Digital Adaptive Control of a Winged Experimental Rocket

Tomoaki Shimozawa, Shinichi Sagara and Koichi Yonemoto, Kyushu Institute of Technology

Abstract: Since spaceplanes have wide range flight conditions, the values of parameters of the dynamic equation are not constant. Then some adaptive control methods for the spaceplanes have been proposed and digital control systems are suited for digital computers. The mathematical model of spaceplanes considering actuator dynamics become non-minimum phase system depending on values of discretizing zeros. In this paper, for a winged experimental rocket an adaptive flight control system considering non-minimum phase characteristic is designed and the simulations are performed to verify the effectiveness of the control system.

ある有翼ロケット実験機のディジタル適応飛行制御

九州工業大学　下沢 智啓　相良 憲一　米本 浩一

1. Introduction

Reusable space transportation systems are necessary for the space development and should correspond to various missions to advance the space development. Since reusable launch vehicles (RLV) have wide range flight conditions, the values of parameters of the dynamic equation of the RLVs are not constant. Therefore, a gain scheduling control method 1) has been examined. However, when the air traffic window expands, the design work becomes complex and correspondence to the abort flight is difficult.

On the other hand, adaptive control methods have been researched for the change of the dynamics of the RLVs that originates in the change in the flight conditions 2), 3). However, when the order difference between the denominator and the numerator of the transfer function for the continuous time models is two or more even if the zeros of the transfer function is steady, it is known that there is a possibility that unstable zero is caused when discretization it. The model of the fluid dynamics that influences RLV is uncertain and, it cannot be declared that there is no possibility that the zeros of the controlled systems become unstable because the linearized equation of motion of RLV is used in the control system design. Systems having unstable zeros are said to non-minimum phase systems. Therefore, applying adaptive control methods using zero-pole cancellation such as a model reference adaptive control (MRACS) have the problem.

In this paper, we apply a design method of MRACS for non-minimum phase systems 4) to a winged experimental rocket. To validate the effectiveness of the control systems, simulations for longitudinal and lateral-directional motions are done. The simulations results show that the control system has a good control performance.

2. Model of Winged Rocket

Fig. 1 shows an outline of our developing winged experimental rocket and the parameters are shown in Table 1. It has two elevons and two rudders as aerodynamic control surfaces.

Since the configuration of the winged rocket shown in
5) The pulse transfer function of the winged rocket including actuator dynamics can be expressed as follows:

\[
\alpha(k) = z^{-1} \frac{B_\alpha(z^{-1})}{A_{\text{lon}}(z^{-1})} \delta_{ec}
\]

where

\[
A_{\text{lon}}(z^{-1}) = 1 + a_{\alpha 1}z^{-1} + \cdots + a_{\alpha 4}z^{-4},
\]

\[
B_\alpha(z^{-1}) = b_{\alpha 0} + b_{\alpha 1}z^{-1} + \cdots + b_{\alpha 3}z^{-3},
\]

and \(\delta_{ec}(k)\) is the elevator command, \(\alpha(k)\) is the angle of attack.

In a similar manner, separating a lateral-directional motion to two subsystems, that are the bank and the side-slip systems, the following pulse transfer functions can be obtained:

\[
\phi(k) = z^{-1} \frac{B_\phi(z^{-1})}{A_{\text{lat}}(z^{-1})} \delta_a + z^{-1} \frac{D_\phi(z^{-1})}{A_{\text{lat}}(z^{-1})} \delta_r,
\]

\[
\beta(k) = z^{-1} \frac{B_{\beta r}(z^{-1})}{A_{\text{lat}}(z^{-1})} \delta_r + z^{-1} \frac{D_{\beta r}(z^{-1})}{A_{\text{lat}}(z^{-1})} \delta_a
\]

where \(\phi\) is the bank angle and \(\beta\) is the side-slip angle, \(\delta_a\) is the aileron angle, \(\delta_r\) is the rudder angle, and

\[
A_{\text{lat}}(z^{-1}) = 1 + a_\phi z^{-1} + \cdots + a_\phi 4z^{-4},
\]

\[
B_\phi(z^{-1}) = b_{\phi 0} + b_{\phi 1}z^{-1} + \cdots + b_{\phi 3}z^{-3},
\]

\[
D_\phi(z^{-1}) = d_{\phi 0} + d_{\phi 1}z^{-1} + \cdots + d_{\phi 3}z^{-3},
\]

\[
B_{\beta r}(z^{-1}) = b_{\beta r 0} + b_{\beta r 1}z^{-1} + \cdots + b_{\beta r 3}z^{-3},
\]

\[
D_{\beta r}(z^{-1}) = d_{\beta r 0} + d_{\beta r 1}z^{-1} + \cdots + d_{\beta r 3}z^{-3}.
\]

3. Control System Design

In this section, a design method of MRACS for non-minimum phase systems 4) is explained. The control method consists of a pole assignment controller and a compensator which compensate unstable zeros of the controlled system.

First, we show a control systems design to the following pulse transfer function,

\[
A(z^{-1})y(k) = z^{-1}B(z^{-1})u(k) + z^{-1}D(z^{-1})\xi(k)
\]

where

\[
A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{n} z^{-n},
\]

\[
B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_m z^{-m},
\]

\[
D(z^{-1}) = d_0 + d_1 z^{-1} + \cdots + d_n z^{-n},
\]

and \(u(k)\) is the input, \(y(k)\) is the output, \(\xi(k)\) is the disturbance.

When the parameters of the controlled system are known and \(\xi(k) = 0\), for the controlled system (4) a pole assignment controller 6) can be constructed to satisfy the following relation:

\[
C(z^{-1})y(k) = z^{-1}B(z^{-1})u_F(k)
\]

where

\[
C(z^{-1}) = 1 + c_1 z^{-1} + \cdots + c_{n_c} z^{-n_c} \quad (n_c \leq n + m)
\]

is a stable polynomial specified for a closed-loop system of Eq. (5) to have desirable poles, and \(u_F(k)\) is the bounded reference input. Here, we introduce

\[
R(z^{-1}) = 1 + r_1 z^{-1} + \cdots + r_{n_r} z^{-n_r} \quad (n_r = m)
\]

\[
S(z^{-1}) = s_0 + s_1 z^{-1} + \cdots + s_{n_s} z^{-n_s} \quad (n_s = n - 1)
\]

that satisfy the following equation:

\[
C(z^{-1}) = A(z^{-1})R(z^{-1}) + z^{-1}B(z^{-1})S(z^{-1}).
\]
For Eqs. (5) and (6), the control input to (4) is defined as

$$u(k) = \frac{1}{R(z^{-1})} \{ u_F(k) - S(z^{-1})y(k) \}.$$  \hfill (7)$$

When the disturbance $\xi(k) \neq 0$, the pole assignment controller can be constructed to satisfy the following equation:

$$C(z^{-1}) = \Delta_p(z^{-1})A(z^{-1})R(z^{-1}) + z^{-1}B(z^{-1})S(z^{-1})$$  \hfill (8)$$

where

$$R(z^{-1}) = 1 + r_1z^{-1} + \cdots + r_nz^{-nr} \quad (n_r = m),$$

$$S(z^{-1}) = s_0 + s_1z^{-1} + \cdots + s_nz^{-ns} \quad (n_s = n),$$

and $\Delta_p$ is a known polynomial introduced to the influence of $\xi(k)$ to be asymptotically 0. For Eq. (8), the control input is defined as

$$u(k) = \frac{1}{\Delta_p R(z^{-1})} \{ u_F(k) - S(z^{-1})y(k) \}.$$  \hfill (9)$$

From Eqs. (4), (8) and (9), we have

$$y(k) = z^{-1}B(z^{-1})u_F(k)$$

$$+ z^{-1} \frac{\Delta_p R(z^{-1})D(z^{-1})}{C(z^{-1})} \xi(k).$$

In the event where the disturbance $\xi(k)$ is stepwise input, if we select $\Delta_p(z^{-1}) = 1 - z^{-1}$, the second term of the right side of Eq. (10) converges to zero and pole assignment for Eq. (5) is satisfied. Moreover, in case where $\xi(k)$ is a ramp input, we select $\Delta_p(z^{-1}) = (1 - z^{-1})^2$.

Next, for the pole assignment control system described above, an augmented system shown in Fig. 2 is introduced. In Fig. 2,

$$F(z^{-1}) = 1 + f_1z^{-1} + \cdots + f_nz^{-nf}$$  \hfill (11)$$

and

$$G_c(z^{-1}) = \frac{y_c(k)}{u_a(k)} = z^{-1} \frac{H(z^{-1})}{C(z^{-1})}$$  \hfill (12)$$

are the serial and the parallel elements of the compensator, respectively, where

$$H(z^{-1}) = h_0 + h_1z^{-1} + \cdots + h_nz^{-nh}.$$  \hfill (13)$$

And

$$y_a = y(k) + y_c(k)$$  \hfill (14)$$

is the output of the augmented system, and $y_c(k)$ is the output of the parallel element of the compensator.

Orders of $F(z^{-1})$ and $H(z^{-1})$, $n_f$ and $n_h$, are selected to ensure that $y_c(k)$ converges to asymptotically zero as $k$ tends to infinity. For example, when $u_a(k)$ is a stepwise input,

$$H(z^{-1}) = (1 - z^{-1})(h_0 + h_1z^{-1} + \cdots + h_nz^{-nh})$$

and $n_f = 1, n_h = m (n_k = m - 1)$.

From Eqs. (11)-(14), the pulse transfer function of the augmented system, $G_a(z^{-1})$, is

$$G_a(z^{-1}) = \frac{y_a(k)}{u_a(k)} = z^{-1} \frac{b_0B_m(z^{-1})}{C(z^{-1})}$$  \hfill (15)$$

where

$$B_a(z^{-1}) = \frac{1}{b_0} \{ F(z^{-1})B(z^{-1}) + z^{-1}H(z^{-1}) \}$$  \hfill (16)$$

is a stable polynomial. Since $F(z^{-1})$ and $H(z^{-1})$ are the unique polynomials for Eq. (16), the zero assignment of the augmented system can be realized.

Here, a reference model of the augmented system is defined as

$$G_M(z^{-1}) = \frac{y_M(k)}{r(k)} = z^{-1} \frac{B_M(z^{-1})}{C(z^{-1})}$$  \hfill (17)$$

where $r(k)$ and $y_M(k)$ are the input and the output of the reference model, respectively. For Eq. (17), the input of augmented system is defined as

$$u_a(k) = \frac{b_m}{b_0} r(k),$$  \hfill (18)$$

and

$$B_a(z^{-1}) = \frac{1}{b_m} B_M(z^{-1}).$$  \hfill (19)$$

where $b_m$ is the constant term of $B_M(z^{-1})$.

From Eqs. (15), (17), (18) and (19), $y_a(k)$ tends to $y_M(k)$ as $k$ tends to infinity. Furthermore, $y(k)$ tends to $y_M(k)$ is satisfied when $y_c(k)$ tends to 0.
Fig. 3 shows the configuration of the control system described above.

In a case of unknown parameters, we introduce the polynomials \( \hat{A}(z^{-1}) \) and \( \hat{B}(z^{-1}) \) those coefficients are estimated values of the coefficients in \( A(z^{-1}) \) and \( B(z^{-1}) \), respectively. The coefficients in \( \hat{A}(z^{-1}) \) and \( \hat{B}(z^{-1}) \) are estimated by using adaptive algorithms. Then, the pole assignment controller is designed to satisfy the following equation corresponding to Eq. (6).

\[
C(z^{-1}) = \hat{A}(z^{-1}) \hat{R}(z^{-1}) + z^{-1} \hat{B}(z^{-1}) \hat{S}(z^{-1})
\]  

(20)

where \( \hat{R}(z^{-1}) \) and \( \hat{S}(z^{-1}) \) are polynomials for \( \hat{A}(z^{-1}) \) and \( \hat{B}(z^{-1}) \).

Similarly, the control input and the polynomials of the compensator are obtained by following equations:

\[
u(k) = \frac{1}{\hat{R}(z^{-1})} \{ u_F(k) - \hat{S}(z^{-1}) y(k) \}
\]

(21)

\[
B_a(z^{-1}) = \frac{1}{b_0} \{ \hat{F}(z^{-1}) \hat{B}(z^{-1}) + z^{-1} \hat{H}(z^{-1}) \}
\]

(22)

Now, we apply the control method described above to the winged rocket described in Section 2. The angle of the attack control system shown in Fig. 4 consists of the pole assignment and zero compensator. On the other hand, the bank angle control system and the side-slip angle control system shown in Fig. 5 have bilateral interference. Therefore, the pole assignment controller considering disturbance is used to both the control systems. Moreover, the zero compensator is applied to the bank angle control system. Since the reference output of the side-slip angle is 0, the side-slip angle control system consists of the pole assignment only.

4. Numerical Simulation

To validate the adaptive control system described in Section 3, computer simulations for a 6-DOF nonlinear winged rocket model considering the atmospheric fluctuation are performed. Simulation condition is follows. When the winged rocket is climbing, a trouble is happened at about 10000[m] in altitude. Then the winged rocket changes the thrust power from 3000[N] into 0[N] for changing the route. That is an abort flight. At the start of the simulation, the altitude of the winged rocket is 5000[m], the velocity is 200[m/s], and the pitch angle is 70[deg]. The aerodynamic coefficient of the winged rocket uses the value obtained from the wind tunnel examination result. The actuators of elevons is used that the attenuation coefficient \( \zeta = 0.7 \) and the natural frequency \( \omega_n = 72[\text{rad/s}] \). The sampling period of the control system is \( T = 0.01[s] \). The polynomials of augmented systems are \( C_\alpha(z^{-1}) = (1 - 0.5z^{-1})^7 \), \( B_{\alpha M}(z^{-1}) = 1, C_\phi(z^{-1}) = (1 - 0.5z^{-1})^8 \), \( B_{\phi M}(z^{-1}) = 1 \) and \( C_\beta(z^{-1}) = (1 - 0.9z^{-1})^2 \). The parallel compensator is select for a stepwise input. A constant trace algorithm with width \( \delta = \pm 0.0001 \) dead zone \( 7) \) is utilized.

Figs. 6-9 show the time histories of the simulation result. In all figures, the dashed lines are the start time of the abort flight and 0[s] on the time axis is set at the time the altitude was about 7000[m]. First, the altitude and the thrust are shown in Figs. 6 and 7, respectively. From these figures, it can be seen that the winged rocket have the abort flight. Next, Figs. 8 and 9 show the input and the output, respectively. In Fig. 9, the dotted lines show the output of the reference model. From Fig. 8, we can see that the input commands to the actuators is almost smooth. And from Fig. 9, it can be seen that the angle of attack and the bank angle follow to the reference output well, and the side-slip angle is suppressed to 1[deg] or less. From the simulation result, it can be confirmed that the applied control method is effective.
for the flight control of the winged rocket.

5. Conclusion

In this paper, MRACS for a non-minimum phase system was applied to a winged rocket. From the numerical simulation, we showed that the control system of the winged rocket has a good control performance.

References


2) Eric N. Johnson, Anthony J. Calise: “Limited


