A Study on Trajectory of Round Trip between Earth and Mars

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Abstract: Space Agencies in several countries, they plan Mars exploration, like sample return mission and human exploration mission. For achieve these projects, we need to study trajectory design of round trip between the earth and the mars. The trajectories are required to be not only maximizing transportation capability, in other words minimizing fuel consumption, but also minimizing mission duration. However, these requests are basically opposed.

In this paper, we discuss trajectory of round trip between the earth and the mars, using chemical propulsion. Then we make comparison for mission capability estimation in some mission duration.

1. Introduction

The Mars is the most interesting planet in solar system and has been explored it for many years by several space agencies. Recently Phoenix, the explorer by NASA, examined Martian surface. And decade ago, Nozomi, the explorer by JAXA, tried to observe Martian atmosphere.

On another front, sample return mission and human exploration mission have received attention in recent years. These missions will be applied to Mars exploration. Many researchers try to examine these assignments.

To achieve the mission, like sample return and human exploration, we need to study round trip trajectory planning. The trajectories are required to be not only maximizing transportation capability, in other words minimizing fuel consumption, but also minimizing mission duration. However, these requests are basically opposed.

So this paper represents discuss trajectory of round trip between the earth and the mars, using chemical propulsion. Then we make comparison for mission capability estimation in some mission duration.
Previous studies of round trip between Earth and Mars were examined. In 2004, the round trip trajectory that is from Earth to the Mars, Venus, and Jupiter was studied by James R Weltz\cite{1}. In addition, the round trip by combined chemical and electrical propulsion and the property of round trip trajectory was studied A.Miele et al\cite{2}\cite{3}\cite{4}. And the mission opportunities for human exploration mission were studied by L.Casalino et al \cite{5}.

2. **Hohmann Transfer**

In this paper, we assumed that planetary revolving orbits are circular and co-planar orbits and the spacecraft flights only by sun gravitational field.

In circular and co-planar cases, hohamnn transfer is a trajectory known as minimum energy transfer. The following is a sample of one-way transfer between Earth and Mars. For earth departure

\[ \Delta V_p = \frac{\mu_s}{r_p} \left( \sqrt{\frac{2r_p}{r_a + r_p}} - 1 \right) \]

\[ \approx 2.94 \text{[}km/s\text{]} \] (2.1)

where \( \mu_s \) is sun gravitational constant which is \( 1.327 \times 10^{11} \text{ km}^3/\text{s}^2 \), \( r_p \) is perihelion which is 1 AU, and \( r_a \) is aphelion which is 1.524 AU.

In addition, for mars arrival

\[ \Delta V_a = \frac{\mu_s}{r_a} \left( 1 - \frac{2r_p}{r_a + r_p} \right) \]

\[ \approx 2.64 \text{[}km/s\text{]} \] (2.2)

Therefore, total delta-V is approximately 5.58 km/s from equation (2.1) and (2.2).

And the flight time is represented as follows.

\[ t = \frac{1}{2} \sqrt{\frac{\mu_s}{a^3}} \] (2.3)

Therefore, hohmann transfer time is approximately 256 days.

On the other hand, round trip hohmann transfer, (for the examples, see ref. [1]) we have

Total flight time: \( \approx 971 \) days
Wait time at mars: \( \approx 453 \) days
Total delta-V: \( \approx 8.22 \text{ km/s} \)

Fig.1 shows round trip hohmann transfer.

![Fig.1 hohmann transfer round trip](image)

3. **Lambert Problem**

To determine transfer between Earth and Mars, the lambert problem is employed \cite{7}\cite{8}. The lambert problem that is effective method for obtain trajectory between two planets, can solve providing we have already two planetary locations and the flight time.

The calculate process is represented as follows.

1. Obtain the parameter;

\[ s = \frac{r_1 + r_2 + c}{2} \] (3.1)

\( r_1 \) is distance from the departure planet to the sun, \( r_2 \) is distance from the arrival planet to the sun, and \( c \) is distance between planets.

2. Obtain parameter \( \alpha \) and \( \beta \) from following equation.
3. Obtain eccentricity to $a$, which is supposed, from following equation.
4. Do iterative calculation for $2 \sim 3$ by operate $a$.
5. Obtain orbital six elements of transfer and velocity at departure and arrival point according to obtained conclusive $a$.

This approach can solve no lap trajectory around the sun. More details of calculate method leaves out.

4. Approach

This section presents the calculation routine that is to obtain minimum delta-V in some stay duration at mars.

1. We consider that outgoing trajectory is hohmann transfer with either stay duration, because it can move with minimum expenditure of energy as indicated above.
2. Set stay duration at mars.
3. Set return time to Earth. And obtain the transfer orbit by solving Lambert problem. Therefore we can obtain delta-V for return to Earth.
4. To obtain minimum delta-V in fixed stay duration at Mars, return time is moved from 100 days to 700 days every 10 days about third process.
5. Then, choice minimum delta-V by results in fifth process.
6. Repeat from second to fifth process while stay duration time is moved from 30 days to 750 days (approximately one Mars year) every 30 days.

Fig.2 shows the conceptual figure with respect to using chemical propulsion.

Next step is to examine the mission capability. The mission capability is evaluated by payload mass. The payload mass is derived from the initial mass (the wet weight) by loss of the fuel mass.

$$m_{pl} = m_0 - m_p$$  \hspace{1cm} (4.1)  

$m_0$ is the initial mass which is 1000 kg, $m_{pl}$ is the payload mass, and $m_p$ is represented as follows.

$$m_p = m_0 \left[1 - \exp\left(-\frac{\Delta V}{g I_{sp}}\right)\right]$$  \hspace{1cm} (4.2)  

$g$ is the gravitational acceleration which is 0.0098 km/s$^2$, $I_{sp}$ is the specific impulse which is 300 s.

5. Results

Fig.3 shows the total delta-V to the total flight time and the stay duration at Mars. The smallest delta-V in some flight time is 8.1823 km/s at 969 days on the total flight time and 450 days on the stay duration at Mars. These results indicate hohmann transfer round trip, and give close agreement with the analysis results that was indicated in second section.

Then, Fig.4 shows the payload mass to the total flight time and the stay duration at Mars. The largest payload mass in some flight time is 9.258 kg at 969 days on the total flight time and 450 days on the stay duration at Mars. These results indicate hohmann transfer round trip and that the fuel mass accounts for more 99 % of the total mass.
Fig. 3 total delta-V versus flight time. The flight time involves total flight time and stay duration at Mars.

Fig. 4 payload mass versus flight time. The flight time involves total flight time and stay duration at Mars.

6. Summary

We discuss trajectory of round trip between the earth and the mars, using chemical propulsion. Then we make comparison of each case for mission capability estimation in some mission duration in this paper.

The hohmann transfer round trip can gain the largest payload mass in some stay duration. But its mass ratio is more 0.99; therefore it is not distant idea for carrying out sample return mission and human exploration mission.

For several years, low-thrust flight that is using electrical propulsion is adopted in the deep-space interplanetary mission and its research is examined actively.

The characteristics of electrical propulsion is to flight with low thrust. Compared to using chemical propulsion, thrust force is less than it. But electrical propulsion can actuate for a long time. Then when employing equivalent amount of propellant, electrical propulsion can more accelerated to high speed than using chemical propulsion.

One of the successful examples of electrical propulsion is HAYABUSA. HAYABUSA, the Japanese asteroid spacecraft, adopts ion engine.

In this paper, the focus is using chemical propulsion, we need to examine round trip between Earth and Mars by using electrical propulsion in the future.

7. References