mu}) = -\dot{h}\boldsymbol{\mu} \tag{10}$$

$$\begin{aligned}\frac{d}{dt}(\boldsymbol{\phi}_{20} - \boldsymbol{\phi}_{21}) =& \boldsymbol{\omega}_{\mathbf{20}} + \frac{1}{2}(\boldsymbol{\phi}_{20} - \boldsymbol{\phi}_{21}) \times \boldsymbol{\omega}_{20} \\ &+ \Theta[(\boldsymbol{\phi}_{20} - \boldsymbol{\phi}_{21}) \times \{(\boldsymbol{\phi}_{20} - \boldsymbol{\phi}_{21}) \times \boldsymbol{\omega}_{20}\}]\end{aligned} \tag{11}$$

2-2 Control law

Consider a control law of the wheel rotation. The control law of HAYABUSA is PD control using angular velocity and angle about z-axis of $[a_1]$.

That is to say, the feedback values for the PD control are the z-component of the angular velocity and the angle observed in $[a_1]$, or ω_{10z}, ϕ_{10z}. We can write the magnitude of control torque $\dot{h} = |\dot{\boldsymbol{H}}_w|$ as

$$\dot{h} = k_1\omega_{10z} + k_2\phi_{10z} \tag{12}$$

where k_1, k_2 are the feedback gains.

We define a vector $\boldsymbol{\lambda}^T = (\lambda_1, \lambda_2, \lambda_3)$ such as

$$A_{12} = \begin{pmatrix} * & * & \boldsymbol{\lambda} \end{pmatrix} \tag{13}$$

This $\boldsymbol{\lambda}$ describes products of inertia in $[a_1]$. Using $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$, we can express the tilt of the wheel (or wheel array error) and the products of inertia independently(Fig.2). With above equations about transform of the references, $\dot{h}$ is written as

$$\dot{h} = k_1\boldsymbol{\lambda}^T\boldsymbol{\omega}_{20} + k_2\boldsymbol{\lambda}^T(\boldsymbol{\phi}_{20} - \boldsymbol{\phi}_{21}) \tag{14}$$

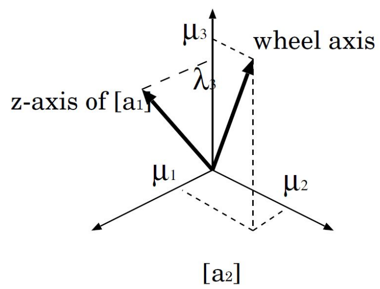

Fig.2: Description of the tilt and the products of inertia in the principal axes-fixed frame

2-3 Characteristic equation

To obtain the characteristic equation, we linearize Eqs.(10)(11) by restricting a range of motion properly.

Defining

$$\begin{aligned}J &= J_2 \\ \boldsymbol{\omega} &= \boldsymbol{\omega}_{20} \\ \boldsymbol{\phi} &= \boldsymbol{\phi}_{20} - \boldsymbol{\phi}_{21}\end{aligned} \tag{15}$$

we obtain the below system of differential equations.

$$J\dot{\boldsymbol{\omega}} + \tilde{\boldsymbol{\omega}}(J\boldsymbol{\omega} + h\boldsymbol{\mu}) = -k_1\boldsymbol{\mu}\boldsymbol{\lambda}^T\boldsymbol{\omega} - k_2\boldsymbol{\mu}\boldsymbol{\lambda}^T\boldsymbol{\phi} \tag{16}$$

$$\dot{\boldsymbol{\phi}} = \boldsymbol{\omega} + \frac{1}{2}\tilde{\boldsymbol{\phi}}\boldsymbol{\omega} + \Theta\tilde{\boldsymbol{\phi}}^2\boldsymbol{\omega} \tag{17}$$

Consider restricting the range of angular velocity, angle and the wheel rotation as near its zero, a certain angle $\boldsymbol{\phi}_0$ and bias angular momentum h_0, respectively. Namely we suppose

$$\begin{aligned} \boldsymbol{\omega} &= \delta\boldsymbol{\omega} \\ \boldsymbol{\phi} &= \boldsymbol{\phi}_0 + \delta\boldsymbol{\phi} \\ h &= h_0 + \delta h \end{aligned} \tag{18}$$

Substituting to Eqs.(16)(17) and neglecting δ^2 terms, we obtain the below system of linear differential equations about $\boldsymbol{\omega}, \boldsymbol{\phi}$.

$$J\dot{\boldsymbol{\omega}} + h_0\tilde{\boldsymbol{\omega}}\boldsymbol{\mu} = -k_1\boldsymbol{\mu}\boldsymbol{\lambda}^T\boldsymbol{\omega} - k_2\boldsymbol{\mu}\boldsymbol{\lambda}^T\boldsymbol{\phi} \tag{19}$$

$$\dot{\boldsymbol{\phi}} = \boldsymbol{\omega} + \frac{1}{2}\tilde{\boldsymbol{\phi}}_0\boldsymbol{\omega} + \Theta_0\tilde{\boldsymbol{\phi}}_0{}^2\boldsymbol{\omega} \tag{20}$$

Its matrix expression is

$$\frac{d}{dt}\begin{pmatrix} \boldsymbol{\omega} \\ \boldsymbol{\phi} \end{pmatrix} = \begin{pmatrix} -J^{-1}(k_1\boldsymbol{\mu}\boldsymbol{\lambda}^T - h\tilde{\boldsymbol{\mu}}) & -J^{-1}k_2\boldsymbol{\mu}\boldsymbol{\lambda}^T \\ E + \frac{1}{2}\tilde{\boldsymbol{\phi}}_0 + \Theta\tilde{\boldsymbol{\phi}}_0{}^2 & 0 \end{pmatrix} \tag{21}$$

Finally, we obtain the characteristic equation

$$det\begin{pmatrix} sE + J^{-1}(k_1\boldsymbol{\mu}\boldsymbol{\lambda}^T - h\tilde{\boldsymbol{\mu}}) & J^{-1}k_2\boldsymbol{\mu}\boldsymbol{\lambda}^T \\ -E - \frac{1}{2}\tilde{\boldsymbol{\phi}}_0 - \Theta\tilde{\boldsymbol{\phi}}_0{}^2 & sE \end{pmatrix} = 0 \tag{22}$$

where E is a unit matrix.

3.Stability analysis of the system

In this section, we analyze the stability of the system $(\boldsymbol{\omega}, \boldsymbol{\phi})$ by eigenvalue analysis using Eq.(22).

3-1 Conditions about stability

Though Eq.(22) is a sixth-order equation about s, it has double root at $s = 0$. Its eigenvectors are

$$\begin{aligned} x_1^T &= \begin{pmatrix} 0 & 0 & 0 & \lambda_3 & 0 & -\lambda_1 \end{pmatrix} \\ x_2^T &= \begin{pmatrix} 0 & 0 & 0 & \lambda_2 & -\lambda_1 & 0 \end{pmatrix} \end{aligned} \tag{23}$$

which means some deviation about angle remain and the double root do not affect the stability. We can consider the stability by analysis of the other 4 eigenvalues.

The characteristic equation is now fourth-order like

$$s^4 + a_1s^3 + a_2s^2 + a_3s + a_4 = 0 \tag{24}$$

Using *Hurwitz criterion*, we can say all real parts of the eigenvalues are negative if and only if[2)]

$$a_i > 0 \tag{25}$$

and

$$\Delta_3 = det\begin{pmatrix} a_1 & a_3 & 0 \\ 1 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{pmatrix} > 0 \tag{26}$$

3-2 Approximation by supposing magnitudes

It is quite hard to analyze the sign of a_i, Δ_3 because actual expanded forms are quite complex and large with some characteristic constants and the angle $\boldsymbol{\phi}_0$, where characteristic constants mean $J, \boldsymbol{\mu}, \boldsymbol{\lambda}, k_1, k_2$ and h_0. So we suppose the magnitudes of these constants and extract only dominant terms in a_i and Δ_3.

When the tilt of the wheel and the products of inertia is small, $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ is approximated as

$$\begin{aligned} \boldsymbol{\mu}^T &= \begin{pmatrix} \mu_1 & \mu_2 & 1 \end{pmatrix} \\ \boldsymbol{\lambda}^T &= \begin{pmatrix} \lambda_1 & \lambda_2 & 1 \end{pmatrix} \end{aligned} \tag{27}$$

where μ_1, μ_2, λ_1 and λ_2 is also approximated small. Besides we suppose $|\boldsymbol{\phi}_0| < 1[rad]$. Based on Θ at Eq.(4) as the basis of the magnitudes, we suppose

$$\begin{aligned} \mu_i &\approx \lambda_i \\ \Theta\phi_i &\approx \mu_i \\ \frac{h_0}{J_i} &\approx \Theta \\ k_1 &\approx J_i \\ k_2 &\approx h_0 \end{aligned} \tag{28}$$

and neglect terms which are less or equal to μ_i^3.

With these suppositions, a_i is always positive. Meanwhile, Δ_3 is not necessarily positive. It is written as first-order form about $\boldsymbol{\phi}_0^T = (\phi_{0x}, \phi_{0y}, \phi_{0z})$ like

$$\Delta_3 = A\phi_{0x} + B\phi_{0y} + C\phi_{0z} + D \tag{29}$$

where

$$
\begin{aligned}
A &= \frac{h_0 k_1 k_2}{2 J_y J_z}\Big[\alpha_{xz}\beta_2\mu_1 + \alpha_{yz}\frac{\beta_1}{J_x}\mu_2\Big] \\
B &= -\frac{h_0 k_1 k_2}{2 J_x J_z}\Big[\alpha_{yz}\beta_2\mu_2 + \alpha_{xz}\frac{\beta_1}{J_y}\mu_1\Big] \\
C &= -\frac{h_0 k_1 k_2}{2 J_z}\Big[\frac{\alpha_{xz}\beta_2}{J_y}\mu_1^2 + \frac{\alpha_{yz}\beta_2}{J_x}\mu_2^2 + \frac{h_0 k_1}{J_x J_y J_z}\alpha_{xy}\mu_1\mu_2\Big] \\
D &= \frac{k_1 h_0 \lambda_1}{J_z}\Big[\frac{h_0^3 k_1 \mu_1}{J_x^2 J_y^2}\alpha_{xz} + \Big(-\frac{h_0^2 k_1^2}{J_x^2 J_y J_z} + \frac{h_0^2 k_2}{J_x^2 J_y} - \frac{k_2^2}{J_x J_z}\Big)\alpha_{yz}\mu_2\Big] \\
&\quad + \frac{k_1 h_0 \lambda_2}{J_z}\Big[\frac{h_0^3 k_1 \mu_2}{J_x^2 J_y^2}\alpha_{yz} + \Big(\frac{h_0^2 k_1^2}{J_x J_y^2 J_z} - \frac{h_0^2 k_2}{J_x J_y^2} + \frac{k_2^2}{J_y J_z}\Big)\alpha_{xz}\mu_1\Big] \\
&\quad - \frac{h_0^2 \beta_1^2}{J_x J_y}\Big[\frac{\alpha_{xz}}{J_y}\mu_1^2 + \frac{\alpha_{yz}}{J_x}\mu_2^2\Big] + \frac{h_0 k_1 \alpha_{yx}\mu_1\mu_2}{J_z^2}\Big[\frac{J_z\beta_1^2}{J_x J_y} - k_2\beta_2\Big]
\end{aligned}
\tag{30}
$$

where

$$
\begin{aligned}
\alpha_{ij} &= \frac{1}{J_i} - \frac{1}{J_j} \\
\beta_1 &= \frac{h_0 k_1}{J_z} \\
\beta_2 &= \frac{k_2}{J_z} - \frac{h_0^2}{J_x J_y}
\end{aligned}
\tag{31}
$$

We can not determine the sign of Δ_3 unique, which means a boundary about the stability exists depending on the constants in Eq.(29). Some examples of the boundaries are shown at Fig.3. We describe them only on a plane (ϕ_{0x}, ϕ_{0y}) approximately, because C is smaller than A and B. The boundary varies its shape depending on the signs of A and B.

Based on above analysis, a system (or an attitude) of certain satellite with $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ may go into stable or unstable depending on its $\boldsymbol{\phi}_0$ and it may converge or diverge.

We can say that the cause of these phenomena is the composition of the feedback value. It includes x and y-angle (see Eq.(14)) which does not relate to control low if the satellite does not have some misalignment μ_i, λ_i, and this results changing the dynamics of the system. The biased feedback value about angle takes place these boundaries. So we can also say that the way of the definition of $\boldsymbol{\phi}_0$,or the definition of inertial frame varies the dynamics.

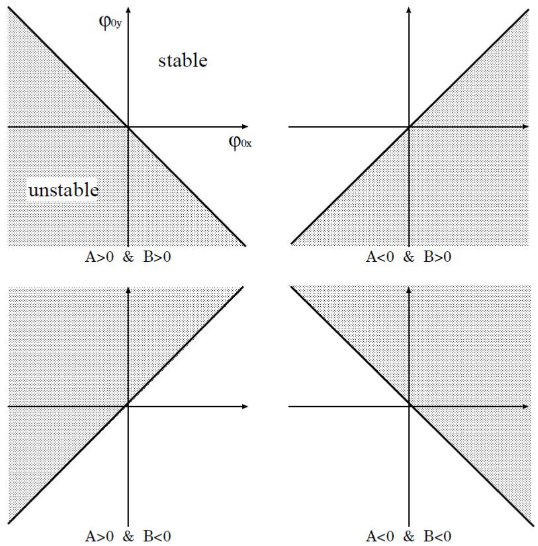

Fig.3: Examples of boundary about stability

4.Influence of the stability upon the motion

4-1 Analysis of the divergent motion of HAYABUSA

Based on the above stability analysis, we investigate the divergent motion of HAYABUSA shown in Section 1. When this phenomenon happened, HAYABUSA's characteristic constants were

$$
\begin{aligned}
J &= \begin{pmatrix} 352.4 & 0 & 0 \\ 0 & 268.2 & 0 \\ 0 & 0 & 428.3 \end{pmatrix} [kgm^2] \\
h_0 &= -2.90 [kgm^2/s] \\
k_1 &= 114 \\
k_2 &= 15.35 \\
\boldsymbol{\lambda}^T &= \begin{pmatrix} 0.0823 & -0.0100 & 0.9966 \end{pmatrix} \\
\boldsymbol{\mu}^T &= \begin{pmatrix} 0.0823 & -0.0100 & 0.9966 \end{pmatrix}
\end{aligned}
\tag{32}
$$

Here, $\boldsymbol{\lambda} = \boldsymbol{\mu}$ means the wheel axis is equal to the z-axis of $[a_1]$. And it happened after an attitude maneuver about the x-axis, its rotation vector was

$$
\boldsymbol{\phi}_0 = \begin{pmatrix} 0.393 & 0.021 & 0.000 \end{pmatrix} [rad] \tag{33}
$$

In this settings, Δ_3 in Eq.(29) is calculated as

$$
\begin{aligned}
\Delta_3 &= A\phi_{0x} + B\phi_{0y} + C\phi_{0z} + D \\
&= -3.34 \times 10^{-8}\phi_{0x} + 6.37 \times 10^{-9}\phi_{0y} \\
&\quad + 2.81 \times 10^{-9}\phi_{0z} + 1.58 \times 10^{-13} \\
&= -1.30 \times 10^{-8}
\end{aligned}
\tag{34}
$$

which means the boundary shape is like upper right of Fig.3, and the angle $(\phi_{0x}, \phi_{0y}) = (0.393, 0.021)$ is placed in the unstable region. We can say this is why the unstable nutation divergence occurred.

To compare a numerical calculation using Eqs.(16)(17)(32)(33) with HAYABUSA's angle record, we show the result of the calculation at Fig.4. The uppers of Fig.4 is the records, the lower is the calculation. These graph consist of time biased to zero when the divergence occurred for the transverses and z-angle described by rotation vector for verticals.

Compared with the record, the calculation corresponds well.

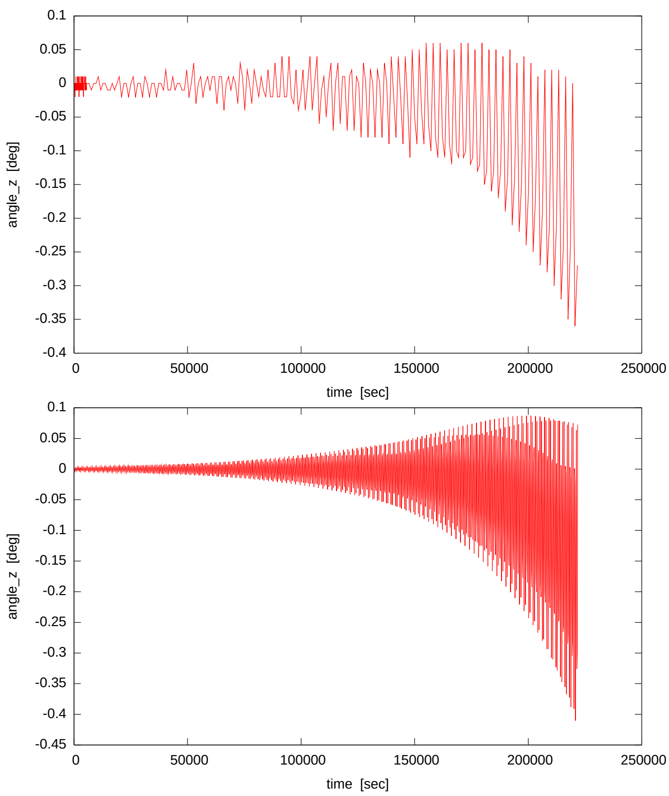

Fig.4: Comparison of HAYABUSA and simulation about angle z

4-2 Nutational damping using the stability

We showed unstable motion was caused because its angle was placed in the unstable region. In reverse, it may be possible to damp some nutational motion when we keep its angle in the stable region.

To show numerically, we place the angle as

$$\boldsymbol{\phi}_0 = \begin{pmatrix} -0.800 & 0.800 & 0.000 \end{pmatrix} [rad] \quad (35)$$

and run a numerical calculation with the constants Eq.(32) again. We saw that Eq.(32) draws its stability like upper right of Fig.3. And Eq.(35) instead of Eq.(33) means the angle is in the stable region. For example, the result is shown in Fig.5, where nutation damping can be observed. It can be said that one momentum wheel can act as a nutation damper if the attitude is kept in the stable region.

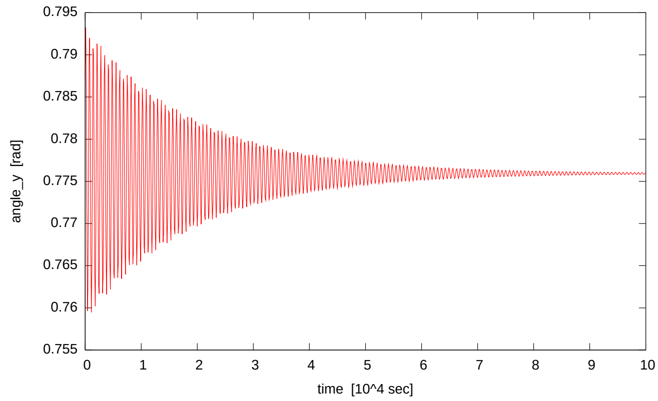

Fig.5: simulation in stable region

4-3 Stabilization strategy at usual operation of HAYABUSA

The unstable motion of HAYABUSA occurred after the large attitude maneuver. As is seen in section 4-2, the targeted angle $\boldsymbol{\phi}_0$ was in the unstable region and the attitude motion diverged. This is an unusual case, and at usual operation, a target angle is set to zero after every attitude maneuver, and the difference from zero is recognized as error angle to be feedbacked. This section shows that the unstable motion can be avoided around this angle ($\boldsymbol{\phi}_0 = 0$).

Consider the stability in this case. Eq.(29) is now written as

$$\Delta_3 = D \quad (36)$$

The stability depends on only the sign of D. $D = 1.58 \times 10^{-13}$ on HAYABUSA means stable at $\boldsymbol{\phi}_0 = 0$.

We consider the magnitude of angle d which expresses the distance between $\boldsymbol{\phi}_0 = 0$ and the boundary. It is written as

$$d = \frac{D}{\sqrt{A^2 + B^2 + C^2}} \quad (37)$$

Here, $d = 2.65 \times 10^{-4} [deg]$ for HAYABUSA, and this means HAYABUSA is always quite close to the unstable region. This means it is possible that the angle goes into the unstable region easily because the zero $\boldsymbol{\phi}_0 = 0$ is always near the boundary since generally $|D| \ll |A|, |B|$.

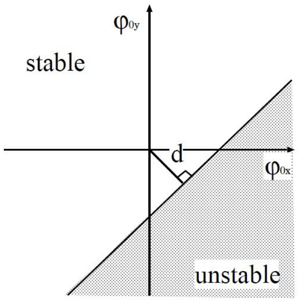

Fig.6: the distance between $\boldsymbol{\phi}_0 = 0$ and the boundary

However, the speed of divergence or convergence is quite slow around the boundary. For example, some numerical calculations show the amplitude of the nutational motion grows only less than $1[deg]$ in one week.

At usual operation, HAYABUSA is not be kept left for a week without attitude correction. Based on this assumption, the amplitude of the nutational motion does little increase or decrease around $\boldsymbol{\phi}_0 = 0$, and thus HAYABUSA will be stabilized enough if its angle is reset to $\boldsymbol{\phi}_0 = 0$ after every attitude maneuver. Here, resetting to $\boldsymbol{\phi}_0 = 0$ means resetting the inertial frame properly, we can do the maneuver easily. It is a guarantee that no problems happened when the angle had been around $\boldsymbol{\phi}_0 = 0$ at usual operation.

5.Use for nutational damper

In section4-2, we saw that taking an intentional angle $\boldsymbol{\phi}_0$ in stable region can damp nutational motion. We also saw the speed of the amplitude of the nutational motion decreasing is very slow around $\boldsymbol{\phi}_0 = 0$. Of course it is suitable as a nutational damper that the speed is fast, and this becomes possible by setting $\boldsymbol{\phi_0}$ at center of the stable region like Eq.(35). However, the result of the numerical calculation at section4-2 showed that the angle converges to around $\boldsymbol{\phi}^T = (-0.800, 0.776, 0.074)[rad]$ (only y-angle is shown at Fig.5) which has no concerns with the initial angle Eq.(35). The reason is that when the system converges, the control torque $\dot{h}$ also converges to zero, or $\boldsymbol{\phi}$ satisfies

$$\boldsymbol{\lambda}^T \boldsymbol{\phi} = 0 \tag{38}$$

(see Eq.(14)). This $\boldsymbol{\phi}$ shapes a plane in the space (ϕ_x, ϕ_y, ϕ_z) and it is possible to converge wherever on the plane. We can not control the convergent point, which takes place some unforeseen errors. It is not suitable for satellites when the onboard equipments require high attitude accuracy. We should consider better method of taking the intentional angle $\boldsymbol{\phi_0}$ to minimize the error.

6.Conclusion

To analyze the nutational divergence on HAYABUSA, we analytically derived the stability condition of a satellite which is controlled by one bias-momentum wheel. We showed that when the wheel is controlled under PD control using the attitude-angle and angular velocity about the wheel axis, the feedback value is biased according to products of inertia, and this results in the change of stability. Because feedbacked attitude-angle varies the stability, its nutational motion diverge or converge depending on its attitude-angle. We showed angle-boundaries about stability, and based on this assumption, we concluded the cause of the divergence on HAYABUSA is that its attitude-angle was placed in the unstable region.

To avoid this unstability, we considered two strategies. One was to set the attitude-angle zero, the other was to set the attitude-angle in the stable region. Setting the attitude-angle is done by changing the definition of the inertial frame. The former resulted eliminating the biased feedback value and the nutational motion does not diverge and converge. The latter, we could see the motion converges, and we proposed one of the way to use one momentum wheel as a nutatinal damper.

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