Preliminary Study on Satellite Formation Flying Control Around L_2 Using Solar Sail Propulsion

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Here at L2, introduced a satellite formation flying consists of two satellites powered by solar sail, the Mirror Spacecraft (MSC) and the Detector Spacecraft (DSC). The two satellites are separated such that the operational detector is at the focus of the mirror. Due to the instability of the collinear Lagrangian points, the desired distance between the two satellites need to be maintained. This paper addresses the control design scheme for formation keeping control. The equation of motion is derived in Hill's frame and inertial frame and then control law formulated mathematically. Halo and Lissajous orbit are considered as options for reference orbit.

Key Words: Formation Flying Control, Lagrangian Libration Point, Solar Sail Propulsion, Restricted Three Body Problem

1. Introduction

Solar sail offers advantage over conventional propulsions systems that missions aren't constrained by ΔV available from stored reaction mass. This advantage enables many novel interesting high-energy space mission concepts.

Higher angular resolution in astronomical images requires increasing apertures of telescope or increasing baselines of interferometers. The mass of the support structure of the telescopes increases accordingly, and propellant for lunch and navigation of long baseline telescopes is about to exceed technical and financial boundaries. Formation flying is a revolutionary idea overcome the mass constraint which combine satellites in autonomous formation flight to behave just like a rigid body[1].

The advantages of a mission near L_2, as Beichman [2] point out are, the simplicity and the inexpensive inserting into Lissajous orbit from Earth. Moreover L_2 is a place with a constant cold temperature with half of the entire celestial sphere available at all times, which suitable for missions with heat sensitive instruments.

More on transfer from Earth to L_2, it gives benefits such as, less energy to achieve required which gives more mass to be delivered, its distance relatively close to Earth compare others Lagrange Points, availability of multiple options due to its nature of flexibility and forgiveness.

Typical missions like XEUS (X-ray evolving-Universe Spectroscopy) mission [3], which consists of two satellites is main motivation of this research. The future of The Gamma Ray Astronomy [4] on the development of significantly higher sensitivity instrumentation, will be solved potentially by formation flying.

Here is proposed, two satellites in a formation flying placed around L_2 point of Sun-Earth system as Mirror Spacecraft and Detector Spacecraft. This formation flying powered by solar sail and analyzed in a Restricted Three Body Problem domain.

2. Solar Sails

Solar consist of large area gossamer structures with a reflective coating which intercepts the solar photon flux. A satellites receives impart momentum on the sail and the reflection results in a reaction force, hence providing double the force which would be imparted to absorbing surface.

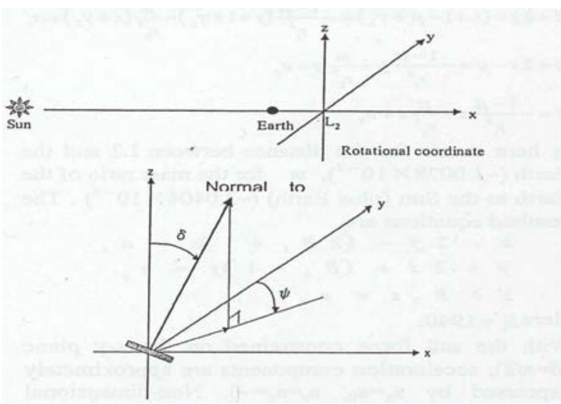

Fig. 1 Solar sail attitude angles

The sail accelerations can be expressed as follows [5],

$$\boldsymbol{\kappa} = \eta \frac{\beta \mu_s}{R_s^2} \cos^2 \delta \cos^2 \psi \mathbf{n} \tag{1}$$

Where η the sail reflectivity, R_s is the distance from the Sun, δ is the pitch angle of the sail normal vector **n** to the Sun-line and Ψ is th yaw angle of the sail of the sail normal vector to the Sun-line and β is the solar sail lightness parameter defined as the ratio of solar radiation pressure to gravitational attraction:

$$\beta = \frac{L_s}{2\pi c \mu_s \sigma} \tag{2}$$

where solar luminosity L_s=3.86x10^{26}W, c is the speed of light, μ_s is the solar gravitational parameter and σ is the ratio of solar sail mass to surface area known as the loading parameter.

3. Circular Restricted Three Body Problem

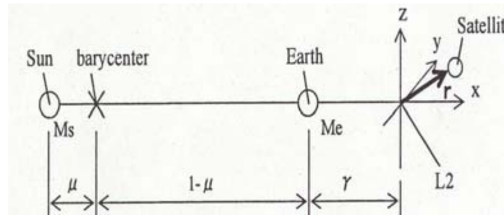

Fig.2 Cartesian coordinate system

Congruently with Farquhar work on translunar libration point [6], the nonlinear equations of motion around L_2 in non dimensional form are expressed as below,

$$\begin{aligned}
&\ddot{x} - 2\dot{y} - (x + 1 - \mu + \gamma_L) = -\frac{1-\mu}{r_1^3}(x + 1 + \gamma_L) - \frac{\mu}{r_2^3}(x + \gamma_L) + a_x \\
&\ddot{y} + 2\dot{x} - y = -\frac{1-\mu}{r_1^3} y - \frac{\mu}{r_2^3} y + a_y \\
&\ddot{z} = -\frac{1-\mu}{r_1^3} z - \frac{\mu}{r_2^3} z + a_z
\end{aligned} \tag{3}$$

The linearized form of the above system of equations [7] are

$$\begin{aligned}
&\ddot{x} - 2\dot{y} - (2B_L + 1)x = a_x \\
&\ddot{y} + 2\dot{x} + (B_L - 1)y = a_y \\
&\ddot{z} + B_L z = a_z
\end{aligned} \tag{4}$$

Where,

$$\mathrm{B_L} = \left\{ \frac{1-\mu}{(1+\gamma_L)^3} + \frac{\mu}{\gamma_L{}^3} \right\}$$

4. Control Strategy Fixed Formation

In the formation flying, the Mirror satellite is put the leader and the Detector Satellite as the follower. The relative motion of the follower respected to the leader are shown below

$$\ddot{\mathbf{r}}_l - \ddot{\mathbf{r}}_f = \ddot{\boldsymbol{\rho}} \tag{5}$$

Hence,

$$\begin{aligned}
&\ddot{\rho}_x - 2\dot{\rho}_y - (2B_L + 1)\rho_x = a_x \\
&\ddot{\rho}_y + 2\dot{\rho}_y + (B_L - 1)\rho_y = a_y \\
&\ddot{\rho}_y + B_L \rho_z = a_z
\end{aligned} \tag{6}$$

For the formation frozen in rotating frame will require that

$$\begin{aligned}
&\ddot{\rho}_x = \dot{\rho}_x = 0 \\
&\ddot{\rho}_y = \dot{\rho}_y = 0 \\
&\ddot{\rho}_z = 0
\end{aligned} \tag{7}$$

Consequently the accelerations needed are

$$a_x = -(2B_L + 1)\rho_x$$
$$a_y = (B_L - 1)\rho_y \quad (8)$$
$$a_z = B_L \rho_z$$

analytically, for the formation to be fixed in inertial frame the accelerations are

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}_I = \begin{bmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -(2B_L+1)\rho_x \\ (B_L-1)\rho_y \\ B_L \rho_z \end{bmatrix} \quad (9)$$

5. Study Cases

Here three type of study cases are examined to derive the control strategy mathematically. The basic assumptions on these cases are that the formation is placed nearby L_2 point and the shadow area around L_2 is not accounted

5.1. The leader has no control capability and the follower has solar sail with two gimbal

The relative position in rotating frame of the follower subsequently can be written as

$$\begin{bmatrix} \rho_x \\ \rho_y \\ \rho_z \end{bmatrix} = \begin{bmatrix} \frac{1}{2B_L-1} & 0 & 0 \\ 0 & \frac{1}{B_L-1} & 0 \\ 0 & 0 & \frac{1}{B_L} \end{bmatrix} \begin{bmatrix} A\sin\delta\sin\psi \\ A\sin\delta\cos\psi \\ A\cos\delta \end{bmatrix} \quad (10)$$

The absolute distance between the leader and the follower is

$$\rho(\delta,\psi) = \sqrt{\rho_x{}^2 + \rho_y{}^2 + \rho_z{}^2} \quad (11)$$

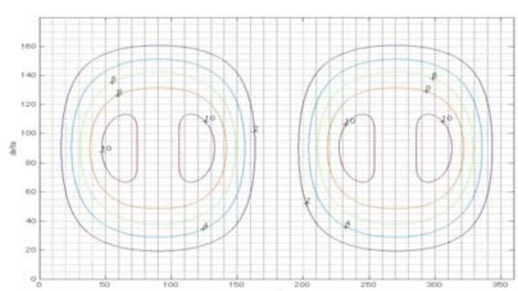

Fig. 3 The contour for the Euclidian distance between spacecraft

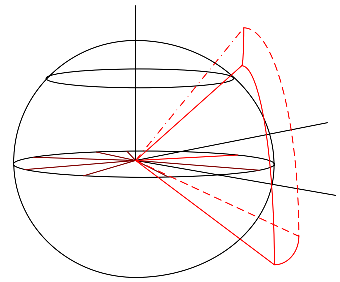

Fig.4 Accessible Space

From the contour graph we can found the accessible space is the limited to

$$\begin{aligned} &\Delta\psi_1 \approx 48^\circ - 72^\circ \\ &\Delta\psi_2 \approx 108^\circ - 132^\circ \qquad \Delta\delta \approx 66^\circ - 108^\circ \\ &\Delta\psi_3 \approx 228^\circ - 252^\circ \\ &\Delta\psi_4 = 288^\circ - 312^\circ \end{aligned} \quad (12)$$

In this case the acceleration of the solar sail approached [8,9] by

$$A \approx a_0 \sin^2\delta \sin^2(\phi + \psi) \quad \text{with } \phi \approx 0 \quad (13)$$

and with

$$P_{L_2} \approx 2.0\times10^{-6}\, N/m^2 \quad \text{and for } \sigma = \frac{10}{6}\, Kg/m^2 \quad (14)$$

for the follower with mass i s 500 Kg, the total area needed for maintaining distance with the leader for 10 m, is 300 m^2.

5.2. Both satellites has solar sail with two gimbal

For this case, the solar sail control accelerations are as follows

$$\begin{aligned} \Delta a_x &= A(\sin\delta_2 \sin\psi_2 - \sin\delta_1 \cos\psi_1) \\ \Delta a_y &= A(\sin\delta_2 \cos\psi_2 - \sin\delta_1 \cos\psi_1) \\ \Delta a_z &= A(\cos\delta_2 - \cos\delta_1) \end{aligned} \quad (15)$$

Hence we can form a cost function for minimizing the drifting motions of the formation as

$$\begin{aligned} H \equiv\ & A^2[(\sin\delta_2 \sin\psi_2 - \sin\delta_1 \sin\psi_1)^2 \\ & + (\sin\delta_2 \cos\psi_2 - \sin\delta_1 \cos\psi_1)^2 + (\cos\delta_2 - \cos\delta_1)^2] \\ & + \lambda_1 \left[a(\sin\delta_2 \sin\psi_2 - \sin\delta_1 \cos\psi_1) - \rho_x \right] + \\ & \lambda_2 \left[b(\sin\delta_2 \cos\psi_2 - \sin\delta_1 \cos\psi_1) - \rho_y \right] \\ & + \lambda_3 \left[(\cos\delta_2 - \cos\delta_1) - \rho_z \right] \end{aligned} \quad (16)$$

The simplest analytical solutions is

$$\delta_1 = 0 \quad or \quad \delta_2 = 0 \tag{17}$$

which for $\delta_2=0$

$$\delta_1 = \cos^{-1}\left(1-\frac{\rho_z}{c}\right)$$
$$\psi_1 = \tan^{-1}\left(\frac{\rho_x}{\rho_y}\frac{b}{a}\right) \tag{18}$$
$$\psi_2 = \tan^{-1}\frac{b}{a}\frac{\left(\rho_x + a\sin\delta_1 \sin\psi_1\right)}{\left(\rho_y + b\sin\delta_1 \cos\psi_1\right)}$$

where

$$a = \frac{A}{2B_L - 1}$$
$$b = \frac{A}{B_L - 1} \tag{19}$$
$$c = \frac{A}{B_L}$$

5.3. The leader has solar sail with one gimbal and the follower has solar sail with two gimbal

In this third case, the formation is fully controllable, because the number degree of freedom is the same with the number of equations available.

6. Conclusions

Here in this paper has been derived the control strategy for formation flying around L_2 point. The visibility of this application also have been shown, that for maintaining the relative motion between the leader and the follower for 10 m approximately need 300 m^2 are of sail.

Future detail study needs to be done with more accurate mathematical modeling of dynamical motions at the restricted three body problem, to include perturbations components. The satellite attitude dynamics also need to be considered for implementing a vigorous formation control.

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