Expanding opportunities for lunar gravity capture

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Abstract

One of the less delta V Earth-Moon transfer trajectories is achieved by using lunar gravity capture. However, whether a spacecraft is captured by the moon or not depends on how the spacecraft approaches the moon. This paper presents how the chance of lunar gravity capture will be expanded using the solar tidal force and low thrust propulsion.

1 INTRODUCTION

The concern with an exploration for the Moon has been growing. For example, the Japanese observation satellite Kaguya has been launched recently and played important role in finding out the origin and the evolution of the Moon.

What is important in a recent lunar exploration is to design as low delta-V Earth-Moon transfer trajectories as possible. When a spacecraft plans to go to the Moon with Hohmann transfer orbit, it needs to have about 0.8 km/s velocity decrease in order to enter a lunar orbit. Hohmann trajectory has the merit of little transfer time but requires a lot of fuel to have a spacecraft send the orbit. It can be gotten by solving the Earth-Spacecraft Two Body problem. But if you want to design a lower delta-V trajectory, you have to consider the solar perturbation, that will influence the movement of a spacecraft during the flight outside the earth gravity field. One of the transfer schemes in which a spacecraft uses solar perturbation effectively is illustrated in fig.1. In this trajectory, a spacecraft swings-by the moon and flights in the far boundary region of the Earth gravity field using the solar perturbation. By using the force of the Sun, it can save so much delta-V, that is, can save fuel to increase or decrease its speed.

Considering a Earth-Moon transfer trajectory like that, whether a spacecraft can be captured or not depends on when it swings-by the Moon and where approaches it from. In other words, the Moon’s age of a lunar swing-by is almost uniquely determined. Therefore, such spontaneous capture is hardly applicable to the flexible and practical transportation systems to the moon. The purpose of this study is to show how the lunar swing-by conditions is relaxed taking the advantage of the artificial acceleration, electric propulsion means.

Figure 1: Earth-Moon transfer trajectory
2 FORMULATION OF THE PROBLEM

2.1 THE DEFINITION OF LUNAR GRAVITY CAPTURE

Gravity Capture is a useful way to achieve the low delta V transfer from the Earth to the Moon. It is a mechanism by which an object from outside the sphere of influence can orbit temporarily around a central body without any other effects such as atmospheric drag. Fig. 2 shows an example of gravity capture trajectory and the potential curve around the Moon, which is calculated in Earth-Moon-Spacecraft Three Body problem. A spacecraft can only move in the restricted area which contains L1 and L2 points, that is, it can approach or leave the Moon only from Earth side or anti-Earth side.

When the spacecraft enters the lunar sphere of influence, if the orbital energy is negative, it moves in elliptical orbit there. Thus, lunar gravity capture is defined as follows in this paper.

- A perilune is in the lunar sphere of influence (R=66300 km).
- The two-body Keplerian energy of the spacecraft to the Moon at perilune is negative.

\[ E_{\text{Perilune}} = \frac{v^2}{2} - \frac{\mu_{\text{Moon}}}{r} < 0 \]  

![Figure 2: a gravity capture orbit (left) and potential curve around the Moon (right)](H. Yamakawa, AAS-92-633)

2.2 The equations of motion

The Circular Restricted Four-Body Problem (CR4BP) is adopted as a model. The Sun and the Moon orbit around the Earth in circular motion in the same plane. The equations of motion of a spacecraft in the geocentric inertial frame are shown in Eq.(2). In these equations the solar perturbation and the lunar perturbation are represented as \( f_{\text{Sun}} \) and \( f_{\text{Moon}} \). To analyze the motion of the spacecraft in CR4BP, both perturbations must be considered.

\[
\frac{dx}{dt} = -\mu_{\text{Earth}} \frac{x}{|x|^3} + f_{\text{Sun}} + f_{\text{Moon}} \\
 f_{\text{Sun}} = -\mu_{\text{Sun}} \left( \frac{x - x_{\text{Sun}}}{|x - x_{\text{Sun}}|^3} + \frac{x_{\text{Sun}}}{|x_{\text{Sun}}|^3} \right) \\
 f_{\text{Moon}} = -\mu_{\text{Moon}} \left( \frac{x - x_{\text{Moon}}}{|x - x_{\text{Moon}}|^3} + \frac{x_{\text{Moon}}}{|x_{\text{Moon}}|^3} \right)
\]  

3 CAPTURE WINDOW IN BALLISTIC FLIGHT

This section describes the feasibility of a ballistic lunar capture. As mentioned above, when a spacecraft swings by the Moon strongly affects whether it will be able to success a lunar gravity capture.
So, I will particularly focus on the relationship between the timing of a lunar flyby and the possibility of a lunar gravity capture. The calculation starts with a lunar swing-by varying specific control variables and targets to a ballistic capture. There are three control variables: the swing-by velocity, the swing-by direction, and the swing-by Moon’s age. The swing-by velocity and the swing-by direction are defined with respect to the Moon’s motion. The swing-by velocity has 0.9 to 1.1 kilometers per second range. The swing-by Moon’s age, which means the Moon’s age a the spacecraft swings by the Moon, has 0 to 180 degrees range (see Fig.3).

Fig.4 shows the capture orbit when the swing-by Moon’s age is 90 degrees. These top three figures show the same trajectory in different frames. Starting from the top left, the trajectory in the Sun-Earth fixed rotating frame, in the inertial frame, and in the Earth-Moon fixed rotating frame. In this case the altitude of the perilune is 10420 km. Below charts show the histories of relative distance, relative velocity and Keplerian energy to the Moon. The apparent orbital energy to the Moon is -0.076 km²/s² at perilune. From these figures you can see that, the spacecraft makes a single orbit around the Moon after arriving at the lunar sphere of influence. Thus this is the success case of the ballistic lunar capture.

Fig.5 indicates the relationship between the swing-by Moon’s age and the two-body Keplerian energy at perilune. It shows that the spacecraft which swings by the Moon when the Moon’s age is between 70 and 130 degrees can reach a perilune with negative energy. This means that the window of ballistic lunar capture is 60 degrees out of 180 degrees. And because of its dynamical symmetric property, the tendencies between 180 and 360 degrees are expected to be the same as those of between 0 and 180 degrees. In fact the values whose initial conditions are 0 and 180 degrees are the same.

Figure 3: Swingby Condition

Figure 4: Ballistic capture example: the swing-by Moon’s age = 90 degrees
In this section, it is proved that using low thrust acceleration can expand the window for lunar gravity capture. In ballistic flight, the range in which the keplerian energy at perilune become negative is only 60 degrees out of 180 degrees and that is the window for capture. Here, I make the orbital energy of other initial flyby conditions decrease to less than zero by using artificial propulsion.

We have an assumption that the spacecraft has 1000 kg constant mass though the flight, and can accelerate with 100 mN propulsion. DCNLP method is adopted to optimize calculations of trajectories. It is a computation method which approximates the continuous problem with parameter optimization problem. And in this study a trajectory is divided into 30 segments and approximated by interpolating polynomials.

Fig.6 represents one of the the results of the optimizations. The spacecraft swings by the Moon when the Moon’s age is 50 degrees and arrives at the perilune with negative energy after 70 days travel. The arrows which are drawn in the top center figure, that is the trajectory seen in the inertial frame, indicate the direction of the thrust force. The main point is that this optimized trajectory has two different parts. One is called 'Thrust Arc', in which the spacecraft uses the propulsion to control its maneuver. Second is 'Coast Arc', in which it flights ballistic. Below right of Fig.6 shows the change of thrust control to the time. You can also see from this graph both Thrust Arc and Coast Arc. In this case, since the trajectory has one Thrust Arc, it is called one burn transfer. As a result the spacecraft succeeds in being captured by the Moon. Fig.7 indicates how much delta V is needed to decrease orbital energy to zero at each initial flyby condition. From this graph you can see that a spacecraft will be able to succeed a lunar gravity capture with the velocity increment of several hundred meters per seconds.
5 DESIGN OF A TRAJECTORY FOLLOWING A PARTICULAR CAPTURE ORBIT

Here I discuss how to design a trajectory following a particular capture orbit. Since the capture state at the Moon is unstable, the backwards integration from perilune is useful to get the capture orbit which has a certain perilune and duration of stay. In the backwards integration the trajectory is integrated backwards in time. Outside the lunar sphere of influence the trajectory is calculated in the forward time integration and optimized to be connected with a following capture orbit on the boundary of the lunar sphere of influence (Fig.8).

Fig.9 shows the example of a capture orbit around the Moon. This trajectory is gotten by integrating from the perilune whose altitude is 5000 km. The spacecraft orbits around the Moon two times and stays 20 days in the lunar sphere of influence. Next, the trajectory from lunar swing-by to re-approach is optimized to follow this capture orbit in DCNLP method.
Fig.10 suggests the result of this optimization. The initial condition is 50 degrees. While the necessary delta-V is 179 m/s under the condition that the Keplerian energy at perilune is zero, it needs 215 m/s velocity increment to make the spacecraft to go on the capture orbit shown in Fig.10. That is, the better capture orbit you want to get, the more velocity increment the spacecraft needs.

6 CONCLUSION

First, I showed that the spontaneous capture is difficult to achieve. The window for the gravity capture is 60 degrees out of 180 degrees from the stand point of the swing-by Moon’s age. But by using electric propulsion only several hundred meters per second velocity increments can makes a spacecraft sent to a lunar gravity capture orbit. In spite of the swing-by Moon age, the two body Keplerian energy at perilune decrease to less than zero. And the backwards integration is useful when you want to assume a particular capture orbit. You can design various capture orbits as long as the propulsion allows. With this merit, the combination of low thrust propulsion and gravity capture is powerful way for future lunar exploration.

References

