Orbit Determination of Small Solar Sail Demonstration Spacecraft

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This study investigates the trajectory analysis of small solar sail demonstration spacecraft IKAROS considering the uncertainty of solar radiation pressure. Estimation of solar sail force model in space is the key factor for successful solar sail navigation because the solar sail have large uncertainty due to the flexible membrane. Since the sail wrinkles after the deployment and its surface will suffer from degradation, the solar sail force model is difficult to develop on the ground. In this paper, a practical analysis of estimating the solar sail force model from Doppler and range observable is investigated. This is demonstrated by orbit determination including parameter estimation of solar sail model. Some examples are described to investigate better parameters to estimate the solar sail force model.

1. Introduction

Recently, the target of space explorations become further and further from Earth. For instance, the interest in small celestial bodies are going to move on main-belt and Trojan asteroids from near Earth objects. Since these space explorations require huge energy to achieve their objectives, high efficient propulsion system (i.e. electric propulsion, solar sail, etc.) have developed. Hayabusa\textsuperscript{1)} and Dawn spacecraft are now cruising deep space with electric propulsion system. Although more efficient propulsion system are required to expand deep space explorations.

Solar sail have a capability to receive solar photon momentum and obtain acceleration without fuel. Many application for the innovative space mission are investigated and planned using solar sail.\textsuperscript{2)}–\textsuperscript{5)} Also several concept and project are going on. UrtraSail is the solar sail concept which has several “brades” of solar sail firm material with a microsattelite on the tip.\textsuperscript{6)} This system uses a solar sail firm of densities approaching 1 g/m\textsuperscript{2} for a very large, and high acceleration due to 1 km\textsuperscript{2} solar sail areas. Cosmos 1 was a project to test the solar sail technology, funded by Planetary Society.\textsuperscript{7)} Although, the 600 m\textsuperscript{2} mylar sail was planed to launch into the Earth orbit with inclination of 80 deg by converted missile, the launch was failed and the spacecraft failed to reach the orbit.

Japan Aerospace Exploration Agency (JAXA) is planning “solar power sail”, which is a hybrid system using ion-propulsion engines and solar sail. JAXA is now studying two missions to demonstrate solar power sail. One is small-size solar sail demonstration spacecraft named IKAROS (Interplanetary Kite-craft Accelerated by Radiation Of the Sun), which has 20 m solar sail with solar cells on the membrane. The minimum success criteria of this spacecraft are the demonstration of the solar sail deployment system and power supply from the thin-film solar cell. The full success criteria of the spacecraft are acceleration and navigation using the solar sail in the interplanetary trajectory. Another spacecraft is the medium-sized solar power sail, which has ion propulsion system with solar sail of diameter 50 m. This spacecraft is planning the launch at mid-2010s and its destinations are the Jupiter and the Trojan asteroids.

A precise solar sail force model is the key factor of...
successful navigation for the solar sail spacecraft. Many prelaunch experiments have held to confirm the performance of solar sail deployment and stability.\textsuperscript{5,9} Although conventional spacecraft model can be based on ground-based measurements, solar sail force model needs to be refined after launch. Since a practical solar sail requires large area to obtain a force from solar radiation pressure, a sail needs to be deployed in space. The deployment may occur some wrinkles and deformation from nominal shape that expected before launch. Also the uncertainty becomes much larger since the sail surface will suffer from degradation. There is some studies about the on-orbit calibration of solar sail model. Wharton and Hoot shows the technique of astronomical observation for solar sail orbit determination, thrust performance verification, and optical model validation.\textsuperscript{10,11} Heaton and Brinckerhoff present the solar sail model validation from NASA historic Echo balloons trajectories.\textsuperscript{12} The Echo balloons are comprised of aluminized Mylar, were in low Earth orbits. However, these method is available if the spacecraft orbits around Earth and this strategy cannot adapt for IKAROS spacecraft. Ríos-Reyes and Scheeres investigated a methodology for estimating the force and moment generated by arbitrary shape using generalized sail model based on in-flight data.\textsuperscript{13} However they assumed ideal in-flight data measured using inertial measuring units.

This paper investigates the simulation of solar sail force estimation using measurement with Doppler and range measurement based on realistic sequence of IKAROS. Since an actual navigation data is not available right now, pseudo navigation data is generated using the multi plane model that considers the deformation and detail optical properties of the solar sail. Several simulations are demonstrated to investigate the better parameter for estimating the solar sail force model for IKAROS.

2. Solar Sail Model Estimation

Estimation of solar sail force means a parameter estimation of solar sail model. Since IKAROS’s solar sail is composed by flexible membrane, the sail may have a large uncertainty due to the deployment after the launch. The complex solar sail model like generalized sail model\textsuperscript{13} could adapt to the uncertainties by estimating a lot of parameters, though it requires the long term tracking data including various attitude of the sail with respect to the Sun. It’s because some of the parameters could not distinguish without the deviation of solar sail force with respect to the sail’s attitude. Long duration is required to obtain the various solar sail force, because IKAROS spacecraft could not change its attitude quickly due to its large moment of inertia and the capability of attitude control system. The main measurement to estimate solar sail model is radiometric observables (range and Doppler observable), and those observables detect small maneuvers due to the attitude control and the trajectory correction. The maneuvers affect the solar sail model estimation because the maneuvers also have large uncertainties. Since those small maneuvers are required during the whole mission sequence, it’s difficult to obtain long period tracking data without any maneuvers.

The estimation of solar radiation pressure (SRP) force is implemented by two stages to avoid the maneuver uncertainties. In first stage, SRP acceleration is estimated using simple solar sail model with short period orbit determination (OD). In this phase, single plate solar sail model is used to estimate the solar sail force. Short period tracking data is not available to estimate SRP force with complex solar sail, because it only include the limited amount of information about solar sail force in terms of sail’s attitude. In next stage, precise solar sail model is estimated using the results of several SRP accelerations from short period OD. This two-stage estimation allows us to exclude the bad noise due to small maneuvers from the tracking data. This study focus on the first stage estimation in terms of the impact of solar sail uncertainty. The suitable estimated parameter for the first stage estimation is investigated to produce precise result for final solar sail model.

2.1. Single plane solar sail model

The single plane solar sail model is implemented to estimate SRP acceleration with local state estimation (first stage estimation). This model assumes a solar sail as a plane composed by a unique material. The acceleration is modeled as following equation, considering absorbing and reflecting component of SRP.

\[
\ddot{r}_{SRP} = - P \frac{1AU^2}{r^2} \frac{A}{m} \cos \theta \left\{ (1 - \epsilon) e^{SUN} + \left( 2 \epsilon \cos \theta + \frac{2}{3} \right) n \right\}
\]

where \( P \) is the solar radiation pressure at sun distance \( 1AU \), \( r \) is sun-spacecraft distance, \( A \) is the effective cross section area, \( m \) is the spacecraft mass, \( \epsilon \), \( \tau \) is the solar sail specular and diffusive reflectivity, respectively. \( n \) is the normal vector of the solar sail and \( e^{SUN} \) is the unit vector of the sun direction with respect to the spacecraft, \( \theta \) is called as solar cone angle and calculated by \( \cos \theta = n^T e^{SUN} \), and those vectors are described in Fig. 1. The spacecraft bass area is neglected assuming the solar sail area is larger enough than the area of the spacecraft bass system. In this model, spacecraft area and specular and diffusive reflectivity can be estimated parameter, because those parameters may have large uncertainty due to the deployment or degradation. However it is difficult to estimate all those parameters in short period OD, because it requires a measurement with various solar cone angles to distinguish those parameters. Therefore we need to select best parameter in terms of estimating SRP force. In this research, specular reflectivity or spacecraft area are selected as an estimate parameter.
The covariance of the SRP acceleration $P_{SRP}$ is quantified by covariance of estimate parameter $P_X$ using following equation.

$$P_{SRP} = \left( \frac{\partial \ddot{r}_{SRP}}{\partial X} \right) P_X \left( \frac{\partial \ddot{r}_{SRP}}{\partial X} \right)^T$$  \hspace{1cm} (2)$$

where, $\partial \ddot{r}_{SRP}/\partial X$ is a partial derivative of SRP acceleration with respect to a estimate parameter $X$.

### 2.2. Multi plane solar sail model

Multi plane solar sail model is implemented as a "true" trajectory of the solar sail spacecraft. This model considers the sail deformation and several optical properties due to multiple materials. The deformation of the sail is caused by SRP and centrifugal force. Tsuda\(^{14}\) developed the analytical solution of deformation for rotating membrane, and the deformation described as following equation.

$$z_b = \frac{4P}{(3 + \nu) \mu \Omega^2} \log \left( \frac{\sqrt{x_b^2 + y_b^2}}{r_a} \right)$$  \hspace{1cm} (3)$$

where $x_b, y_b, z_b$ are the components of the solar sail fixed coordinate $(X_b, Y_b, Z_b)$, respectively. $X_b-Y_b$ plane is the plane perpendicular to the spin axis, and $Z_b$ axis is the direction of angular momentum vector. $\nu$ is Poisson’s ratio, $\mu$ is the plane density, $\Omega$ is the rotation rate of the spacecraft, and $r_a$ is the inner radius (the boundary between the membrane and the spacecraft hub). In this research, Poisson’s ratio $\nu$ is 0.3, the plane density $\mu$ is 0.01 kg/m\(^2\), rotation rate $\Omega$ is 1 rpm, inner radius $r_a$ is 0.75 m and deformation is described in Fig. 2. The out-plane deformation is about 10 cm.

The multi plane model is implemented by 3 categories, 48 nodes. 3 categories indicate aluminized membrane, solar cells and liquid crystal. Each category has own optical parameters (specular/diffusive reflectivity and absorbing coefficient) and each node has own area and normal vector considering the deformation. The optical properties of multi plane solar sail model are summarized in Table 1.

The acceleration of multi plane solar sail model is calculated as a summation of single plane model and described as follows:

$$\ddot{r}_{SRP} = t - P \frac{1AU^2}{r^3} \sum_{k=1}^N \left[ \frac{1}{r^2} \left( 1 - e_k \right) e_{SUN} + \left( \frac{1}{3} \cos \theta_k e_{SUN} + \left( \frac{2}{3} \cos \theta_k + e_k \right) \bar{n} \right) \right]$$  \hspace{1cm} (4)$$

where $N$ is the number of the node. In this study, $N$ is 48 and each node has each normal vector.

### 2.3. Difference of single and multi plane solar sail model

The difference of the two models is calculated assuming the transfer trajectory to Venus from Earth. The material of the single plane model is assumed as aluminized membrane. The normal vector of the sail lies on the trajectory plane of the spacecraft and the solar cone angle of solar sail during the interplanetary space is described in Fig. 3. Since the angle varies only between 0 to 45 degrees, we cannot detect the characteristics of the solar sail in case of shallow insertion of solar radiation pressure. Even the maximum change of the solar cone angle is about 1 degree per day, The plot of trajectory is described in Fig. 4 with Sun-Earth line fixed coordinate frame.
The SRP acceleration with respect to the solar cone angle is described in Fig. 5 as out-plane and in-plane components in spacecraft body fixed frame. The in-plane acceleration is caused by the absorbing component of solar radiation pressure. The acceleration due to solar radiation pressure is shown in Fig. 6. The components in J2000.0EQ are described separately. Comparing the acceleration of multi plane solar sail model with single plane solar sail model, the tendency of the acceleration is same, though the amplitude of the acceleration is different. The effect of the deformation will effect as scaling down of the area, because we assume only symmetric deformation. However the deformation of the sail is small (see previous section), the main factor of this difference in acceleration is the difference of optical properties.

The uncertainty effect of the attitude determination is evaluated using the single plane model. The uncertainty of attitude is described as the uncertainty of solar cone angle and the uncertainty is assumed as 0.5, 1.0 and 5.0 degrees. The effect is shown in Fig. 7. The effect is illustrated with the ratio of the covariance error and the magnitude of SRP acceleration. Since the solar cone angle varies 0 to 45 deg according to IKAROS’s mission sequence, it is found that the attitude accuracy need to be less than 0.5 deg to determine the SRP acceleration with 1 % accuracy.

3. Solar Sail Force Short Period Orbit Determination

Since the actual navigation data is not available yet, the pseudo tracking data is defined using the multi plane solar sail model which described in the previous section. The short period orbit determination is investigated assuming actual mission sequence of IKAROS spacecraft.

3.1. Condition of pseudo tracking data

The pseudo tracking data is generated using multi plane solar sail model. The epoch of the orbit determination is at the head of tracking data. The interplanetary trajectory is assumed as a direct transfer to Venus. The tracking station is assumed as Usuda deep space center 64 m antenna. A set of short period tracking data is generated by 3 days observation. 5 hours operation per day is assumed and 5 hours Doppler and an hour range observable is obtained in the operation. A set of tracking data is obtained every 2 weeks, and total 11 sets of
Figure 7. Uncertainty of SRP acceleration caused by the attitude uncertainty (Out-plane).

Table 2. Condition of estimation.

<table>
<thead>
<tr>
<th>Observable</th>
<th>X/X 2-way Doppler</th>
<th>X/X 2-way range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimation method</td>
<td>Weight least squares</td>
<td></td>
</tr>
<tr>
<td>Force model</td>
<td>Sun and planetary gravity Solar radiation pressure</td>
<td></td>
</tr>
<tr>
<td>Earth orientation model</td>
<td>IAU 2000A CIO based</td>
<td></td>
</tr>
<tr>
<td>Ground station</td>
<td>Usuda 64 m</td>
<td></td>
</tr>
<tr>
<td>Observable accuracy</td>
<td>Doppler 1.0 mm/s Range 100 m</td>
<td></td>
</tr>
</tbody>
</table>

tracking data is calculated for this research. The geometry of OD epoch is shown in Fig. 4. The 1-sigma white noise is assumed as 0.5 mm/s and 10 m for Doppler and range observable, respectively. These assumption is determined by the heritage of Hayabusa operation.

3.2. Condition of estimation

The estimation method is conventional weight least squares. Planetary perturbation using JPL ephemeris DE405 is considered for trajectory propagation. Single plane solar sail model is implemented for SRP acceleration. The Earth orientation model for terrestrial to celestial coordinate transformation is compliant with IAU 2000A CIO based. These assumption is summarized in Table 2.

Three estimation cases are calculated to demonstrated the impact of the uncertainty of solar sail parameters. The uncertainties of the solar sail are described in sail’s area and its specular reflectivity of the surface. The consider parameter deals with the uncertainty of unestimated parameter, and its value is given as a gaussian distribution. The deviation of the solar sail parameters are decided by the ground based experiment and the capability of onboard camera. The three set of estimated and consider parameter for the simulation is summarized in Table 3.

| Case 1 State vector Reflectivity Sail’s area (20 %) |
| Case 2 State vector Reflectivity Sail’s area (20 %) |
| Case 3 State vector Reflectivity Sail’s area None |

Figure 8. Difference between the X component of true and estimated SRP acceleration (Case 1).

3.3. Results

The estimate parameters are evaluated by the difference between true and estimated solar radiation pressure acceleration and described in Fig. 8 to 10. The result for case 1 and 2 includes the variation due to the consider parameter. The variation of case 1 and 2 are about 10 % of SRP acceleration, however some of the phase have a significant error in the estimation. The worse phase depends on the estimated parameter. This is caused by the relationship between the sensitivity of SRP acceleration with respect to the estimated parameter and line of sight direction. When those vector is orthogonal, the observable cannot obtain the information about the SRP acceleration. The estimation of case 3 shows better results than case 1 and 2. There is a phase with large error, though the amplitude is smaller than other cases. The common characteristic for all three cases is shown in the solution around 120 to 140 days. The solar cone angle are zero in this phase, so the sensitivity of SRP acceleration with respect to both reflectivity and area are same direction. In this phase, the estimated SRP acceleration is accurate, though the two parameters couple and the estimated values becomes unreliable.
4. Conclusion

The trajectory analysis of small solar sail spacecraft IKAROS in terms of estimating solar radiation pressure force was investigated based on the force represented by multi-plane solar sail model. The multi-plane solar sail model presents the solar sail force including the impact of sail’s deformation and non-uniform optical properties. The uncertainty effect of the attitude determination is evaluated and it is found that attitude accuracy need to be less than 0.5 degrees to determine the SRP acceleration with 1% accuracy.

Short period orbit determination is discussed considering the mission sequence of IKAROS to find the better combination of the estimate parameter. It is found that orbit determination with estimating both specular reflectivity and spacecraft area is suitable for the estimate parameter. Some phase are confirmed to provide unreliable result due to the relationship between sensitivity of solar radiation pressure acceleration with respect to estimated parameter and line of sight direction.

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Reference