Experiment of Orbit Determination Method

using Amateur Radio Ground Station

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ABSTRACT

The Ground Station Network (GSN) organized by the University Space Engineering Consortium (UNISEC) has been active to construct a network-based ground station system for satellite operations of university satellites. Kyushu University and Tohoku University participate in GSN and are developing the orbit determination system for this network. In order to perform orbit determination, we measure the Doppler frequency of the satellite’s beacon signal with equipment of GSN and convert this frequency into a range rate measurement. The satellite’s orbital elements are estimated using the recursive least square method. We report the progress of the development and the results of orbit determination for CUTE-I manufactured by Tokyo Institute of Technology.

アマチュア無線地上局を用いた

軌道決定手法実証実験

概要

国内の衛星を開発している大学では、アマチュア無線を利用した地上局を所有している。NPO法人大学宇宙工学コンソーシアム(UNISEC)の作業部会の1つであるGround Station Network (GSN)では、これら各大学が所有する地上局をインターネットを介して相互に利用できるようにする活動を行っている。このプロジェクトの中で、これらGSNの設備を利用し、衛星の軌道決定を行う機能を開発中である。本発表では、現在までの開発状況とこれまでに行ったcubesatを対象とした観測実験の結果について述べる。

1. Introduction

Many small satellites developed at universities are launched in space in last five years. In the cases when these satellites conduct science observation missions, we need the orbit determination function.

The University Space Engineering Consortium (UNISEC) has been organizing a ground station network (GSN). The main purpose of the GSN is to share ground station facilities among the satellites developers in universities.

The UNISEC/GSN working group has proposed and developed the low-cost orbit determination system using amateur radio stations as an additional function of GSN. This system uses only time-series data of Doppler frequency of the beacon signal which can be measured with a spectrum analyzer. This paper shows a summary of the orbit determination system and a few of the experimental results in which the reasonable accuracy has been achieved.

2. Orbit Determination Method

2.1. Coordinate System

The satellite coordinate system shown in Fig. 1 is used in this paper. The \( R \) axis is parallel to the position vector. The \( S \) axis is normal to the position vector in the orbit plane and the \( W \) axis is normal to the orbit plane.

![Fig. 1 Coordinate System](image-url)
The range vector is represented as follows:
\[ \mathbf{p} = \mathbf{r} - \mathbf{R}_\oplus \]  
where \( \mathbf{p} \) is parallel to the line between the satellite and the ground station. \( \mathbf{r} \) is the satellite’s position vector. \( \mathbf{R}_\oplus \) is the ground station position vector from the center of the earth. The distance between the satellite and the ground station is
\[ \rho = ||\mathbf{p}|| \]  
(2)
The range rate which is the time derivative of the range vector is
\[ \dot{\rho} = \dot{\mathbf{r}} - \dot{\mathbf{R}}_\oplus \]  
(3)
From the derivative of \( \rho^2 = \mathbf{p} \cdot \mathbf{p} \), we find:
\[ 2\rho \dot{\rho} = 2(\mathbf{p} \cdot \dot{\mathbf{p}}) \]  
(4)
Therefore eq. (4) is converted to:
\[ \dot{\rho} = \frac{\mathbf{p} \cdot \dot{\mathbf{p}}}{\rho} \]  
(5)
This value is used in the orbit determination method.

2.2. Measurement and Observed Value
The orbit determination method uses only the Doppler frequency. Almost all satellites manufactured by universities transmit a beacon signal using continuous wave (CW). We observe the Doppler frequencies of the CW beacon signal for orbit determination. Doppler frequencies are measured by a spectrum analyzer. The relationship between range rate and Doppler frequency is represented as follows:
\[ f_d = \left(1 - \frac{\dot{\rho}}{c}\right)f_0 \]  
(6)
with \( c \) the velocity of light. The Eq. (6) is converted to:
\[ \dot{\rho} = c \left(1 - \frac{f_d}{f_0}\right) \]  
(7)
The relation between the standard deviation of the observation value and the standard deviation of the measurement is represented by:
\[ \sigma_\rho = \frac{c}{f_0} \sigma_f \]  
(8)

2.3. Measurement Noise
The measurement values include some noise effects. We assume that the measurement values include the following noise sources:
1. Random and bias noise from the spectrum analyzer
2. Random and bias noise from the transmitter on the satellite
3. Time measurement error of the Doppler frequency
4. Measurement error of the position and the velocity of the ground station
5. Effect of the ionosphere
In these experiments, the noise sources 3, 4 and 5 are ignored because they are smaller than the effects from 1 and 2.

2.4. Orbit Determination Filter
For determining the satellite orbit, we use the recursive least square method. Figure 2 shows the diagram of the orbit determination filter.

Fig. 2 Orbit Determination Filter

When the satellite position and velocity vectors are defined as \( \mathbf{r} = \{x, y, z\}^T \) and \( \mathbf{v} = \{\dot{x}, \dot{y}, \dot{z}\} \), the state vector is defined as:
\[ \mathbf{X} = \{x, y, z, \dot{x}, \dot{y}, \dot{z}\}^T \]  
(9)
When the satellite is observed at times \( t_1, t_2, \ldots, t_n \), the series of the observation values are defined as \( \dot{\rho}_1(t_1), \dot{\rho}_2(t_2), \ldots, \dot{\rho}_n(t_n) \), and the observation vector \( \mathbf{Z} \) is
\[ \mathbf{Z} = \{\dot{\rho}_1(t_1), \dot{\rho}_2(t_2), \ldots, \dot{\rho}_n(t_n)\}^T \]  
(10)
The estimated observation vector is defined as follows:
\[ \bar{\mathbf{Z}} = h(\mathbf{X}_{\mathbf{p}}, t_0, t_1, \ldots, t_n) \]  
(11)
h is a function for obtaining the estimated observation vector. The residual between estimated and real observation vector is:
\[ \Delta \mathbf{Z} = \mathbf{Z} - \bar{\mathbf{Z}} \]  
(12)
Because the real observation values include the measurement noise, the true observation vector \( \mathbf{Z} \) is defined as follows:
\[ \mathbf{Z} = \bar{\mathbf{Z}} + \mathbf{v} \]  
(13)
\( \mathbf{v} \) is the observation noise vector.
We assume that the average of observation noise is zero and the variance of the observation value is \( \sigma^2 \). The observation covariance matrix \( \mathbf{R} \) is defined as follows:
The state covariance matrix $\mathbf{P}$ is calculated as follows:

$$\mathbf{P} = (\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H})^T$$

Hence the modification of the state matrix $\Delta \mathbf{X}_0$ is

$$\Delta \mathbf{X}_0 = \mathbf{P}\mathbf{H}^T\mathbf{R}^{-1}\Delta \mathbf{Z}$$

Equation (17) is used for updating the state vector. This filter calculates Eq. (17) recently until the $\Delta \mathbf{X}_0$ becomes zero. The $6 \times 6$ state covariance matrix $\mathbf{P}$ is represented as:

$$\mathbf{P} = \begin{bmatrix} 
\sigma_x^2 & \sigma_{xy} & \ldots & \sigma_{xz} \\
\sigma_{yx} & \sigma_y^2 & \ldots & \sigma_{yz} \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{xz} & \sigma_{yz} & \ldots & \sigma_z^2 
\end{bmatrix}$$

For the simple method, the standard deviation is estimated as:

$$\sigma_x = \sqrt{\mathbf{P}(1,1)} \quad \sigma_y = \sqrt{\mathbf{P}(2,2)} \quad \sigma_z = \sqrt{\mathbf{P}(3,3)}$$

$$\sigma_a = \sqrt{\mathbf{P}(4,4)} \quad \sigma_b = \sqrt{\mathbf{P}(5,5)} \quad \sigma_c = \sqrt{\mathbf{P}(6,6)}$$

Only the diagonal elements are used and the other elements are neglected. The measurement noise value is defined as:

$$\sigma_p = \sqrt{\frac{\sum_{i=1}^{n}(\Delta \rho_i)^2}{n}}$$

3. Observation Equipments

Kyushu University Ground Station (KUGS) is a ground station using amateur radio equipment. The specification of KUGS is shown in Table 1 and Figs. 3 - 4. In this experiment, Doppler frequencies were measured by the spectrum analyzer. The specification of the spectrum analyzer is shown in Table 2.

KUGS is controlled by the Ground Station Management Service (GMS) developed by GSN/UNISEC. GMS automatically controls all equipment of the ground station except for the spectrum analyzer. GMS also receives the status of ground station and downlink data transmitted from the satellites to the TCP/IP port. In this experiment, only the function of controlling antenna angle is used.

The spectrum analyzer is controlled by the software developed at Kyushu University. This software sets the conditions of the measurements and measures the Doppler frequencies. These results are also recorded to the ground station PC.
Table 1 Specification of KUGS

<table>
<thead>
<tr>
<th>Location</th>
<th>N33.59 deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitude</td>
<td>E130.21deg</td>
</tr>
<tr>
<td>Altitude</td>
<td>90m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequency</th>
<th>144-146MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downlink</td>
<td>430-440MHz</td>
</tr>
</tbody>
</table>

| Antenna           | MASPRO WH32N (Cross Yagi Antenna) |

<table>
<thead>
<tr>
<th>Antenna Gain</th>
<th>Uplink 10 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Downlink 12.5 dB</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Half Band Width</th>
<th>144 MHz 33 deg</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>430 Mhz 27 deg</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Low Noise Preamp</th>
<th>Gain 20dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NF 0.4 dB</td>
</tr>
</tbody>
</table>

Table 2 Specification of Spectrum Analyzer

<table>
<thead>
<tr>
<th>ADVANTEST R3261C</th>
<th>9 kHz - 2.6 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>-130dBm +25dBm</td>
</tr>
<tr>
<td>Input Range</td>
<td>30 Hz</td>
</tr>
<tr>
<td>Maximum Resolution</td>
<td>30msec - 1000s</td>
</tr>
</tbody>
</table>

4. Results of Single Ground Station

4.1. Observation Conditions

For the purpose of confirming the performance of the orbit determination, we observed a satellite at KUGS. The target satellite was CUTE-I as shown in Table 3. This satellite always transmits the CW that we use for the orbit determination.

The observation period was from October 31, 2007 to December 26, 2007. The satellite is observed every morning and every evening. The latest 3 days of measurement data was used for orbit determination.

Table 3 Target Satellite

<table>
<thead>
<tr>
<th>Satellite Name</th>
<th>CUTE-I (CO-55)</th>
</tr>
</thead>
<tbody>
<tr>
<td>University</td>
<td>Tokyo Institute of Technology</td>
</tr>
<tr>
<td>Launch Date</td>
<td>June 30, 2003 (JST)</td>
</tr>
<tr>
<td>Size</td>
<td>10cm × 10cm × 10cm</td>
</tr>
<tr>
<td>Frequency (CW)</td>
<td>436.8375MHz</td>
</tr>
<tr>
<td>Transmitter Power</td>
<td>100mW</td>
</tr>
</tbody>
</table>

4.2. Reducing Large Noise Observation Values

For the purpose of getting the low-noise measurement values, we use the filter as shown in Fig. 5. OOBS means Orbit Observation Simulator which generates the quasi-measured values. ODEF means Orbit Determination Filter which estimates the orbit of the observed satellite. Both software programs are also developed at Kyushu University. When this filter is used for this orbit determination method, noise value is assumed to reduce as shown in Fig. 6. The accuracy of orbit determination is improved by this filter as shown in Table 4.

![Fig. 5 Sequence of Modification of Observation Values and Orbit Determination](image)

![Fig. 6 Effect of Modification of Observation Values](image)
<table>
<thead>
<tr>
<th>Effect of Modification of Observation Values</th>
<th>Unit</th>
<th>No Modified</th>
<th>Modified</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS of frequency</td>
<td>Hz</td>
<td>1221</td>
<td>19.7</td>
</tr>
<tr>
<td>Standard Deviation of Observation Values</td>
<td>km</td>
<td>75499</td>
<td>1.97</td>
</tr>
<tr>
<td>Along</td>
<td>km</td>
<td>39359</td>
<td>1.03</td>
</tr>
<tr>
<td>Cross</td>
<td>km</td>
<td>62514</td>
<td>1.65</td>
</tr>
<tr>
<td>of Position</td>
<td>km</td>
<td>9.5</td>
<td>2.6</td>
</tr>
<tr>
<td>Error</td>
<td>deg</td>
<td>0.52</td>
<td>0.013</td>
</tr>
<tr>
<td>i</td>
<td>deg</td>
<td>0.49</td>
<td>0.011</td>
</tr>
<tr>
<td>Ω</td>
<td>deg</td>
<td>0.32</td>
<td>0.0079</td>
</tr>
</tbody>
</table>

### 4.3. Estimated Error of Single Ground Station

In this paper, the reference position is defined by:

\[
\mathbf{r}_{\text{ref}} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{r}_{\text{estimate},i} = \frac{1}{n} \left[ \sum_{i=1}^{n} x_{\text{estimate},i} \mathbf{i} \right] = \frac{1}{n} \left[ \sum_{i=1}^{n} y_{\text{estimate},i} \mathbf{j} \right] = \frac{1}{n} \left[ \sum_{i=1}^{n} z_{\text{estimate},i} \mathbf{k} \right] \tag{21}
\]

The residual between this position and the estimated position is defined as error of this orbit determination.

Figures 7 - 9 are the results of the observations. The horizontal axis is the residual divided by the estimated standard deviation, \( \Delta r/\sigma \). The vertical axis is the rate of the cumulative number of residuals less than \( \Delta r/\sigma \). If this distribution follows the standard normal distribution, the ratio \( \Delta r/\sigma \) within three should be 99.73%. However, the ratio \( \Delta r/\sigma \) reaches the limit of within 99.73%, when the \( \Delta r/\sigma \) is much larger than three in each direction. Therefore, this distribution does not follow the standard normal distribution.

In this distribution, the maximum error of this orbit determination is shown in Table 5. Hence, the maximum position error is estimated as 38 km.

This result fulfills the requirements of tracking satellites because the half-band width is wide. However, this result does not fulfill the requirements of a science mission. For examples, the QSAT mission requires the orbit determination accuracy as shown in Table 5. Therefore, the observation at several ground station is needed for improving the performance of this orbit determination method.

<table>
<thead>
<tr>
<th>Table 5 Difference between the results and requirements</th>
<th>Error</th>
<th>Tracking</th>
<th>QSAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial</td>
<td>7.7 km</td>
<td>-</td>
<td>0.984 km</td>
</tr>
<tr>
<td>Along-Track</td>
<td>11 km</td>
<td>150 km</td>
<td>9.64 km</td>
</tr>
<tr>
<td>Cross-Track</td>
<td>24 km</td>
<td>150 km</td>
<td>12.7 km</td>
</tr>
<tr>
<td>Distance</td>
<td>38 km</td>
<td>214 km</td>
<td>16.0 km</td>
</tr>
</tbody>
</table>
5. Observation by Two Ground Station

For the purpose of observing the satellite at several ground station, the spectrum analyzer controller via the Internet was developed. The main ground station has the client software and the remote ground station has the server software as shown in Fig. 10. The client sends the command to the server in the remote ground station via the Internet. The server controls the spectrum analyzer as the single ground station. The antenna control is conducted by the Ground Station Management Service. The results of measurements are sent to the client by the server.

We plan to observe CUTE-I at Tohoku University Ground Station and KUGS using this system.

6. Conclusions

This paper shows the summary of the orbit determination method using amateur radio ground station via the Internet. The results of orbit determination are shown in this paper. We are able to get reasonable results of orbit determination for tracking satellites. However, these results cannot be used for science observation missions like QSAT. We plan to observe the satellite at two ground stations and do confirm the improvement of the orbit determination performance.

References


