Design of Loose Formation Flying in the Vicinity of Halo Orbits Based on Initial Set Values

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Abstract

In this paper we address natural formation flying around $L_2$ point of the Sun-Earth Circular Restricted Three-Body Problem (CR3BP) systems. Halo orbit is chosen as the reference orbit for the leader and the follower is conceived to follow loosely. This type of formation will be useful in future for constructing space based ports, long-baseline space telescope, sun-shielded astronomical spacecraft, etc. We approach this problem by utilizing Dynamical System Theory (DST).

1 Introduction

Two spacecrafts are assumed to fly in a formation flying around $L_2$ point of the Sun-Earth Circular Restricted Three-Body Problem (CR3BP) systems. Orbit of choice for reference (leader’s orbit) is halo orbit and the follower is conceived to follow loosely in a range of set of maximum and minimum relative distances from the leader and possible geometrical constraints, e.g. orientation of the formation in the Sun-Earth rotating frame. This type of formation could be useful in future for constructing space based ports, space telescope, astronomical spacecraft requiring sun shield, and with more constituents, spacecraft swarm missions.

What particularly considered in this paper are exclusively natural formations which designed solely based on set initial values. In other words, the formations subjected in this study are established as product of carefully chosen initial set of velocities and positions. In this proposed scheme, firstly a Halo orbit is searched by means of differential correction method \cite{1} and consecutively its Monodromy matrix is used for initial set values analysis. This current work will be integrated with future work about control strategies of impulsively controlled formation for maintaining and reconfiguring the formation.

It is well known that Monodromy matrix of a Halo orbit has six eigenvalues which can be grouped into three pairs. The first pair has self-product of one, the second pair is real and has magnitude of one, and the third pair is complex with magnitude of one. The first pair represents stable and unstable manifold, so if the initials are set in the direction of this pair’s eigenvectors, the shape and the magnitude of the formation will be changed radically therefore unsuitable for establishing formation which stays nearby after one period. Inspecting the other four eigenvalues reveals two modifications are needed to their eigenvectors to construct basis for the formation. Firstly, the eigenvectors of the complex pair eigenvalues are linearly combined to form two real vectors. Secondly since eigenvalue of one has eigenvector only one \cite{2}, algebraic multiplicity is two but geometric multiplicity one, means the monodromy matris is defective, hence a generalized eigenvector which represents the change of energy is introduced\cite{3}. So basically the available principle direction, therefore the degree of freedoms, for designing the formation are limited to only four.

In brief, what this paper presents is how to utilize the four vectors as design parameters to establish loose formation flying within the scope of future space exploration interest. Special attention is given to institute design method for various useful natural formation allowed by combinations of these principal vectors.
2 Motion around of Halo Orbits

2.1 Equations of Motions

The motion of spacecrafat under investigation here is modeled mainly under the influence of the Sun and the Earth gravity. Perturbation forces such as solar radiation pressure (SRP), perturbation by other planets are ignored. Since the mass of spacecraft is much smaller than the other two primaries and the motion of the Earth relative to the Sun is nearly circular, then Circular Restricted Three Body Problem (CR3BP) mathematical model is accurate enough as the preliminary model.

In a rotating, barycentric, dimensionless coordinate system with the smaller primary (the Earth) on the positive x-axis, the differential equations of motion for the circular three-dimensional restricted problem are given as

\[ \ddot{X} - 2\dot{Y} = \Omega_x \]  
\[ \ddot{Y} + 2\dot{X} = \Omega_y \]  
\[ \ddot{Z} + Z = \Omega_x \]  

where

\[ \Omega = \frac{1}{2}(x^2 + y^2 + z^2) + \frac{1 - \mu}{r_1} + \frac{\mu}{r_2} + \frac{1}{2}\mu(1 - \mu) \]  

and

\[ \Omega_i = \frac{\partial \Omega}{\partial i} \]  
\[ r_1^2 = (X - \mu)^2 + Y^2 + Z^2 \]  
\[ r_2^2 = (X + 1 - \mu)^2 + Y^2 + Z^2 \]  
\[ \mu = 3.0037588749187072 \times 10^{-6} \]

2.2 Halo Orbits around L₂ Point

L₂ point offers some great advantages to used for space missions. They are temperature stability for sensitive harware, uninterrupted observations since the Sun, Earth, and Moon reside within the orbit of L₂ point, full access to the entire celestial sphere during whole year, and requires less eneregy for orbit maintenance due to weak gravity environment. Henceforth the study proposed here takes L₂ point as an attractive location for the formation to be placed.

Circular restricted three body problem (CR3BP) model has five equilibrium points. Of these, three are collinear points (L₁, L₂, L₃) and two are triangular points (L₄, L₅). The linearized model shows that collinear point has center x center x saddle stability. However when we move to nonlinear region high enough, the unstable dynamics can be decoupled from the center dynamics hence gives us a good approximation of the dynamics in the center of manifold. In this region there exits nonlinear, 3D, periodic orbits, which in poincare section appears to be fixed points, called Halo Orbits. As stated previously, the Halo Orbits here are found numerically as described in [1]. The period of these orbits are approximately 6 months. Figures 1 & 2 show a Halo Orbit in normalized coordinate.
2.3 Loose Formation Flying

Interest on formation flying could be grouped into two type: precise formation flying and loose formation flying. The major difference between the two type is how important the accuracy versus fuel cost required for the formation to function as designed. When a formation flying demands precision in distance or geometry above all, it is categorized as precise formation flying. But when the formation resort to fuel cost instead of restrictive distance and geometry then it is included in the group of loose formation flying. Up to this present most of works in formation flying are focused on precise formation than loose
formation flying. Furthermore considering the potential usage of loose formation in future, we choose to address this subject in the present work.

The scenario addressed here is a formation where the leader spacecraft orbits a halo orbit and the follower spacecraft follows in nearby. The formation in the interest is a natural formation. This means to this stage we study a loose formation where it’s purely obtained by selecting appropriately the initial conditions, velocities and positions. This formation does not account the use of propulsions, therefore fully rely on the natural dynamics.

3 Design Method

The design method taken in this present study is summarized as follows. The design parameters are obtained from the monodromy matrix of the orbit reference, the leader. Since the size of the formation within present interest is very small compared to the unit distance, we can use the monodromy matrix to approach the motion nearby halo orbit. The monodromy matrix can be consecutively obtained numerically when the reference Halo Orbit is found. As briefly explained in the Introduction, we have only four design parameters. They are four independent vectors which allow motion after one period stay close nearby initial state space.

The four vectors are:

1. $\bar{v}_3 = \ker (A - \lambda_3 I)^2$
2. $v_4$
3. $\bar{a} = \frac{1}{2} (v_5 + v_6)$
4. $\bar{b} = \frac{1}{2i} (v_5 - v_6)$

The relative position between the leader and the follower spacecraft in the rotating frame is simply expressed by

$$\delta = r_{\text{leader}} - r_{\text{follower}}$$  \hspace{1cm} (8)

For the follower to have relative motion stays in the subspace span by the four vectors above, its initial condition must be also lies in the same subspace. Since analytical solution is not available for the CR3BP, we use fourier series approximation to model the relative motion and then investigate exploitable feature of linear combination of the these four vectors. Overview of motion in one period with displacement in the direction of $a, b$ are shown in Fig.3 and Fig.4 respectively.
Figure 3: Inplane Motion of \textbf{a}

Figure 4: Inplane Motion of \textbf{b}

Two models of approximation used to model the relative motion, pure Fourier series and modified Fourier series. Fourier series, Eq. 9, is chosen since the halo orbit is a periodic orbit so as motion around its neighborhood, therefore suitable to model with. The modified Fourier series, Eq. 10, is needed to approach motion of with initial in the direction \textbf{a} \& \textbf{b} for its non periodic nature in one orbit.

\[
\delta v_3(t) = a_0 v_3 + a_{n_{v_3}} \sum_{n=1}^{\infty} \cos(2f_n \pi \frac{t}{T}) + b_{n_{v_3}} \sum_{n=1}^{\infty} \sin(2f_n \pi \frac{t}{T})
\] (9)
\[ \delta x_a(t) = a_0 + a_n \sum_{n=1}^{n=2} \cos(2f_n \pi \frac{t}{T}) + b_n \sum_{n=1}^{n=2} \sin(2f_n \pi \frac{t}{T}) + \eta_{x_a}(\frac{t}{T}) \] (10)

The accuracy of Fourier series and modified Fourier series approximation is found to be accurate enough up to first order as shown in Fig. 5 & 6.

Figure 5: Approximation Accuracy of \( a \)
With this approximation, we can construct the relative motion as linear combination of motion of the four vectors as

$$\delta(t) = \alpha \delta v + \beta \delta a + \gamma \delta b + \kappa \delta v_4$$

where, $\delta = [\delta_x, \delta_y, \delta_z]^T$

### 4 Example Case: Supressing Oscillation in the x-axis Direction

By mathematical manipulations, we can obtain expression for the combined motions. E.g:

$$\delta x(t) = C_{1x} + C_{2x} \sum_{n=1}^{\infty} \sin(2f_n \pi t' + \varphi_x) + C_{3x} t'$$

To supress the oscillation then $C_{2x}, C_{3x} = 0$.

$$C_{2x} = \sqrt{(a_{nx} n v_3 + b_{nx} n a + c_{nx} n b)^2 + (a_{nx} n v_3 + b_{nx} n a + c_{nx} n b)^2} = 0$$

$$C_{3x} = 0$$
Figure 7: Suppressed Motion in the x Direction
In this paper, results of surpressing for the first order are shown in Fig. 7 and 8.

5 Conclusions & Remarks

In this paper design of natural loose formation flying around halo orbits of the Sun-Earth system has been presented. The proposed design method was successfully applied into surpressing oscillation in the x axis direction. In future works, other type of natural motions e.g. considering formation orientation will be explored. Additionally control strategies to optimize fuel cost for maintaining and reconfining formation also will studied.

References

