Small Halo Orbit at the Sun-Earth L2 Point by Solar Sail

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To maintain Halo orbits around L2 point in the Sun-Earth system using conventional chemical propulsion, the size of the orbits is limited to about more than 650,000 km. Therefore small halo orbits in the vicinity of L2 point by solar sails, which can produce continuous acceleration by utilizing photons from the sun and consume no fuel, are studied in this paper.

I. INTRODUCTION

To maintain Halo orbits around L2 point in the Sun-Earth system using conventional chemical propulsion, the size of the orbits is limited to about more than 650,000 km. Therefore small-halo orbit in the vicinity of L2 point has been studied by using continuous thrust control.

Tarao assumed the thrusters as ion engines and discussed the way to realize small-halo orbit with a constant magnitude of thrust and constraining the thrust direction in a certain plane rotated at L1/L2 point.1 Morimoto defined an artificial equilibrium point (AEP) and generated periodic orbit around AEP with constant control acceleration which is needed to create AEP. And small halo orbit was found on the line connecting Sun and the Earth.2

Both control low assumes low thrust propulsion systems such as electric propulsion and solar sail. However both can not be adapted for small halo orbit using solar sail because the direction of acceleration by solar sail is constrained to the opposite side to the sun.

II. ACCELERATION BY SOLAR SAIL

Solar sails are propulsion systems utilizing large membrane mirrors. The spacecraft deploys a large, lightweight sail which reflects photons from the Sun or some other light sources. Solar radiation pressure is small and also inversely proportional to the square of the distance from the Sun. Although the thrust is small, solar sails require no fuel (i.e., Isp is considered as infinity). Tilting the reflective sail at the steering angle from the Sun produces the thrust angle that halves the steering angle.

In this paper we use a simple solar radiation model as acceleration by solar sail. When solar sail is inclined at an angle \( \alpha \) (Fig. 1), the acceleration by solar sail \( \mathbf{a}_s \) can be expressed as follows (Ref. 3).

\[
\mathbf{a}_s = -P_{\text{SUN}} \frac{14U^2}{r_{\text{SUN}}^2} A m \cos(\alpha) \left[1 - e + 2e \cos(\alpha)a \right] \quad (1)
\]
\( P_{\text{SUN}} \) is solar radiation pressure which is given by \( P_{\text{SUN}} = \Phi/c \). \( \Phi \) is the solar flux, and in the vicinity of the Earth this amounts to \( \Phi \approx 1367 \text{Wm}^{-2} \). \( c \) is the velocity of light, \( r_{\text{sun}} \) is the distance from the sun, \( m \) is mass of spacecraft and \( A \) is surface area of solar sail. Vector \( e \) points into the direction of the sun and \( n \) is the normal vector of the surface, then \( \alpha \) is calculated from \( \cos \alpha = n^T e \) . \( \varepsilon \) (0 \( \leq \varepsilon \leq 1 \) ) is reflectivity.

\[

\nabla U(r_0) - a(r_0) = 0.
\]

Therefore, a non-equilibrium point \( r_0 \) is changed into an artificial equilibrium point (AEP) with low thrust acceleration, \( a(r_0) = \nabla U(r_0) \) (see Ref. 2 in detail). The components \( (a_{0x}, a_{0y}, a_{0z}) \) of \( a(r_0) \) \( (r_0 = (x_0, y_0, z_0) ) \) in the rotating frame \( (x, y, z) \) are

\[

a_{0x} = -x_0 + \frac{1-\mu}{r_1^3}(x_0 + \mu) + \frac{\mu}{r_2^3}(x_0 - 1 + \mu) \quad (4a)
\]

\[

a_{0y} = -y_0 + \frac{1-\mu}{r_1^3}y_0 + \frac{\mu}{r_2^3}y_0 \quad (4b)
\]

\[

a_{0z} = \frac{1-\mu}{r_1^3}z_0 + \frac{\mu}{r_2^3}z_0 \quad (4c)
\]

where \( r_1 \) and \( r_2 \) are two separate distances between the spacecraft and two primary bodies and \( \mu \) is mass ratio (Ref.4). The spacecraft can orbit around the AEPs \( (r_0) \) periodically with continuous constant acceleration which is taken as \( a(r_0) \) (Ref. 2).

The equations are linearized at an AEP. Using \( r = r_0 + \delta \), the linearized equation of motion is obtained as

\[

\ddot{\delta} + 2\omega \times \dot{\delta} + \left[ \frac{\partial}{\partial r} \nabla U(r) \right]_{r_0} \delta = 0.
\]

This is an equation of motion for dynamics about an AEP \( (r_0) \) with constant acceleration \( a(r_0) \) in the rotating frame. By this equation of motion, periodic orbit with constant control acceleration around artificial equilibrium point can be generated.
When we consider dynamics of spacecraft about an AEP which is on $r_0 = (X_0, 0, 0)$ (Fig. 2), Eq. (5) is rewritten as follows

\[
\begin{align*}
\ddot{\delta}_x - 2\dot{\delta}_y - (2P + 1)\dot{\delta}_x &= 0 \\
\ddot{\delta}_y + 2\dot{\delta}_x + (P - 1)\dot{\delta}_y &= 0 \\
\ddot{\delta}_z + P\dot{\delta}_z &= 0
\end{align*}
\]  

(6)

$P$ is defined as

\[
P = \mu|X_0 + \mu - 1|^3 + (1 - \mu)|X_0 + \mu|^3.
\]  

(7)

By $\mu > 0$ and $1 - \mu > 0$, we have

\[
P(X_0, \mu) > 0.
\]  

(8)

This is very similar form as linearized equation of motion about L2 point which is better known as

\[
\begin{align*}
\ddot{\delta}_x - 2\dot{\delta}_y - (2B_L + 1)\dot{\delta}_x &= 0 \\
\ddot{\delta}_y + 2\dot{\delta}_x + (B_L - 1)\dot{\delta}_y &= 0 \\
\ddot{\delta}_z + B_L\dot{\delta}_z &= 0
\end{align*}
\]  

(9)

where

\[
B_L = \mu|X_L + \mu - 1|^3 + (1 - \mu)|X_L + \mu|^3
\]  

(10)

It can predict that Tarao’s method is applied. In Ref 1 ion engines, which is basically not constraint about direction of acceleration, is assumed and set the equation of motion with control acceleration $(a_x, a_y, a_z)$ as,

\[
\begin{align*}
\ddot{\delta}_x - 2\dot{\delta}_y - (2B_x + 1)\dot{\delta}_x &= a_x \\
\ddot{\delta}_y + 2\dot{\delta}_x + (B_x - 1)\dot{\delta}_y &= a_y \\
\ddot{\delta}_z + B_x\dot{\delta}_z &= a_z
\end{align*}
\]  

(11)

Then it is assumed that constant control rotate at a constant angular rate on the plane which is inclined by $\phi$ on the $\delta_y$ axis from $\delta_y - \delta_z$ plane, while the small halo orbit is on the plane which is inclined by $\theta$ on the $\delta_y$ axis (Figs. 3a and 3b).

Here, when we set solar sail and attitude as shown in Fig. 4, the acceleration by solar sail is decomposed to that on the $\delta_x$ axis and that on the $\delta_y - \delta_z$ plane.

We apply Morimoto’s control low to $a_x$ of control acceleration ($\delta_x$ direction) and Tarao’s low to $a_y$ and $a_z$ (on the $\delta_y - \delta_z$ plane). By $a_x$, the sail orbit must shift to the sun so as to satisfy $a(r_0) = \nabla U(r_0) \cdot a_y$ and $a_z$ in $\delta_y - \delta_z$ orbital plane correspond to the case that $\phi = 0$ in Fig. 3a.
Therefore, the equation of motion by solar sail about AEP is expressed as

\[
\begin{align*}
\ddot{\delta}_x - 2\dot{\delta}_y - (2P+1)\delta_x &= 0 \\
\ddot{\delta}_y + 2\dot{\delta}_x + (P-1)\delta_y &= a_y \\
\ddot{\delta}_z + P\delta_z &= a_z
\end{align*}
\]

(12)

Note that \( a_x \) is already included in the equation as \( a(x_0) \) which is shown in Eq. (4a).

Then the small halo orbit around \( r_0 = (X_0,0,0) \) which is obtained from linearized equation of motion, is

\[
\begin{align*}
\delta_x &= \beta \delta_{y0} \cos \theta \sin \omega t \\
\delta_y &= \delta_{y0} \cos \omega t \\
\delta_z &= \delta_{z0} \cos \theta \sin \omega t
\end{align*}
\]

(13)

where \( \beta \) and \( \omega \) must satisfy the following equation simultaneously,

\[
\begin{align*}
\omega^4 + (4P-2)\omega^2 + (2P+1)^2 \beta^2 - 12\omega P \beta + 2\omega^2 + 2P - 1 &= 0 \\
2\omega - (\omega^2 + 2P - 1)\beta &= 0
\end{align*}
\]

(14)

(15)

Necessary acceleration \( a_y \) and \( a_z \) are obtained by

\[
\begin{align*}
a_y &= \delta_{y0} \cos \omega t \left( -\omega^2 + P - 1 + 2\omega \beta \right) \\
a_z &= \delta_{y0} \sin \omega t \left( -\omega^2 + P \right)
\end{align*}
\]

(16)

See Ref. 1 in detail.

IV. NUMERICAL EXAMPLE

\( a_x \), \( a_y \) and \( a_z \) are depends on \( X_0 \). In addition \( a_y \) and \( a_z \) are also depend on the size of the halo orbits (Eq. 16) while \( a_x \) is independent of it. When we set \( X_0 \) as 1.010064 AU from the common center of the Earth and sun, which is about 1646 km to the Earth from the L2, required \( a_x \) creating for AEP is \( 5.8915 \times 10^{-7} \text{ m/s}^2 \) by Eq. (4a). From Eqs. (7), (14) and (15), \( \omega = 1.7225 \) and \( \beta = 0.2902 \) are obtained. When \( \delta_{y0} \) is set to 15000 km, the magnitude of acceleration on \( \delta_y - \delta_z \) plane is \( 5.8322 \times 10^{-7} \text{ m/s}^2 \). For simplicity of calculation, we consider reflectivity \( \varepsilon = 1 \). This is assumed full reflecting surface (actually reflectivity of solar sail is about 0.85 or so). As the magnitude of acceleration \( a_x \) and that on \( \delta_y - \delta_z \) plane is very similar, we consider they are the same magnitude. Then \( \alpha = 45^\circ \) and Eq. (1) becomes simply

\[
a_s = -P_{\text{SUN}} \frac{1AU^2 \ A}{r_{\text{SUN}}^2} \mathbf{n}
\]

(17)

\( r_{\text{sun}} \) is also considered as constant an it is set to the center of small halo orbit in this paper, because the range of it in small halo orbit is very small.

When mass of spacecraft is 1000 kg, necessary area of solar sail is \( A \approx 185 \text{ m}^2 \). Figs. 5a-5c show an orbit calculated in non-linearized equation of motion. The orbit still have slight shift for 76.24 km to the Sun from the center (1.010064 AU).
V. CONCLUSION

A control low of a small halo orbit in the vicinity of L2 point by solar sails is studied. By combination of two lows studied before we can generated small halo orbit successfully, even though each low can not be applied to solar sails by itself.

References