Observation of the impact of rubble piles by use of N-body simulation

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Abstract

The formation of rubble pile after a collision of two primary rubble piles is discussed, changing an initial collision velocity, its directions, and their initial radius. At initial, the two primary sub-kilometer rubble piles which have same mass and radius are collided with each other. Since rubble piles are shaped by the aggregation of many fragments affected by their gravity, the N-body simulation combining with the spring-dumper system is used to analyze the motion of each fragment. In this model, each fragment shall be a mass particle. All frictions which serve forces in the circumferential direction are neglected. Additionally, the analysis is done in 2-dimensional field. Any study conclusions is derived from comparisons of simulation data sets statistically. This analysis leads to the result that the initial collision velocity and its directions have an important role in the motion of particles. In the range from about 0.25 to 0.5 [m/sec] of the collision velocity, an intensive motion change occurs. On the other hand, changing the direction of the collision causes the result that the motion of particle has similar trends expect for that near to a glancing collision. In the vicinity of a glancing collision, the trend of formations is quite different from others. These results show same tendencies, even though the radius of the primary rubble pile is changed in the collision of sub-kilometer rubble piles.

1. Introduction

In recent years, many kinds of small asteroids have been observed owing to developments of observation methods like not only ground radar observations but also direct searching using spacecraft. From these observations, asteroids may be mainly classified by two categories; the main belt asteroid and the near earth asteroid (NEA). Celes [1], Vesta [2], and IdaeDactyl [3] can be described as the famous main belt asteroid. On the other hand, Eros [4], Toutatis [5], Golevka [6], and Iokawa [7] are good example for NEAs. Discoveries of these asteroids have contributed to growing interests to small asteroids. In recent years, researches focusing on small asteroids have challenged to uncover their mechanism by some approaches like image techniques, geology, simulation and theoretical analysis. In theoretical analysis, Scheeres studies orbits around small asteroids, modeling them as an ellipsoidal shape body [8]. On the other hand, in numerical analysis, researchers try to analyze their orbits [9], [10]. Then, in [11], [12], and [13], the authors have studied the formation mechanisms of rubble piles, a collection of small fragments attracted by their own gravity. Benz [11], and Michel et al [12] have analyzed aggregations after a devastating collision of two primary rubble piles. On the other hand, Richardson [13] has investigated rubble pile’s shapes just
after affected by tidal forces from big planets. All of them have tried to study for a few kilometer size asteroids to hundreds kilometer asteroids. However, from the detailed observation of Itokawa [7], investigating sub-kilometer rubble piles has become important. From these results, in this paper, using the N-body simulation, we focus on how rubble piles are shaped after a collision of two primary sub-kilometer rubble piles. Here, the velocity when the rubble piles collide with each other and the collision directions are defined as the impact velocity and the impact parameter, respectively. These values and the radius of the primary rubble piles are parameterized in our simulation. In each simulation, we have some assumptions here as follows: all frictions are neglected; all fragments shape spherical body with same mass and radius; the primary rubble piles have also spherical body with same mass and same radius. Additionally, the analysis is done in 2-dimensanal field. To prevent any confusion, a technical term is defined. Since rubble piles are made by small fragment collection, calling these components of rubble piles as ‘fragment’ may be valid. In this study, however, we name this ‘fragment’ ‘particle’ in terms of the analysis using the N-body simulations. We start our discussion from modeling of a rubble pile. Then, the validation of our simulations is described. Last, the simulation results are shown.

2. Modeling of a rubble pile

A rubble pile is defined as an aggregate attracted by its own gravity. Modeling rubble piles is executed by the N-body simulation. In this study, the spring-dumper system is also used to model the collision of each particle.

2.1. Expression of collision between particle

Since each particle flies around in the gravity field affected by other particles, the motion of $i$th particle is described in Eq. (1) by using the N-body problem.

$$\frac{d^2\mathbf{r}_i}{dt^2} = -\sum_{j=1, j \neq i} G m_j \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

Here, $G$, $m$, and $\mathbf{r}$ are the gravitational constant, the mass of a particle, and the position vector, respectively.

In order to investigate how particles move in an effect of gravity forces from others, we have to model the particle collision occurring frequently. In this paper, this collision between particles is described by using the spring-dumper system. In order words, we model the collision by bounces proportional to difference of position vectors and that of velocity vector between particles. As $c$ is defined as the dumper coefficient and $k$ is the spring coefficient, the force when a particle contacts with another in the collision is written by,

$$f_{sd} = -c \frac{dx}{dt} - k(x - d)$$

where $x$ indicates the position in the arbitrary direction and $d$ is the reference indicating the difference between the particles.

2.2. The Equation of Motion

Considering the previous section, we derive the equation of the motion of each particle. Combing Eq.(1) with Eq.(2), the simulation model may be easily obtained. However, the simulation results diverge easily due to calculation errors when taking into account the gravitational force between the particles directly attaching with each other. In order to avoid this problem, in this study, we neglect the gravity by which the particles collide with each other. Here, using Eq. (2) and Eq. (3), now we can describe the equation of motion as,

$$\frac{d^2\mathbf{r}_i}{dt^2} = -\sum_{j=1} G m_j \frac{\mathbf{r}_i - \mathbf{r}_j}{d_{ij}^3} \mathbf{e}_{ij}$$

$$- \sum_s \left[ k \left( \mathbf{r}_i - \mathbf{r}_s \right) \cdot \mathbf{e}_{is} - d_{is} \right] \mathbf{e}_{is} + c \left( \mathbf{v}_i - \mathbf{v}_s \right) \cdot \mathbf{e}_{is} \mathbf{e}_{is}$$

In this equation, $s$ is the letter indicating the particle which attaches with $i$th particle. Then, $\mathbf{e}$ is the unit vector and $\mathbf{v}$ is the velocity vector (Fig. 1).

![Fig. 1: The relation between particles.](image-url)

$s$ th particle collides with $i$th particle, but $j$th particle does not. The gray zone indicates the shape of particles.

2.3. Assumption

In the description of the equation of motion written in Eq. (3), we have some assumptions of the motion of particles. This section introduces these assumptions.

First, all of particles have a same mass and a same shape. In particular, it defines that these shape are spherical shape.

Second, all frictions are neglected in attachments of particles. It means that only forces in the radial direction are considered.

Third, we discuss in 2-dimensional field. While the symmetry from the second assumption, the disregard of
all frictions, makes it possible to discuss in 2-dimensional (2D) field, this simplicity may cause to deviate from real motion in 3-dimensional (3D) field. The difference of particle’s mass may lead some different results, for instance. From this reason, we define the mass of the particle again, comparing density in 3D with in 2D. The densities in 3D and 2D are now described by $\rho'$ and $\rho$, respectively. Here, the mass of a particle in 2D shall correspond to that in 3D. This relation is shown in Eq. (4),

$$\frac{4}{3} \pi R_p^3 \rho' = \pi R_p^2 \rho$$  \hspace{1cm} (4)$$

where $R_p$ indicates a radius of each particle.

2.4. Parameters in Simulation

Characterizing the collision of the two primary sub-kilometer rubble piles, we bring in the two parameters; the impact velocity and the impact parameter. The impact velocity indicates the magnitude of the relative velocity just before the collision of the two primary rubble piles. The impact parameter, on the other hand, is defined as the value which indicates the direction of collision. Here, we define the value of the impact parameter in the range from 0 to 1. When a head-on collision occurs, this value is 0. Then, at a glancing collision, the impact parameter describes 1 (Fig. 2). Note that the impact velocity is described as a dimensionless value.

$$b$$ is the impact parameter, and $v_0$ is the impact velocity. The grey circle indicates the rubble pile in the initial condition.

3. Validation of simulation

3.1. Run Time

In this section, we show the validity of the run time. Compared with the previous study [14], however, our run time may be relatively short. Now, one simulation example is shown in Fig. 3. In this simulation, the impact velocity is 0.1 m/s and the impact parameter is 0. It shows the number of particle against the time. According to this figure, the rapid change of the number of the small rubble piles shaping after the collision of the two primaries finishes in 7.6 [h]. This result means that the serious phase is done in less than 7.6 [h] after the collision of the two primary rubble piles. This trend is seen in other cases. This shows the investigating the motion of particles for 7.6 [h] is valid for our study.

This figure shows the run time vs. the number of particle. The vertical line is the end of simulation, 7.6 [h].

3.2. Physical parameter

Since the spring-dumper system is used to express the collision of particles, we need to investigate the validity for their coefficient. Additionally, the particle size should be determined, too. However, since the particle size is different from each other in real motions, we may not be able to seek the valid size in this study. Thus, in this section, we introduce the validation of these parameters.

A. The coefficient expressing particle collisions

To express the collision of particles, this study uses the spring-dumper system. This utilization leads an uncertainty of the magnitude of the spring coefficient and of the dumper coefficient. In this section, we compare simulation results, using two different sets of values for the spring coefficient and for the dumper coefficient.

At first, the comparison of the spring coefficient is done. Fig. 4 shows the simulation example indicating a conclusion of the comparison. The distribution and the diffusion process are almost same in the two simulations.

Then, two values for the dumper coefficient are compared in Fig. 5. We can also get a result that even though the dumping coefficient is changed, the motion have almost same trends in the two simulations.

B. The size of the particle

The size of the particle is an important factor to determine its restitution and kinetic energy. As mentioned above, we assume the size of all particles shall be same. Here, two simulations example for different values of particles’ size is shown in Fig. 6. From this comparison, we can see that the two simulations get almost same conclusion.

Note that ‘same’ does not mean that the position of particles in one simulation absolutely corresponds to that in another, but indicates that the distribution of particles in both simulation is similar. It is because each simulation has its own initial condition. Even if the initial velocity, the initial parameter, the size of initial rubble piles, and other parameters are set beforehand, some values such as position of particle composing the primary rubble piles
are different from each case.

From these result, it concludes that changing the particle size, the spring coefficient, and the dumper coefficient does not have any effects in characteristics of the motion of particles in our simulations.

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**Fig. 4: Comparison of the spring coefficient**

The spring coefficient for the small dot is 2.5 times bigger than that for the open circle. These cases are computed by using the particle radius 35[m]. The left figure shows the condition at the beginning of simulation, the mid figure shows the motion at 3.8 [h], and the right figure is for 7.6 [h].

**Fig. 5: Comparison of the dumper coefficient**

The dumper coefficient for the small dot is 2.0 times bigger than that for the open circle. Similar to the comparison in Fig. 5, the particle radius is set to 35 [m]. The figures are shown in the same manner in Fig. 4.

**Fig. 6: Comparison of the particle size**

This shows two simulations for two different particle of which radius is 19 [m] (open circles) and is 35 [m] (small dot). Each figure shows the time profile. The figures are shown in the same manner in Fig. 4.
4. Simulation results

In this study, 55 cases are computed. Now we define the radius of the primary rubble pile as $R_r$. Among them, 26 cases is for $R_r = 260[m]$ and $\rho = 1.42[kg/m^3]$. Other 29 cases is for $R_r = 380[m]$ and $\rho = 1.34[kg/m^3]$. In these simulations, initial conditions are set as follows: the initial impact velocity velocities are 0 to 1.25 [m/sec]; the initial impact parameters are the range from 0 to 0.82.

Fig. 7 shows the relation of the impact velocity, the number of small rubble piles shaping after the collision of the two primaries, and the maximum mass of them. Note that we use dimensionless values for the number of the same rubble piles and the maximum mass of them. They are obtained when divided by the total number of particle and the sum of the mass for the two primary rubble piles, respectively. According to all figures, the results for the
6 [m] radius rubble pile and that for the 380 [m] indicates almost same trends. The gradient of the distribution becomes sharp at the impact velocity 0.25 to 0.5 [m/sec]. This trend is shown in all figures, and indicates that the intensive change occurs in this range of the impact velocity. This means that various motions are shown in this condition as some small rubble pile are generated and then these rubble piles are absorbed by others again and again. This also indicates that once the condition is deviated from this condition, the motions keep only a simple trend. In other words, smaller impact velocities cause that the primary rubble piles only attach softly with each other, and then only a few rubble piles are shaped. In bigger impact velocities, on the other hand, the particles no longer attached with each other and then fly away from the center of mass. From these simulation results, we can see that only small velocities are needed to shape small rubble piles again when the same size of sub-kilometer rubble piles collide with each other.

Then, Fig. 8 indicates the relation with the impact parameter. Similar to Fig. 7, Fig. 8 describes that the result for the 260 [m] radius rubble pile is almost same as that for the 380 [m] radius rubble pile. This also shows that the trend with the impact velocity 0.82, indicating a clash near to a glancing collision, is quite different from other magnitudes such as 0, 0.42, and 0.56. This means that the trend in the impact parameter nearly 1.0 has different characteristics with others. However, the boundary where the trend is changed is still unclear. To analyze this point is the future works.

5. Conclusion

In this paper, we simulated the collision of two rubble piles. The N-body simulation was used, and then spring dumper system was applied for modeling the collision and the attaching of particle. Totally, 55 cases were simulated. In simulations, we focused on the sub-kilometer rubble pile and investigated the relation of the impact velocity, the impact parameter, and its radius. When the impact velocity is 0.25 to 0.5 m/sec, the trend of collision changed intensively. If the impact velocity more than about 0.5 [m/sec], most particles fly away from the center of mass. This result led the fact that a very small impact velocity is need to shape rubble piles after the collision again.

Then, comparing the results by changing the impact parameter showed that the rubble pile crash near to the glancing collision causes different motions from others with lower impact parameters.

6. References