Effects of Velocity Increment Uncertainty in Optimal Trajectories for Deflecting Potentially Hazardous Asteroids

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This paper investigates the interplanetary trajectories associated with the impulsive deflection of a potentially hazardous asteroid (PHA) considering the uncertainty of velocity increment that spacecraft gives to the PHA at the time of collision. The velocity increment is assumed to have uncertainties of the magnitude and direction due to the estimation errors of the asteroid shape and mass distributions. The uncertainty is modeled using a convex model under the assumption that the magnitude and the direction are independently varied. The effect of the uncertainty is assessed by evaluating the worst (i.e. minimum) value of the closest approach distance between the PHA and the Earth. It is shown that the worst value of closest approach distance can be determined analytically without searching the whole convex hull. The optimal spacecraft trajectory is designed by maximizing the worst value of the closest approach distance in terms of the Earth departure date and the asteroid arrival date of the spacecraft under C3 (Earth departure energy) constraint. Through numerical example using a fictitious asteroid, the importance of considering the velocity increment uncertainty is demonstrated by comparing the optimum trajectory with the deterministic optimum trajectory. The uncertainty of the velocity increment direction is shown to have a significant effect on the deflection of the PHA.

地球接近小惑星の軌道変更ミッションにおける速度増分不確定性を考慮した軌道設計

本研究では，地球接近小惑星が地球に衝突することを回避する手段として，宇宙機を小惑星に衝突させて，小惑星の軌道を変更する手法を検討し，宇宙機が小惑星に与える速度増分の不確定性に着目した軌道設計法を提案する．まずは，小惑星と宇宙機が完全非弾性衝突すると仮定し，宇宙機の地球出発時期や小惑星到着時期など，小惑星の地球最接近距離におよぼす影響を明らかにする．しかし，実際には小惑星の形状や質量分布，表面構成が未知なことから，宇宙機が小惑星に衝突する時の衝突形態を確定することが困難である．そこで，宇宙機が小惑星に衝突する速度のうち，小惑星の軌道変更に有効な速度増分の大きさおよび方向が一定の範囲内で確率的に変動するものと仮定し，その最悪な場合でも小惑星の地球最接近距離が最大となるように，地球出発時期や小惑星到着時期などのシークエンスを設計する．地球最接近距離が最大となる宇宙機の地球出発日，および小惑星到着日がこれらの不確定性を考慮する場合としない場合とで異なることから，軌道設計において速度増分の変動を考慮することの重要性を明らかにする．

1. Introduction

Most of the asteroids exist between Mars and Jupiter orbits called the main-belt, while some of the asteroids approach and cross the Earth orbit. They are called potentially hazardous asteroids (PHAs), which have possibility of impacting the Earth. Many scientists have studied the PHA orbit propagation and quantified the hazards of the Earth impact[1] .

Several strategies indicate that, in order to prevent the asteroid collision with the Earth, deflection of the PHAs by kinetic energy is more effective than fragmentation of the asteroid body itself[2] . Park and the coauthors[3)–5] investigated the optimal magnitude and direction of velocity increment required for deflecting the asteroids, but the interplanetary trajectory of the spacecraft has not been discussed. Ivashkin[6] showed that a spacecraft with eight tons at launch could deflect the asteroid Toutatis, whose radius is less than 270 m. He also presented some models for spacecraft impact on the asteroid, which has a large difference in required momentum for deflecting the asteroids. Other papers[7)–9] studied the asteroid deflection missions using electric propulsion or solar sail technology. Izzo[10] studied the asteroid deflection mission for 99942 Apophis that approaches the Earth in 2029, which states that the asteroid deflection should be performed at least three years before the collision to the Earth using a multiple gravity assist.

A mission, named Don Quijote, proposed by ESA will investigate the effect of the impulsive asteroid deflection by using two spacecraft; one will impact on the asteroid and the other will investigate the asteroid to determine the collision point and then observe the change in the asteroid orbit by rendezvousing with the asteroid[11) . The achievements of this mission will be utilized for future asteroid deflection missions.

The asteroid deflection mission requires orbital parameters, mass, shapes and the surface construction. However, it is difficult to determine exact values due to limitation of measurement accuracy. Therefore, the asteroid deflection mission should consider uncertainties of the parameters. Vasile[12)] studied the optimal trajectory for asteroid deflection, including the uncertainty of orbital elements and mass of the asteroids. However, he didn't consider the uncertainties of the velocity increment that a spacecraft gives to the asteroid.

This paper investigates the optimal interplanetary trajectory for deflecting PHAs under the uncertainty of the velocity increment. The uncertainty of velocity increment is modeled as a convex model[13)] under the assumption that the uncertainties of the magnitude and the direction of the velocity increment are independently varied. The optimal trajectory is designed by maximizing the worst (minimum) value of the Earth closest approach distance in the uncertain range of the velocity increment. The effect of the velocity increment uncertainty is investigated through numerical examples.

Chapter 2 describes the outline of the asteroid deflection mission treated in the present study. Chapter 3 defines the orbit design problem in consideration with the uncertainty of velocity increment. Chapter 4 describes some examples of the optimal trajectories with and without the velocity increment uncertainty. Comparing these examples, the importance of considering the uncertainty of velocity increment is demonstrated.

2. Outline of Asteroid Deflection Mission

The spacecraft of this mission is assumed to follow a ballistic interplanetary trajectory after leaving the Earth, and to intercept an asteroid. Then, the spacecraft gives velocity increment $\Delta \boldsymbol{V}$ to the asteroid by perfectly inelastic impact model, where the mass of spacecraft m is assumed to be much smaller than that of the asteroid M ($m \ll M$) and negligible. The mission constrains the upper limit to $C3$, the square root of escape velocity at the Earth departure, due to the launcher performance. The spacecraft and the asteroid orbits are calculated by the patched-conic approximation in this study, assuming an unperturbed elliptic orbit around the Sun for the interplanetary trajectory of the spacecraft and asteroid, and hyperbolic orbit around the Earth in the vicinity of the Earth for the asteroid.

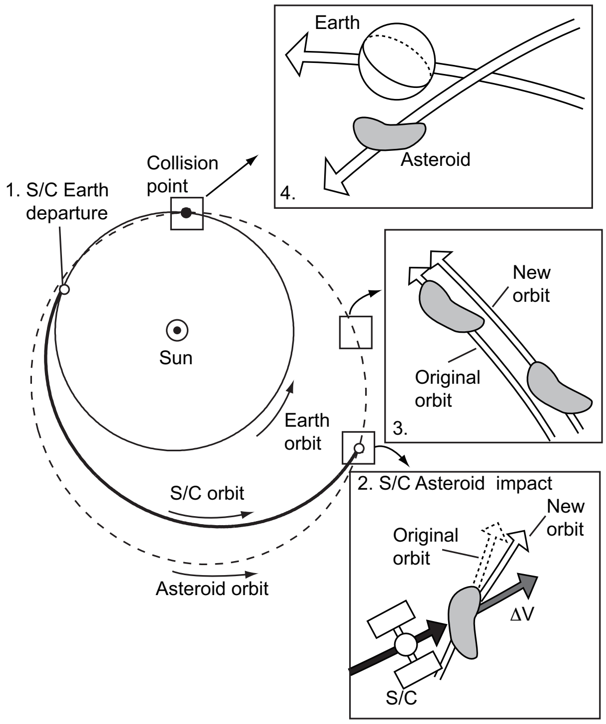

Figure 1. Outline of asteroid deflection mission

Figure 1 shows the strategy of asteroid deflection mission, which can be detailed as follows.

1. An asteroid is assumed to hit the center of the Earth.
2. The spacecraft heliocentric orbit is determined from the Earth departure time t_{dep} and asteroid arrival time t_{arr} using the Lambert method. $C3$ can be calculated from the Earth orbital velocity $\boldsymbol{V}_{ear}$ and spacecraft Earth departure velocity $\boldsymbol{V}_{dep}$ as follows:

   $$C3 = |\boldsymbol{V}_{dep} - \boldsymbol{V}_{ear}|^2 \tag{1}$$

   If multiple-revolution trajectories are obtained, the trajectory with the smallest value of $C3$ is adopted.
3. The spacecraft impacts on the asteroid with the asteroid arrival velocity $\boldsymbol{V}_{arr}$. Then the velocity increment $\Delta \boldsymbol{V}$ added to the asteroid can be approximated as follows:

   $$\Delta \boldsymbol{V} = \frac{m}{M+m} \approx \frac{m}{M}\boldsymbol{V}_{arr} \tag{2}$$

4. The velocity increment $\Delta \boldsymbol{V}$ makes the asteroid orbit changed, and hence the asteroid orbital period is increased as shown in Figure 1.
5. The asteroid flies by the Earth delayed from the original orbit, and the collision with the Earth is avoided. Figure 2 shows the hyperbolic trajectory of the asteroid approaching near the Earth, and

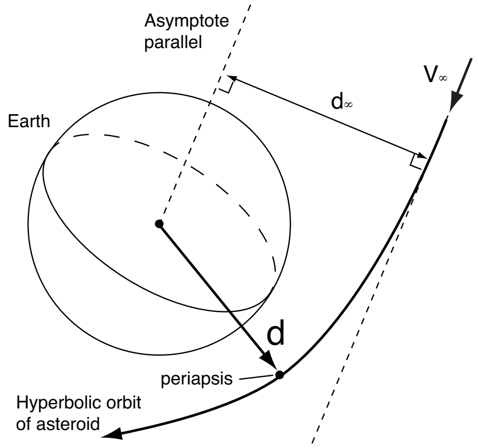

Figure 2. Hyperbolic trajectories of the asteroid in avoiding the collision to the Earth

the Earth closest approach distance d can be obtained from the following equation.

$$d = \frac{\mu_e}{|\boldsymbol{V}_\infty|}\left(\sqrt{1+\frac{|\boldsymbol{V}_\infty|^4 d_\infty^2}{\mu_e^2}}-1\right) \tag{3}$$

where μ_e is a gravity constant of the Earth, $\boldsymbol{V}_\infty$ is the asteroid relative velocity with respect to the Earth, and d_∞ is the distance from the Earth center to the asteroid incoming asymptote.

3. Orbit Design Problem under Uncertainties

3.1. Uncertainty model of velocity increment

It is difficult to estimate the velocity increment that the spacecraft gives to the PHA at the time of collision exactly because shape, surface material property and mass distribution of the PHA are not known exactly. The velocity increment is considered to be varied from the case of perfectly inelastic collision, that is, the magnitude and the direction are considered to have uncertainties. In this research, the quantity of the magnitude and the direction change of the velocity increment are modeled through independent uncertainty parameters.

The uncertainty parameter of the velocity increment magnitude is denoted as α that represents the ratio between the effective momentum and the momentum in the case of perfectly inelastic collision. The range of the magnitude uncertainty parameter α is determined from the average and the standard deviation of $\boldsymbol{\Delta V}$ magnitude ($\mu_\alpha,\ \sigma_\alpha$).

The uncertainty parameters of the direction change of the velocity increment are defined as $d\theta$ and $d\phi$ that describe the changes of vertical and parallel component of the velocity increment direction as shown in Figure 3. The velocity increment under the uncertainty, $\boldsymbol{\Delta V}'$, is

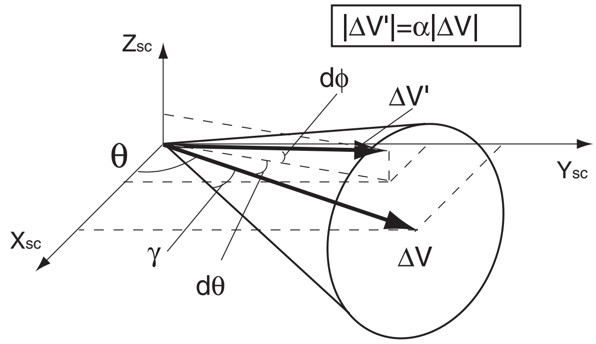

Figure 3. $\boldsymbol{\Delta V}$ uncertainty model

then described as follows:

$$\boldsymbol{\Delta V}' = \alpha|\boldsymbol{\Delta V}|\begin{pmatrix}\cos d\phi\cos(\theta+d\theta)\\ \cos d\phi\sin(\theta+d\theta)\\ \sin d\phi\end{pmatrix} \tag{4}$$

where θ is the angle between X_{sc} axis of the spacecraft-center reference frame and $\boldsymbol{\Delta V}$ as shown in Figure 3. Considering the impact of the spacecraft with the asteroid, uncertainty parameters of $\boldsymbol{\Delta V}$ direction $d\theta$ and $d\phi$ are dependent on each other. It is natural to assume $\boldsymbol{\Delta V}$ spreading as a conic. Therefore, the following relationship is assumed:

$$d\theta^2 + d\phi^2 \le \gamma^2 \tag{5}$$

where γ is the upper limit of the uncertainty range that is determined from the standard deviation σ_γ. Therefore, the uncertainty range of the velocity increment ($\alpha,\ d\theta,\ d\phi$) is modeled by a convex hull as shown in Figure 4. The effect of considering uncertainty of the velocity increment is evaluated by the worst value d' corresponding to the minimum closest approach distance between the Earth and the asteroid. The worst value for the prescribed interplanetary trajectory determined from the Earth departure time and the asteroid arrival time is evaluated by the following optimization problem:

$$\begin{aligned}\text{Minimize}:\ & d' = d\ (\alpha, d\theta, d\phi) \\ \text{subject to}:\ & \mu_\alpha - 3\sigma_\alpha \le \alpha \le \mu_\alpha + 3\sigma_\alpha \\ & d\theta^2 + d\phi^2 \le \gamma^2\end{aligned} \tag{6}$$

If d' is represented as a convex function in term of the uncertainty parameters, the minimum value of d' lies along the boundary of the uncertainty convex hull. That is called a convex model[13] that the minimum value is easier to find. In this study, the worst value is determined analytically without searching in the convex hull[14] .

3.2. Optimal trajectory design problem under uncertainty

In the present spacecraft trajectory optimization, the Earth departure time t_{dep} and asteroid arrival time t_{arr}

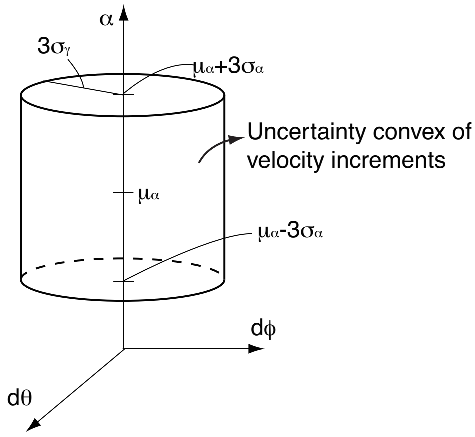

Figure 4. Convex model of uncertainty of the velocity increment

are chosen as design variables in order to maximize the worst value of the Earth closest approach distance d' under $C3$ constraint, that imposes an upper limit $C3^u$ for the mission feasibility. Then, the optimal trajectory problem is formulated as follows:

$$\begin{aligned} \text{Maximize}: \ & d' \ (t_{dep}, t_{arr}) \\ \text{subject to}: \ & C3 \leq C3^u \\ & t^l_{dep} \leq t_{dep} \leq t^u_{dep} \\ & t^l_{arr} \leq t_{arr} \leq t^u_{arr} \end{aligned} \quad (7)$$

where, t^l_{dep}, t^u_{dep}, t^l_{arr} and t^u_{arr} are the lower and upper limits of the Earth departure date and the asteroid arrival date, respectively. Considering the problems (6), the optimal trajectory problem with uncertainty is formulated as max-min problem.

4. Results and Discussions

4.1. Mission Conditions

The asteroid is assumed to impact the Earth on January 1, 2060 at the ascending node. The asteroid shape is assumed to be a perfect sphere with 150 m diameter and 5.3 Mton mass, whose average density is 3.0 g/cm^3. This orbit is determined referring to 3361 Orpheus that has average orbital parameters for PHAs. The orbital elements of the fictitious asteroid for the present study and Orpheus are listed in Table 1. The upper limit of $C3$ is determined as 50 km^2/s^2 corresponding to the capability of the average of HII-A and M-V launchers. The mass of spacecraft is assumed to be 1.0 ton when it impacts with the asteroid. The average and standard deviation of uncertainty parameters are listed in Table 2.

4.2. Optimal trajectories without uncertainty

At first, the optimal spacecraft trajectory to maximize the closest approach distance without uncertainty of $\Delta \boldsymbol{V}$ (asteroid velocity increment at spacecraft impact)

Table 1. Orbital elements of asteroids

	Fictitious asteroid	Orpheus
a [AU]	1.283	1.209
e	0.3226	0.3226
i [deg]	2.683	2.683
Ω [deg]	100.4	189.7
ω [deg]	301.6	301.6
ν_0 [deg]	58.42	346.0
Epoch	1/1/2060	9/13/2000

Table 2. Assumptions of fictitious asteroid, spacecraft, $\Delta \boldsymbol{V}$

Earth collision date of the asteroid	1/1/2060
Asteroid diameter [m]	150
Asteroid density [g/cm^3]	3.00
Asteroid mass [ton]	5.30 $\times 10^6$
Spacecraft mass [ton]	1.00
C_3 limit at Earth [km^2/s^2]	50.0
Average of $\Delta \boldsymbol{V}$ magnitude μ_α	0.60
Standard deviation of $\Delta \boldsymbol{V}$ magnitude σ_α	0.10
Average of $\Delta \boldsymbol{V}$ direction μ_α [deg]	0.00
Standard deviation of $\Delta \boldsymbol{V}$ direction σ_γ [deg]	10.0

is obtained to investigate the effect of the spacecraft arrival time at the asteroid on the asteroid deflection.

We treat the "nominal $\Delta \boldsymbol{V}$ " case where the uncertain parameter is set to $(\alpha, \gamma) = (\mu_\alpha, 0)$. Under this deterministic condition, the closest approach distance is known to change periodically with respect to the asteroid arrival date of the spacecraft, and that local maximum value of the approach distance is reduced as the asteroid arrival date is delayed.[3)–5)] The closest approach distance is evaluated assuming the range of the Earth departure date [1/1/2008, 1/1/2012] and the asteroid arrival date [1/1/2010, 1/1/2014], where the flight time is smaller than six years. The closest approach distance distribution is illustrated in Figure 5, and the asteroid periapsis dates in the range, 6/22/2010, 12/6/2011 and 5/19/2013 are indicated by dashed lines. It is found that the local maximum values of the closest approach distance are taken at these asteroid periapsis dates. This result is identical to Park and coauthors' research.[3)–5)] The global maximum closest approach distance 22492.06 km is obtained at $(t_{dep}, t_{arr}) = $ (7/16/2010, 12/6/2011) as a deterministic optimum design. The optimum trajectory is illustrated in Figure 6.

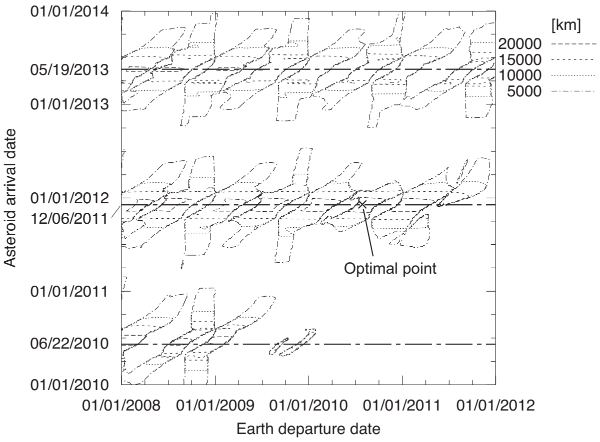

Figure 5. Distribution of closest approach distance

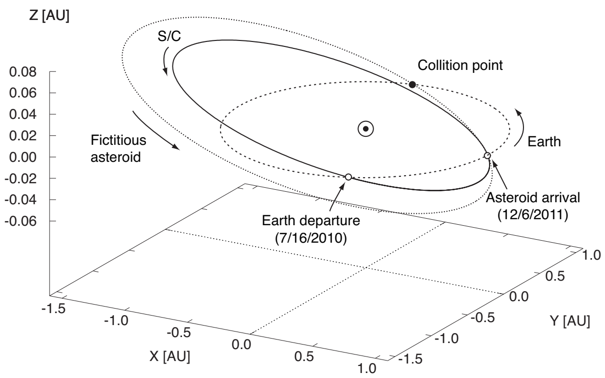

Figure 6. Optimal trajectory for nominal $\Delta\boldsymbol{V}$ case

4.3. Optimal trajectories under uncertainty

Optimal interplanetary trajectories are now designed considering the velocity increment uncertainties ("worst $\Delta\boldsymbol{V}$" case), and the effects of the uncertainties of the velocity increment on the closest approach distance are investigated. From the deterministic results illustrated in section 4.2, the range of the Earth departure date and the asteroid date are selected as [1/1/2008, 3/30/2008] and [6/1/2010, 8/29/2010].

Figures 7 and 8 illustrate the distribution of closest approach distances with and without uncertainties. The cross points (sign "+") in the figures indicate the optimal Earth departure date and asteroid arrival date of the spacecraft. Figure 9 illustrates the optimal trajectories corresponding to the cross points with and without uncertainties. These figures indicate that the distribution of the closest approach distance with uncertainty is different from that without uncertainty.

The effect of the uncertainty of the $\Delta\boldsymbol{V}$ direction is discussed here. The components of the asteroid velocity direction on the spacecraft impact with and without uncertainties are described as follows.

$$\Delta\bar{V}_{ast} = |\Delta\boldsymbol{V'}|\cos\psi \tag{8}$$

$$\Delta V'_{ast} = |\Delta\boldsymbol{V'}|\cos(\psi + 3\sigma_\gamma) \tag{9}$$

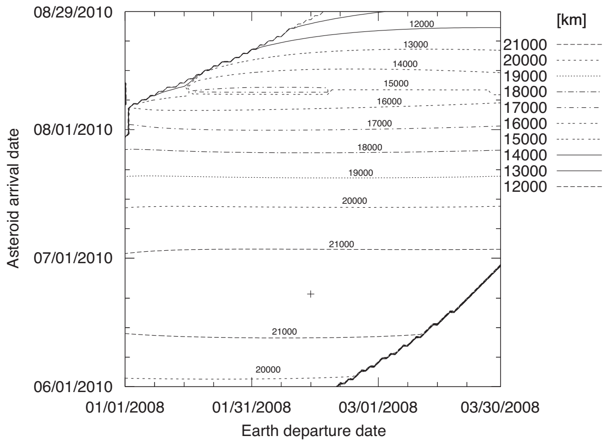

Figure 7. Closest approach distance distribution for nominal condition

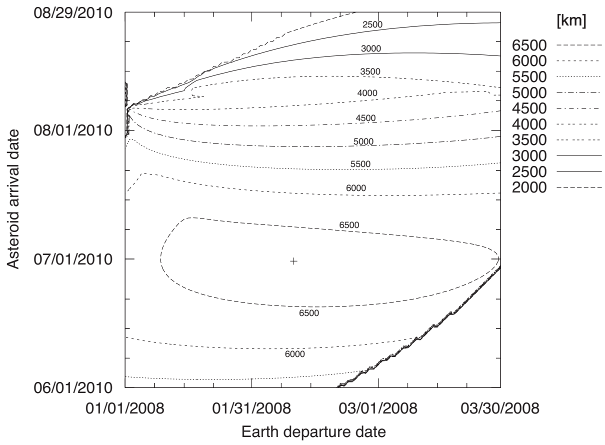

Figure 8. Closest approach distance distribution for worst condition

where $\Delta\bar{V}_{ast}$ is the component of the asteroid velocity direction of the nominal $\Delta\boldsymbol{V}$ (deterministic) case, $\Delta V'_{ast}$ is that of the worst $\Delta\boldsymbol{V}$ (uncertain) case, and ψ is an angle between the spacecraft velocity and the asteroid velocity vectors. If the $|\Delta\boldsymbol{V'}|$ takes a constant value, the ratio between $\Delta\bar{V}_{ast}$ and $\Delta V'_{ast}$ indicates the uncertainty effect of $\Delta\boldsymbol{V}$ direction. Hence, the effects of the uncertainty to the $\Delta\boldsymbol{V}$ and the closest approach distance can be described to ψ.

To investigate the effect of the optimal trajectories due to the standard deviation of $\Delta\boldsymbol{V}$ direction uncertainty on the optimal trajectories, the optimal departure date and arrival date are calculated in the range of σ_γ = [0, 10] deg. The optimal departure dates and arrival dates of corresponding uncertainty range, σ_γ = [0, 10] deg is illustrated in Figure 10 that also shows the distribution of an angle between the spacecraft and the asteroid velocities, ψ. It is found that the optimal departure and arrival dates are shifted to the smaller angle of ψ, as the standard deviation of the velocity increment direction is larger. Figure 11 illustrates the ratio of the closest approach distance with and without uncertainties. This distribution is similar to that of

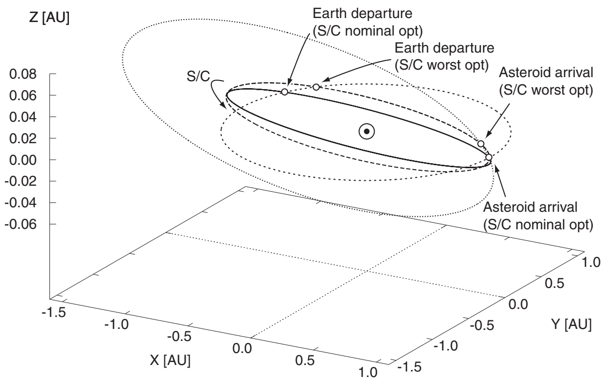

Figure 9. Optimal trajectories under ΔV uncertainty

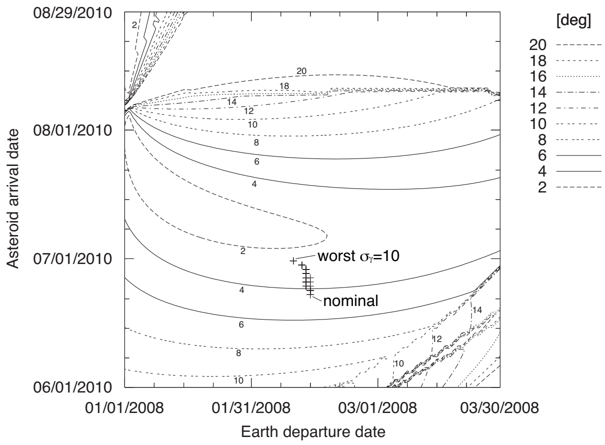

Figure 10. ψ distribution

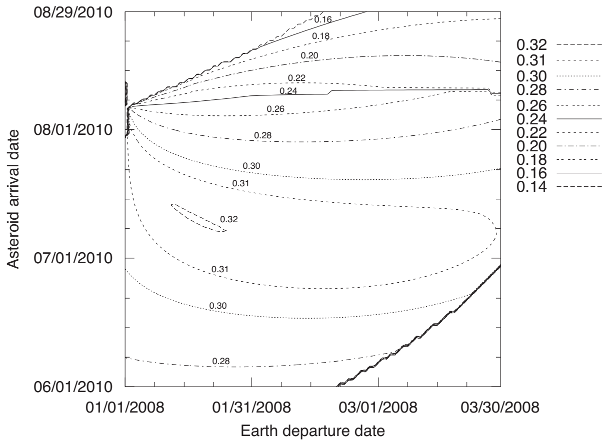

Figure 11. Distribution of closest approach distance ratio

ψ in comparison with Figure 10. Therefore, the effect of $\boldsymbol{\Delta V}$ uncertainty decreases, as the angle between the spacecraft velocity and the asteroid velocity vector ψ decreases. That is, the optimum trajectory under $\boldsymbol{\Delta V}$ uncertainty is selected as that gives a small angle between the asteroid and spacecraft velocities at the collision.

5. Conclusion

This paper describes the optimal interplanetary trajectories for deflecting asteroids, including the uncertainty of velocity increment of the asteroid due to the spacecraft impact into account. The uncertainty of velocity increment is modeled using a convex model, where its magnitude and direction are independently varied. Comparing the optimal trajectories between those with and without the velocity increment uncertainties, the importance of including the uncertainty for the asteroid deflection mission design is demonstrated. It is found that the optimal trajectories of the spacecraft are sensitive to the uncertainty of the velocity increment direction. The effect of velocity increment direction uncertainty depends on the angle between the velocity vector between the spacecraft and the asteroid.

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