For steering a control moment gyro (CMG)-system which is installed as an actuator system of spacecraft, there are lots of previous researches which aim to avoid and leave from vicinity of the singular points of CMG-system. But some steering laws may not be able to manage to output enough torque during an attitude maneuver phase. Because almost all of these researches do not consider the feature of required torque during an attitude maneuver phase. During an attitude maneuver phase, a large feed forward torque is required in an attitude maneuver direction. So the CMG steering law is required the capability of avoiding not only singular points but also the insufficient torque area near a singular point and provide enough torque in an attitude maneuver direction. In some previous researches, only singular points are taken into consideration. Such a steering law will be useful only during the attitude pointing phase in which a small torque is required. During an attitude maneuver phase, there is an important feature that a large control torque is required in a maneuver direction only, and the control torque perpendicular to the maneuver direction is not of importance. In other words, the torquing capability of the CMG-system can be optimized especially for the maneuver direction during an attitude maneuver phase. There is no research which takes this feature into consideration. From this point of view, in this paper, we propose a new steering law of CMG that generate torque effectively in any required direction. A large torque can be generated in the required torque direction with this method. Therefore, this is the method of utilizing the CMG system resource most effectively. This method provides an improved use of the torquing capability of the CMG-system especially during attitude maneuver of spacecraft.
INTRODUCTION

Difficulty in a steering CMG-system

The difficulty in a steering CMG-system results from its “nonlinearity and time varying nature” and “singularity” which are the major features of a CMG-system. In case of Reaction Wheel (RW) system, it is easy to solve the control input for a required output torque. This is originated from the fact that the output torque of a RW system is linear and time-invariant against the input, therefore the inverse kinematics can be easily solved. In case of CMG-system, however, input solution depends on gimbal angles $\phi$ and cannot be solved as easy as a RW-system.

In Figure 1, one of the most popular CMG system called “pyramid type single gimbal CMG system” is shown. Basically CMG-system output torque is proportional to input gimbal rates $\dot{\phi}$. However, nearby at singular points, a CMG-system cannot generate sufficient torque in a certain direction. Not to speak, at the point of singularity, the system cannot generate any torque at all even when it is driven with infinite gimbal rates.

The ratio of the output torque against the input gimbal rates is called “input-output gain”, described by $\|T\|/\|\dot{\phi}\|$, where $\|X\|$ means a norm of a vector $X$. This input-output gain is a function of gimbal angles. This gain’s shape is an ellipsoid as shown in Figure 2, because there are directions along which it is easy to output a torque and directions along which it is difficult.

Figure 1  CMG system

- $g_i$: Unit vector in gimbal axis direction
- $h_i$: Unit vector in angular momentum direction
- $c_i$: Unit vector in output torque direction = $g_i \times h_i$
- $\beta$: Angle between X-Y plane and $c_i$ vector plane
This ellipsoid shape of the gain is dependent on gimbal angles. Now it is supposed that some torque is required in a certain direction. The input-output gain for the direction changes not only near at singular points but also at all area as in Figure 3, which shows that the gain changes as a function of the gimbal angles $\phi$. If a required gimbal rate becomes larger than the hardware limit of a CMG, due to the insufficiency of the input-output gain, the required control torque cannot be produced and the desired attitude control of a spacecraft may not be achieved. Such a phenomenon can be anticipated more often in the attitude maneuver phase than in the attitude pointing phase, because required torque at the attitude maneuver phase is larger than that at attitude pointing phase. Thus the effectiveness of a CMG-system is reduced in attitude maneuver of a spacecraft.

A1: Area in which input-output gain is insufficient even when required torque is small.
A2: Area in which input-output gain is insufficient when required torque is large.

Figure 2 Shape of input-output gain

Figure 3 Continuous change of input-output gain for torque direction
Previous Researches

Many researches have been conducted on how to avoid and leave from the vicinity of the singular points of a CMG-system. Among these, the so-called gradient method (GM) has a feature of being able to avoid singular points, while keeping the input-output gain as large as possible. The algorithm of the GM is as follows; Select a volume of the input-output gain as a cost function and drive the gimbals using the so-called null motion so as to make its time differential term positive. With this algorithm, an effective singular point avoidance can be easily realized. For this reason, GM is the most popular method and becomes the base of every CMG steering research. However it is known that there is a weakness of GM. There are two kinds of singular points in a CMG-system. One is called “passable singular point” from which it is possible to escape and get away by using a null motion. The other is called “impassable singular point ” from which it is impossible to escape once caught. Other notations “hyperbolic / elliptic” and “indefinite / definite” are used in some researches.

The GM cannot guarantee the proper motion around the impassable singular points which exist in single gimbal CMG (SG-CMG) system consisting of 4 SG-CMGs arranged in so-called pyramid configuration. The GM can manage to go through passable singular points by using null motion, but the output torque becomes zero temporarily.

Some methods which manage to avoid or get away from the impassable singular points have been proposed. One is SR-inverse method , the other is Generalized SR-inverse method . These algorithms generate the dither torque at the impassable singular point to get away from the situation. But the dither disturbance torque deteriorates the attitude pointing accuracy.

The constrained steering law uses a simple constraint to the gimbal motion so as for the gimbals not to get to impassable singular points. However, the target of this steering law is to avoid impassable singular points only. So it is possible to get into not only the insufficient torque area near a singular point but also passable singular points.

Therefore, with the steering law which aim to avoid the impassable singular points only, it may not be able to manage to output enough torque during an attitude maneuver phase in which a large feed forward torque is required. Such a steering law will be useful only during the attitude pointing phase in which a small torque is required.

It is known that planning gimbal motions in advance is effective to avoid singular points. But the previously proposed off-line planning method is complicated. Therefore, if sufficient time and computing power were available, a method like a complicated path planning could be utilized.

ANISOTROPICALLY WEIGHTED GRADIENT METHOD

Steering Law

In this section, an improved steering law for a CMG-system having one or more redundant CMG is proposed and formulated. We will call the steering law Anisotropically Weighted Gradient Method (AWGM), because the idea of the proposed steering scheme is based on the GM. According to the proposed algorithm, the CMG-system can generate a large control torque most effectively in any required direction while avoiding singular points. This feature enables to make the most of the torquing capability of the CMG, and thus has the advantage over other algorithms previously proposed, in spacecraft control that need high agility to fulfill their missions.

According to the GM, the volume of the input-output gain as defined in the previous section is chosen as a cost function for avoiding the singularities, and the volume is tried to be maximized. That is to say, the ellipsoid gain is tried to be made spherical, meaning that the gain is maintained isotropically equal in every and all direction. However, in an actual large angle attitude maneuver of a spacecraft, a large control torque is required in a certain direction which is given by maneuver control law. The torquing capability of the CMG-system perpendicular to the required direction is not of importance. That is to say, sufficient input-output gain should be guaranteed in only the required direction. From this point of view, it can be said that the strategy of the GM is not an optimal way for assigning the torques to individual CMG during an attitude maneuver phase.
According to the scheme we propose, we try to make the ellipsoid of the input-output gain anisotropically warped or expanded along the direction, into which a large control torque is required, in order to achieve a desired fast attitude maneuver of a spacecraft.

Besides, the amplification factor of the output torque along the required direction, with respect to the torque generated by the GM, can be designed in a quantitative manner. Because of this feature, the method provides an improved use of the torquing capability of the CMG-system without causing the torque saturation problem, especially in attitude maneuver planning phase of spacecraft. In addition, this method is a real time steering law which does not need the off-line gimbal motion planning in advance. Therefore it does not cause any software computing problem which often happens by the complicated off-line planning algorithm as mentioned before.

Let us explain the AWGM theory with the images shown in Figure 4. At first, we consider the effect of isotropically expanding input-output gain by using GM as a reference, it means that a sphere of radius 1 is used to express the effect of GM shown in Figure 4(a). Supposed that we want to maximize the gain in a main torque direction given from the maneuver profile by a factor $k_{\text{main}} (>1)$ as compared with the gain for GM. On the other hand, we reduce the gain in the other torque direction perpendicular to the main torque direction by a factor $k_{\text{other}} (<1)$. This concept is shown in a visual manner in Figure 4(d) by using ellipsoid. To achieve the input-output gain of Figure 4(d), we design the virtual input-output gain which is weighted small in the main torque direction with parameter $1/k_{\text{main}}$ and large in the other torque direction with $1/k_{\text{other}}$, shown in Figure 4(b). We apply the AWGM by using this virtual weighted gain as a cost function, and the AWGM makes the virtual cost function spherical as shown in Figure 4(c). As a result of applying this scheme, the actual input-output gain can be made ellipsoidal as shown in Figure 4(d).

![Figure 4](image)
According to the Figure 4, $k_{\text{main}}$ is a parameter for the direction of the major axis of the ellipsoid, and $k_{\text{other}}$ is for the minor axis of that. These parameters should be designed considering a distribution of the required torque for each direction. The required torque during the attitude maneuver is composed of the main torque around the maneuver axis and the other torque which is perpendicular to the axis including gyro moment effect and disturbance torque etc.

First, we design the $k_{\text{main}}$ considering how much we want to expand the input-output gain in the main torque direction. After that, we design the $k_{\text{other}}$ by using the ratio between main torque and the other torques as explained above.

However, input-output gain cannot be made large without restriction. The CMG control results in failure with excessive value parameters. The final design of these values will have to be confirmed with the detailed numerical simulations.

Formulation

In the following derivation of the proposed AWGM, we assume a CMG-system consisting of 4 SG-CMGs with the pyramid configuration. However, this derivation is not limited to be applied only to such a system. The equation of motion is given as follows;

$$\begin{align*}
T_{\text{cmg}} &= J(\phi) \cdot \dot{\phi},
\end{align*}$$

where $T_{\text{cmg}}$ : CMG output torque $\in R^{3 \times 1}$

$\dot{\phi}$ : gimbal angle rate $\in R^{4 \times 1}$

Now, the required torque during an attitude maneuver is designated as $T_{\text{mov}}$. Then, the input $\dot{\phi}$ to realize the required torque output is given by

$$\begin{align*}
\dot{\phi} &= J^{-1}(\phi) \cdot T_{\text{mov}} + \left[I - J^{-1}(\phi)J(\phi)\right]k.
\end{align*}$$

$J^{-1}$ is a pseudo-inverse matrix of $J$, $k$ is an arbitrary vector of 4 dimensions. The term of $\left[I - J^{-1}(\phi)J(\phi)\right]k$ is a mapping onto zero space, that is, it represents the null motion which does not produce any effect on output torque $T_{\text{cmg}}$.

We consider that this null motion is utilized to maximize a cost function $p$ below;

$$\begin{align*}
p &= V(\phi),
\end{align*}$$

where $V(\phi)$ is a function of the gimbal angles $\phi$.

This means that the value of $k$ should be decided so as to maximize $p$. Time differential of $p$ is expressed as;

$$\begin{align*}
\dot{p} &= \dot{V}(\phi) = \frac{\partial V(\phi)}{\partial \phi} \cdot \dot{\phi} = \xi(\phi)^T \cdot \dot{\phi},
\end{align*}$$

where $\xi(\phi) = \frac{\partial V(\phi)}{\partial \phi}$.

When the Eq.(2) is substituted in Eq.(4), we get,

$$\begin{align*}
\dot{p} &= \xi(\phi)^T \cdot J^{-1}(\phi) \cdot T_{\text{mov}} + \xi(\phi)^T \left[I - J^{-1}(\phi)J(\phi)\right]k.
\end{align*}$$
Let us assume $k$ in the form given as;
\[ k = \xi(\phi) \cdot k_p, \]  
(6)
where $k_p$ is an arbitrary constant positive scalar.

Then the time differential of $p$ is given by;
\[ \dot{p} = \xi(\phi)' J'(\phi) \cdot T_{aw} + \xi(\phi) \left[ I - J'(\phi) J(\phi) \right] \xi(\phi) k_p. \]  
(7)

As it is already known, $\left[ I - J'(\phi) J(\phi) \right]$ is positive semi-definite, and the second-term of the right-hand side of Eq.(7) always becomes a non-negative number. After all, it is clear that $k$ of the Eq.(6) works to enlarge the cost function $p$.

In the GM, the null motion is used to maximize the volume of the input-output gain. The input-output gain $g$ is defined by Eq.(8) as already mentioned and the gain shape is shown in Figure2.

\[ g(v, \phi) = \frac{\| T \|_2}{\| \phi \|_2}, \]  
(8)
where $T$ is the torque vector, and $T // \nu, \nu$ is radius vector of unit sphere.

In a new steering law we propose here, the idea is that the null motion is used to make input-output gain predominant in the main torque direction given by the maneuver control law. For this purpose, we define a new cost function as follows.

We introduce $T_{aw}$ which is anisotropically weighted virtual torque vector, and the virtual input-output gain $g_{aw}$ defined by Eq.(9) whose gain shape is shown in Figure4(b).

\[ g_{aw}(v, \phi) = \frac{\| T_{aw} \|_2}{\| \phi \|_2}, \]  
(9)
where $\| T_{aw} \| = \| M_w M_d M_T \|_2$, and $T // \nu, \nu$ is radius vector of unit sphere.

$m_r$ : rotational matrix(from body to anisotropic ellipsoid coordinates),  
$m_w$ : virtual weight matrix $\text{diag}(w_1, w_2, w_3) = \text{diag}(1 \ / \ k_{other}, 1 \ / \ k_{other}, 1 \ / \ k_{main})$

We get Eq.(10) by using the pseudo-inverse matrix.

\[ \| \phi \|^2 \geq \| J' T_{aw} \|^2 = \| J(\phi)' M_w M_d M_T \|^2 \]  
(10)

And we obtain the quadratic form as;
\[ \phi^T \left( M_w^{-1} M_j J(\phi) \right) \left( M_w^{-1} M_j J(\phi) \right)^T \]  
\[ = T^T \left( J_{aw}(\phi)' M_j J_{aw}(\phi) \right) T \]  
(11)

We choose the eigen vector $v$ of the symmetric matrix $J_{aw}(\phi)' J_{aw}(\phi)$ as a base vector, and we obtain the quadratic form as;
\[
(g_{av}(v, \phi))^2 = v^T \begin{bmatrix}
\lambda_{av1} & 0 & 0 \\
0 & \lambda_{av2} & 0 \\
0 & 0 & \lambda_{av3}
\end{bmatrix} v,
\] (12)

where \(v\) is eigen vector, \(\lambda_{av1}, \lambda_{av2}, \lambda_{av3}\) are eigen value of the symmetric matrix \(J_{av}(\phi)J_{av}^T(\phi)\).

From the Eq.(12), it is found that \(g_{av}(v, \phi)\) designates the ellipsoid of which principal axis is eigen vector \(v\). Square of the volume of the anisotropically weighted virtual input-output gain \(g_{av}\) is called average gain and defined as follows;

\[
\overline{g}_{av}(\phi) = \lambda_{av1} \cdot \lambda_{av2} \cdot \lambda_{av3} = \det(J_{av}(\phi)J_{av}^T(\phi)).
\] (13)

Here, the volume of the anisotropically weighted virtual input-output gain \(g_{av}\) is chosen as cost function \(p\).

\[
p = V(\phi) = \sqrt{\overline{g}_{av}(\phi)} = \sqrt{\det(J_{av}(\phi)J_{av}^T(\phi))}
\] (14)

The null motion works to maximize this function. It means that the predominant input-output gain in main torque direction is realized.

The \(\xi\) in the Eq.(4) is given by,

\[
\xi = \frac{\partial V(\phi)}{\partial \phi} = \frac{\partial V(\phi)}{\partial f(\phi)} \frac{\partial f(\phi)}{\partial \phi} = \frac{1}{\sqrt{\det(J_{av}(\phi)J_{av}^T(\phi))}} \frac{\partial}{\partial \phi} \left[ \frac{\partial}{\partial \phi} \det(J_{av}(\phi)J_{av}^T(\phi)) \right],
\] (15)

where \(f(\phi) = \det(J_{av}(\phi)J_{av}^T(\phi))\).

The \(\xi\) of Eq.(15) is substituted to Eq.(6), then \(k\) of the Eq.(6) is substituted to Eq.(2), finally the motion of the gimbal rates is obtained as,

\[
\dot{\phi} = J^T(\phi) \cdot T_{mvr} + \left[ I - J^T(\phi) J(\phi) \right] \xi(\phi) k_p,
\] (16)

where \(\xi(\phi) = \frac{1}{2} \frac{\partial}{\partial \phi} \det(J_{av}(\phi)J_{av}^T(\phi)) \frac{\partial}{\partial \phi} \left[ \frac{\partial}{\partial \phi} \det(J_{av}(\phi)J_{av}^T(\phi)) \right].

The value \(k_p\) chooses a suitable scalar which does not make \(\dot{\phi}\) excessive.

**SIMULATION**

We can verify that the effect of AWGM in realizing large input-output gain is more conspicuous in required torque direction than that of the GM during attitude maneuver as shown in simulation results Figure 5 and Figure 6. The angle \(\theta\), the angular velocity \(d\theta/dt\) and the angular acceleration \(d^2\theta/dt^2\) and input-output gain in the attitude change direction are shown in Figure 5, and the gimbal angle velocity \(d\phi/dt\) are shown in Figure 6.

With the GM, more singular points in torque required direction appear than with AWGM. In such a singular situation, enough torque for attitude maneuver can not be generated even with high gimbal rate.
inputs and the angular acceleration is decreased rapidly. On the other hand, with the AWGM, a smaller number of singular points in torque required direction appears and a high agility attitude maneuver is realized.

Figure 5  Maneuver profile

Figure 6  Gimbal rate profile
CONCLUSION

A new steering law of CMG which generate torque effectively in any required direction is proposed. And verification and evaluation of the steering law was executed. As a result, it is confirmed that the input-output gain is amplified to effectively output torque in the main torque output direction by the "anisotropic weighted gradient method (AWGM)". In other words, a confirmation is made that the use of the "anisotropic weighted gradient method (AWGM)" helps to output a larger torque, thereby shortening the time necessary for changing the attitude. In case where the attitude changing time profile is predetermined, it is of course possible that the predetermined attitude time profile or the predetermined torque time profile can be achieved with a smaller gimbal angular velocity input, that is with a smaller energy.

With this method, the large torque can be generated in the required torque direction, so this is the method of utilizing the CMG system resource most effectively.

The proposed steering method is based on the GM. Therefore, it takes over the character of GM basically, and similarly to GM, it is not guaranteed to evade the singularity completely. In order to evade the singularity completely, it is avariable to add a simple gimbal motion planning, for example the preferable initial gimbal angles with which it does not reach to the singularity are chosen.

This method can be applied to not only SG-CMG-system but also Double Gimbal CMG-system, and also can be applied to other than the pyramid type system. In addition, as for the robot manipulator system, the singularity exists same as the CMG system. Besides, the ellipsoid of “manipulability” is also defined for the singularity. Therefore, this method also does work well as an effective drive law of the robot manipulator.

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The method described in this paper is patent pending. Commercial use of these methods requires written permission from NEC TOSHIBA Space Systems, Ltd.

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