Calibration Technique for Attitude Manoeuvres of Spinning Satellites

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Keywords: Spinning Satellites, Attitude Control, Manoeuvre Calibration

Abstract
The paper presents an efficient calibration technique for rhumb-line attitude control manoeuvres for spin-stabilized satellites. The only measurements used in the calibrations are the solar aspect angles generated by a common V-slit Sun sensor. Both the path-length and rhumb-line angle can be established accurately by using the initial and final Sun aspect angles of two independent manoeuvre paths. This technique was applied successfully for the CONTOUR mission in 2002 before executing a full 180-deg flip manoeuvre. The final attitude error at the end of the manoeuvre was below 3 degrees.

INTRODUCTION

We present an efficient practical calibration technique for rhumb-line attitude control manoeuvres of spin-stabilized satellites. This technique was actually employed successfully during the initial operations of the CONTOUR mission in its Earth-orbiting phase in July 2002. The details of the manoeuvre calibration concept that was designed and actually implemented for the CONTOUR mission have not been presented or published previously.

CONTOUR had to perform a 180-degrees manoeuvre just a few days after its separation from the launcher (Ref. 1). At that time, the hydrazine thrusters that were planned to be used for this manoeuvre had not been calibrated. In fact, they had not been used at all. Therefore, there was a significant risk that the satellite attitude would not reach its final target attitude with the necessary precision of only ‘a few degrees’ (imposed by system-level requirements).

The attitude control of a spin-stabilized spacecraft can be accomplished by a forced precession of the spin axis. In practice, this is accomplished by a series of thrust pulses (delivered by one or more thrusters) that are synchronized with the spin phase angle (Refs. 2, 3). The inertial direction of the spin-axis motion can be controlled by introducing a constant delay time in the start of the thruster firings relative to the instant when the Sun crosses the Sun sensor's vertical slit. This strategy makes the spin-axis direction follow a rhumb-line path as illustrated in Figure 1. The angle $\chi$ is the constant rhumb angle relative to the Sun cone (i.e., the local latitude circle) on the unit-sphere with the Sun at its north pole.

![Figure 1 – Spherical Geometry of Rhumb-Line Manoeuvre](image-url)
The manoeuvre path-length $\lambda_f$ is proportional to the number and the magnitude of the applied thrust pulses. Therefore, errors in the intended thrust level lead to errors in the resulting path-length. Similarly, the manoeuvre’s rhumb angle $\chi$ consists of the sum of the delay angle (measured from the Sun slit crossing) and the centroid (i.e., mid-time) of the thrust pulses. Furthermore, the offset between the Sun-sensor position and the thruster location(s) within the satellite reference frame must be taken into account in the calculation of $\chi$. Errors in all of these parameters lead to proportional errors in the effective rhumb angle.

The rhumb-line path length is different from the corresponding minimum great-circle arc-length distance between the initial and final attitude vectors. In practice, however, it is usually fairly close, i.e. within a few percent, especially for small manoeuvres (Ref. 2).

**MANOEUVRE CALIBRATION PARAMETERS**

**Mathematical Formulation**

The mathematical formulation that we employ here is based on the rhumb-line model established as Eqs. (29) in Ref. 2. This model makes careful distinctions between dependent and independent variables:

$$\vartheta_f(\vartheta_i, \lambda_f, \chi) = \vartheta_i - \lambda_f \sin \chi \quad (1a)$$

$$\xi_f(\xi_i, \vartheta_i, \lambda_f, \chi) = \xi_i - \left\{ y(\vartheta_f) - y(\vartheta_i) \right\} / \tan \chi \quad (1b)$$

The logarithmic function $y(\vartheta)$ is defined by:

$$y(\vartheta) = \ln\{\tan(\vartheta / 2)\} \quad (1c)$$

The expression for the variable $\xi_i$ in Eq. (1b) contains hidden dependencies because $\xi_i$ is a function of the independent variables $\vartheta_i$, $\lambda_f$, and $\chi$ as given in Eq. (1a). The system in Eqs. (1) is helpful for analyzing the error propagations over the manoeuvre path (see the results in Ref. 2). The same system will be used for deriving the calibration scheme to be presented here.

For the present objective, Equations (1) are formulated in terms of the actual rhumb-line manoeuvre variables. This means that they refer to the actual values of the initial and final Sun aspect angles $\vartheta_i$ and $\vartheta_f$, respectively, as well as the actual total path length $\lambda_f$ and the actual rhumb angle $\chi$. It should be noted that the actual variables will remain unknown during the manoeuvre planning and during the manoeuvre execution and evaluation.

**Differences of Planned and Actual Manoeuvres**

The planned manoeuvre is designed and prepared prior to the actual manoeuvre execution. For describing the planned manoeuvre, we employ the rhumb-line model given in Eqs. (1) with the understanding that it now contains the planned manoeuvre parameters, i.e. $\vartheta_{i,p}$, $\vartheta_{f,p}$, $\lambda_{f,p}$, and $\chi_p$. In particular, the counterpart of Eq. (1a) can now be written as:

$$\vartheta_{f,p}(\vartheta_{i,p}, \lambda_{f,p}, \chi_p) = \vartheta_{i,p} - \lambda_{f,p} \sin \chi_p \quad (2)$$

We focus now on the differences between the actual and prepared manoeuvre parameters and introduce the associated small difference parameters, i.e. $\Delta a = a - a_p$:

$$\Delta \vartheta_i = \vartheta_i - \vartheta_{i,p}; \quad \Delta \vartheta_f = \vartheta_f - \vartheta_{f,p}; \quad \Delta \lambda_f = \lambda_f - \lambda_{f,p}; \quad \Delta \chi = \chi - \chi_p \quad (3)$$

When subtracting the expressions in Eq. (1a) and Eq. (2) we obtain:

$$\Delta \vartheta_f = \vartheta_i - \lambda_f \sin \chi - \left( \vartheta_{i,p} - \lambda_{f,p} \sin \chi_p \right) = \Delta \vartheta_i + \lambda_{f,p} \sin \chi_p - \lambda_f \sin \chi \quad (4)$$

This expression can be simplified by using a Taylor-series expansion around the known parameters $\lambda_{f,p}$ and $\chi_p$. Finally, we obtain the following approximate first-order result:
\[
\Delta \vartheta_i \approx \Delta \vartheta_i - \left( \sin \chi_p \right) \Delta \lambda_i - \left( \lambda_{f,p} \cos \chi_p \right) \Delta \chi
\]  

(5)

**Measured Manoeuvre Parameters**

After the manoeuvre has been completed, we have the actual in-orbit measurements of the start and end values of the solar aspect angles produced by the Sun sensor, i.e. the angles \( \vartheta_{i,m} \) and \( \vartheta_{f,m} \). Therefore, we can construct the relationships between the measured Sun aspect angles in comparison to the actual rhumb-line parameters appearing in Eq. (1a):

\[
\vartheta_{f,m} - \vartheta_{i,m} = - \lambda_f \sin \chi + m
\]  

(6)

The noise term \( m = m_f - m_i \) represents the difference between the random errors in the final and initial Sun-aspect-angle measurements. We may note that identical biases in the two measurements will cancel by the subtraction. However, any differential bias errors would remain effective but are they are expected to be minor so we ignore those. We also assume that the noise terms \( m_f \) and \( m_i \) are independent and have expected values of zero, i.e. \( E\{m_f\} = E\{m_i\} = 0 \). Finally, we summarize the following statistical properties for the noise term \( m \):

\[
E\{m\} = E\{m_f - m_i\} = E\{m_f\} - E\{m_i\} = 0; \quad E\{m^2\} = \sigma^2
\]  

(7)

The random noise variance is given by \( \sigma^2 = 2 \sigma_\vartheta^2 \) with \( \sigma_\vartheta^2 = E\{\vartheta^2\} \), which is the expected variance of the individual Sun-aspect-angle measurements. In the absence of nutation, the latter are typically of the order of only 0.001 degree, see Ref. [4].

Next, we introduce the notation \( \delta a = a_m - a_p \) for the difference between some general measured and prepared parameter \( a_m \) and \( a_p \). When subtracting the measured and the prepared final solar aspect angles in Eqs. (6) and (2), respectively, we obtain:

\[
\delta \vartheta_f = \vartheta_{f,m} - \vartheta_{f,p} = \vartheta_{i,m} - \lambda_f \sin \chi + m - \left( \vartheta_{i,p} - \lambda_{f,p} \sin \chi_p \right) = \\
= \delta \vartheta_i - \lambda_f \sin \chi + \lambda_{f,p} \sin \chi_p + m = \\
\approx \delta \vartheta_i - \left( \sin \chi_p \right) \Delta \lambda_f - \left( \lambda_{f,p} \cos \chi_p \right) \Delta \chi + m
\]  

(8)

The \( \Delta \) symbol in Eq. (8) was defined in Eq. (3) above and refers to differences between actual and prepared parameters. The final result of Eq. (8) forms the basis for the subsequent calibration analyses to be discussed below.

**COMPUTATION OF CALIBRATION PARAMETERS**

**Rhumb-Line Calibration Equations**

In practical applications, the Sun Sensor typically produces much more accurate measurements than for instance the infra-red horizon Earth sensors. Therefore, it makes sense to perform the manoeuvre calibrations on the basis of the Sun-sensor measurements only.

The initial and final Sun angles \( \vartheta_i \) and \( \vartheta_f \) produced by the Sun sensor do not provide direct information on the rhumb-line parameters \( \lambda \) and \( \chi \). Therefore, we aim at a calibration strategy for estimating the latter parameters from the Sun-aspect-angle measurements.

It should be noted that Eq. (8) provides only one equation for the two unknown errors \( \Delta \lambda_f \) and \( \Delta \chi \). Therefore, we require two separate rhumb-line manoeuvres with different rhumb angles in order to obtain two independent equations for these two unknown parameters. In general, we can write these two equations as:

\[
\delta \vartheta_f^{(1)} - \delta \vartheta_i^{(1)} = - \left( \sin \chi_p^{(1)} \right) \Delta \lambda_f^{(1)} - \left( \lambda_{f,p}^{(1)} \cos \chi_p^{(1)} \right) \Delta \chi^{(1)} + m^{(1)}
\]  

(9a)

\[
\delta \vartheta_f^{(2)} - \delta \vartheta_i^{(2)} = - \left( \sin \chi_p^{(2)} \right) \Delta \lambda_f^{(2)} - \left( \lambda_{f,p}^{(2)} \cos \chi_p^{(2)} \right) \Delta \chi^{(2)} + m^{(2)}
\]  

(9b)
where the superscripts (1) and (2) designate each of these two manoeuvres.

As long as the manoeuvres are performed by the same thrusters(s) and the effects of propellant blow-down and other time-varying influences are negligible, the ratios \( \frac{\Delta \lambda^{(1)}}{\dot{\lambda}_{f,p}^{(1)}} \) and \( \frac{\Delta \lambda^{(2)}}{\dot{\lambda}_{f,p}^{(2)}} \) should be essentially identical. When assuming identical conditions for the thrust centroid and other timing parameters during both manoeuvre paths, also the rhumb-line errors \( \Delta \chi^{(1)} \) and \( \Delta \chi^{(2)} \) should be identical. Therefore, we introduce the abbreviations:

\[
\begin{align*}
y_j &= \left( \frac{\delta \dot{\varphi}^{(j)} - \delta \dot{\varphi}^{(j)}}{\dot{\lambda}_{f,p}^{(j)}} \right), \quad j = 1, 2 \quad (10a) \\
x_1 &= \frac{\Delta \lambda^{(1)}}{\dot{\lambda}_{f,p}^{(1)}} = \frac{\Delta \lambda^{(2)}}{\dot{\lambda}_{f,p}^{(2)}}, \quad x_2 = \Delta \chi^{(1)} = \Delta \chi^{(2)} \quad (10b,c) \\
a_{j1} &= -\sin \chi_p^{(j)}; \quad a_{j2} = -\cos \chi_p^{(j)}; \quad j = 1, 2 \\
m_j &= m^{(j)} / \dot{\lambda}_{f,p}^{(j)}, \quad j = 1, 2 \quad (10f)
\end{align*}
\]

The parameter \( x_1 \) represents the calibration coefficient or scale factor for the manoeuvre path-length and the parameter \( x_2 \) is the calibration constant for the rhumb angle in radians.

Now, we can write the system of equations Eqs. (9) in the compact vector-matrix form:

\[
\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \mathbf{Ax} + \mathbf{m} \quad (11)
\]

The noise terms \( m^{(j)} \) for \( j = 1, 2 \) are in general very small because the Sun-sensor readings are very accurate. Furthermore, the effective noise \( \mathbf{m} \) appearing in Eq. (11) will be reduced when using a relatively large manoeuvre path length as is evident from its definition in Eq. (10f).

**Solutions for Calibrated Parameters**

When ignoring the noise term \( \mathbf{m} \) in Eq. (11) for the time being, we can immediately solve for the unknown parameters \( x_p, j = 1, 2 \) provided that the matrix \( \mathbf{A} \) is non-singular:

\[
\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \mathbf{A}^{-1} \mathbf{y} \quad (12)
\]

The inverse matrix \( \mathbf{A}^{-1} \) can be calculated as:

\[
\mathbf{B} = \mathbf{A}^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{pmatrix} a_{22} - a_{12} \\ -a_{21} & a_{11} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \quad (13)
\]

We calculate the elements of the matrix \( \mathbf{B} \) in explicit terms from the definitions of the elements of \( \mathbf{A} \) in Eqs. (10d,e):

\[
\begin{align*}
\mathbf{B} &= \frac{1}{\sin(\chi_p^{(2)} - \chi_p^{(1)})} \begin{pmatrix} \cos \chi_p^{(2)} - \cos \chi_p^{(1)} \\ -\sin \chi_p^{(2)} & \sin \chi_p^{(1)} \end{pmatrix} \\
\end{align*}
\]

Finally, we can write the following explicit results for the calibration parameters:

\[
\begin{align*}
x_1 &= \frac{1}{\sin(\chi_p^{(2)} - \chi_p^{(1)})} \left[ \frac{\delta \dot{\varphi}^{(1)} - \delta \dot{\varphi}^{(1)}}{\dot{\lambda}_{f,p}^{(1)}} \cos \chi_p^{(2)} - \frac{\delta \dot{\varphi}^{(2)} - \delta \dot{\varphi}^{(2)}}{\dot{\lambda}_{f,p}^{(2)}} \cos \chi_p^{(1)} \right] \quad (15a) \\
x_2 &= \frac{1}{\sin(\chi_p^{(2)} - \chi_p^{(1)})} \left[ -\frac{\delta \dot{\varphi}^{(1)} - \delta \dot{\varphi}^{(1)}}{\dot{\lambda}_{f,p}^{(1)}} \sin \chi_p^{(2)} + \frac{\delta \dot{\varphi}^{(2)} - \delta \dot{\varphi}^{(2)}}{\dot{\lambda}_{f,p}^{(2)}} \sin \chi_p^{(1)} \right] \quad (15b)
\end{align*}
\]
It should be noted that a singularity occurs when the two rhumb angles over the two manoeuvre legs are equal or differ by 180 degrees. In these cases, the two equations are not independent so that unambiguous solutions for the parameters \( x_1 \) and \( x_2 \) cannot be established and different manoeuvre paths should be selected.

The following four special cases are of considerable practical interest:

\[
\chi_p^{(1)} = \pm 90^\circ; \quad \chi_p^{(2)} = 0 \text{ or } 180^\circ
\]  

(16)

Considering for example the first case \((+ 90^\circ, 0)\), we can simplify the results of Eqs. (15):

\[
x_1 = \Delta \lambda_f^{(f)} / \lambda_{f,p}^{(f)} = - (\delta \varphi_f^{(f)} - \delta \varphi_i^{(f)}) / \lambda_{f,p}^{(f)}
\]  

(17a)

\[
x_2 = \Delta \chi^{(2)} = - (\delta \varphi_f^{(2)} - \delta \varphi_i^{(2)}) / \lambda_{f,p}^{(2)}
\]  

(17b)

Similar results (with different signs) can be established for the other three cases of Eqs. (16).

Finally, we can convert the results of Eqs. (17) into the actual calibrated (i.e., updated) parameters \( \lambda_{f,c} \) and \( \chi_c \) for each of the two manoeuvre legs as follows:

\[
\lambda_{f,c}^{(f)} = \lambda_{f,p}^{(f)} + \Delta \lambda_f^{(f)}; \quad \chi_c^{(2)} = \chi_p^{(2)} + \Delta \chi^{(2)}
\]  

(18a,b)

These results form the basis for calibrating the thrust level and the thrust centroid time, respectively, for use in future manoeuvres. In particular, the thrust-level calibration is simply:

\[
T_c = \left(1 + \Delta \lambda_f^{(f)} / \lambda_{f,p}^{(f)} \right) T_p
\]  

(19)

where \( T_p \) and \( T_p \) denote the prepared and calibrated thrust levels, respectively. Because spin-rate changes affect the delay angle between the Sun-sensor signal and the start of the thrust pulses, care should be taken to eliminate their effects from the centroid time calibrations.

The special manoeuvres listed in Eqs. (16) have the properties that the calibrations of the path-length and the rhumb angle are strictly separated for the two manoeuvre legs. During the first manoeuvre leg with rhumb angle \(+90^\circ\), the spin axis moves towards the Sun position in inertial space. This means that the Sun aspect angle decreases continuously. A measured final Sun aspect angle that is larger than the planned final Sun aspect angle implies a thrust-level underperformance. Therefore, the thrust level must be assigned a negative calibration, which is consistent with the minus sign in Eq. (17a). In this case, the measurements of the Sun aspect angle at the start and the end of the manoeuvre leg provide accurate information on the actually achieved path-length, which enables us to perform the path-length calibration.

On the other hand, during the second manoeuvre leg, the spin axis path moves along the local Sun cone circle, i.e. \( \chi_p = 0 \), thereby maintaining a constant Sun aspect angle during the manoeuvre path, at least in the nominal case. If the measured final Sun aspect angle is larger than the expected prepared constant value, this implies that the rhumb angle was actually negative instead of 0 (and vice versa). Therefore, the calibration should assign a negative calibration value to the prepared rhumb angle, which is consistent with the minus sign in Eq. (17b). In this case, the Sun-aspect-angle measurements at the start and end of the manoeuvre provide accurate information about the actual rhumb angle, so that the second manoeuvre leg achieves the rhumb-angle calibration.

As a further potential advantage of the proposed calibration strategy, we mention that the path-length calibration performed during the first leg may benefit the rhumb-angle calibration during the second manoeuvre leg. In practice, this means that, instead of using the initially prepared thrust level, we employ the improved thrust-level knowledge that resulted from the path-length calibration after completion of the first manoeuvre leg.
Summary of Parameters Used in Calibrations

Table 1 summarizes the initial and final designations of the actual, prepared, as well as the measured rhumb-line parameters. Similarly, Table 2 provides the actual and prepared, as well as the calibrated parameters:

- \( \vartheta_i \) and \( \vartheta_f \) represent the actual Sun aspect angle (SAA) at the start \( (i: \text{initial}) \) and end \( (f: \text{final}) \) of the manoeuvre.
- The values \( \vartheta_i \) and \( \vartheta_f \) are unknown but will be approximated by their respective measured values \( \vartheta_{i,m} \) and \( \vartheta_{f,m} \) as provided by the Sun sensor.
- \( \lambda \) represents the unknown actual path length of the rhumb-line manoeuvre as traced on the unit sphere (see Figure 1).
- \( \chi \) represents the unknown actual manoeuvre rhumb angle. This is the manoeuvre heading direction relative to the Sun cone as visualized on the unit sphere (see Figure 1).

### TABLE 1 – MEASUREMENTS of RHUMB MANOEUVRE PARAMETERs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Actual (unknown)</th>
<th>Target (prepared)</th>
<th>Measured</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial SAA</td>
<td>( \vartheta_i )</td>
<td>( \vartheta_{i,p} )</td>
<td>( \vartheta_{i,m} )</td>
<td>( \delta \vartheta_i = \vartheta_{i,m} - \vartheta_{i,p} )</td>
</tr>
<tr>
<td>Final SAA</td>
<td>( \vartheta_f )</td>
<td>( \vartheta_{f,p} )</td>
<td>( \vartheta_{f,m} )</td>
<td>( \delta \vartheta_f = \vartheta_{f,m} - \vartheta_{f,p} )</td>
</tr>
</tbody>
</table>

### TABLE 2 – CALIBRATIONS of RHUMB MANOEUVRE PARAMETERs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Actual (unknown)</th>
<th>Target (prepared)</th>
<th>Calibrated</th>
<th>Calibrations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path length</td>
<td>( \lambda )</td>
<td>( \lambda_p )</td>
<td>( \lambda_c )</td>
<td>( \lambda_c = \lambda_p + \Delta \lambda )</td>
</tr>
<tr>
<td>Rhumb angle</td>
<td>( \chi )</td>
<td>( \chi_p )</td>
<td>( \chi_c )</td>
<td>( \chi_c = \chi_p + \Delta \chi )</td>
</tr>
</tbody>
</table>

**ERROR MODEL**

While recognizing that the prepared manoeuvre parameters are deterministic variables, we can construct the random errors of the \( y_j \) components in Eq. (10a) as follows:

\[
\Delta y_j = \frac{\Delta \vartheta_j^{(i)} - \Delta \vartheta_j^{(j)}}{\lambda_{f,j,p}} \quad j = 1, 2
\]

When assuming that the Sun-aspect angle measurements are independent from each other, we can calculate the covariance matrix of the measurement vector \( y = (y_1, y_2)^T \) as follows:

\[
cov(y) = \mathbb{E}\{\Delta y \cdot \Delta y^T\} = \sigma_\vartheta^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

The errors in the Sun-aspect-angle measurements are known from the observed Sun-sensor performances, i.e. \( \sigma_\vartheta \approx 0.001 \text{ deg} \) (see Ref. 4).

With the help of Eq. (11) we can now calculate the expected errors in the calibrated manoeuvre parameters \( x = (x_1, x_2)^T \) with the help of the pseudo-inverse formula:

\[
\Delta x = \left( A^T A \right)^{-1} A^T \Delta y
\Rightarrow
\mathbb{E}\{\Delta x \cdot \Delta x^T\} = \left( A^T A \right)^{-1} \sigma_\vartheta^2 \left( A^T A \right)^{-1}
\]

\[
\mathbb{E}\{\Delta x \cdot \Delta x^T\} = \left( A^T A \right)^{-1} \sigma_\vartheta^2 \left( A^T A \right)^{-1}
\]
with:

\[
(A^T A)^{-1} = \begin{bmatrix}
    a_{11}^2 + a_{21}^2 & a_{11} a_{12} + a_{21} a_{22} \\
    a_{11} a_{12} + a_{21} a_{22} & a_{12}^2 + a_{22}^2
\end{bmatrix}^{-1}
\]

(22)

When we consider the four special cases listed in Eq. (16), we find that the matrix \((A^T A)\) and also its inverse are equal to the identity matrix in all of the four cases. Therefore, when adopting these manoeuvre calibrations strategies, we find the following expected errors in the calibration parameters from Eq. (21):

\[
E[\Delta x \cdot \Delta x^T] = \sigma_0^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

(23)

It is evident from this result and Eq. (17) that the resulting calibration errors are identical to the solar-aspect angle measurement errors. Therefore, when considering a manoeuvre path-length of about 1 rad (i.e., about 57.3 deg), we find that the resulting random errors in both the calibrated path-length and the rhumb angle will be of the order of just 0.001 deg. Of course, in practice, there will be a number of much larger systematic errors (i.e., biases), in particular variations in the thrust levels and spin-rate changes during and between the manoeuvre legs.

APPLICATION TO CONTOUR

CONTOUR performed (Ref. 1) two dedicated calibration manoeuvres to achieve the calibration of the manoeuvre path-length (i.e., thrust level) and the rhumb angle (i.e., centroid time) in accordance with the strategy proposed in Eqs. (16). These two manoeuvre legs were integral parts of the 180-deg flip manoeuvre that was required for establishing the correct attitude orientation for executing an orbit correction manoeuvre at the time of the second perigee. The calibrations took advantage of the accurate Sun-aspect angle measurements generated by the Sun-sensor. No other sensor measurements were used nor needed.

The first manoeuvre aimed at achieving the thrust-level calibration. It used a manoeuvre path directed normal to the local Sun cone so the Sun sensor could accurately measure the manoeuvre path-length. The second manoeuvre was used to perform the calibration of the rhumb angle, i.e. the effective centroid time of the thrust pulses. It adopted a path along the local Sun-cone circle so that the Sun sensor could accurately measure the deviation from this path. As a result of these two calibration manoeuvres (and diligent accounting for spin rate effects), CONTOUR’s 180º flip manoeuvre achieved its required target attitude to less than 3 degrees (Ref. 1).

Figure 2 provides a visualization of the full 180-deg flip manoeuvre path in the form of a Mercator projection showing the path of the attitude unit-vector traced on the unit-sphere. The complete manoeuvre path consists of 4 manoeuvre legs in total, each with its own path-length and rhumb angle. The first two shorter legs are dedicated to the calibration objectives and the following two larger legs of about 64 deg lengths complete the 180-deg flip manoeuvre. The interruption in the middle may be used to correct for observed spin changes. The five attitude orientations at the start and end of the four manoeuvre legs are identified by three parameters (RA, DE) / SAA, denoting (Right Ascension, DEclination) / Sun Aspect Angle, respectively.

The first leg has a length of about 19 deg long and a rhumb angle of +90 deg. Therefore, the attitude moves (nominally) along a meridian circle in the direction towards the Sun. This results in a pure Sun angle variation so that we can achieve a very precise calibration of the manoeuvre path-length on the basis of the measured change in Sun aspect angle. The second manoeuvre leg is about 57 deg long and follows the direction normal to that of the first leg. The rhumb angle is now 180 deg with the attitude moving (nominally) along the Sun cone at a constant Sun aspect angle. Therefore, any observed change in Sun aspect angle over the manoeuvre leg points to a rhumb-angle error and can be used for its calibration.
Fig. 1 – Visualization of CONTOUR’s Rhumb-Line Manoeuvres (180-deg Flip)

REFERENCES


