Proposal of Terramechanics-based Wheel Dynamics Model on Soft Ground

Noriaki Mizukami (Grad.Univ.Advanced Studies), Tetsuo Yoshimitsu (JAXA), Takashi Kubota (JAXA)

ABSTRACT

Planetary surface is covered by soft soil. Thus wheel slips and gets stuck easily on soft soil. For success of rover mission, it is important to prevent wheel from getting stuck and an increase of wheel slip. This paper presents a wheel dynamics model based on terramechanics theory for preventing wheel from getting stuck and an increase of wheel slip by wheel control. Terramechanics is a study of the interaction between wheel and soil. Terramechanics theory considers a wheel travelling motion in static state of wheel sinkage. There are two problems to formulate wheel dynamics model. One is a wheel sinkage phenomenon, and the other is a wheel slip phenomenon. The authors propose a dynamic normal force model for solving the problem of wheel sinkage phenomenon and propose a shear deformation model for solving the problem of wheel slip phenomenon. Then we verify these proposal models by wheel dynamic simulations.

Key Words: Rover, Terramechanics, Dynamics model

1. Introduction

In planetary exploration by a rover, former Soviet Union’s rovers, Lunokhod 1 and Lunokhod 2 travelled several tens of kilometers on lunar surface and sent a lot of pictures of lunar surface to the earth[1][2]. Moreover, NASA’s Mars rover, Opportunity has travelled several kilometers on Mars until now and discovered evidences of water existed[3]. Planetary surface explorations by rovers have achieved important results.

In many cases, a wheel mechanism is employed as a locomotion system for rovers that travel on a planetary soft surface. However, the wheel slips repeatedly and digs the ground, then gets stuck easily on soft soil. Opportunity and Spirit’s wheel got stuck. Opportunity operated the wheel in top gear and get out it. Spirit gave up travelling and decided to observe around the rover.

The interaction between wheel and soil has been studied in the field of terramechanics. Many researchers focus on investigating the traction mechanism of wheel on soft soil[4]. Recently, the lateral wheel force model based on terramechanics has been developed for steering behaviour of rovers[5]. Moreover, the terrain parameter estimation has been developed for calculating wheel forces[6]. In terramechanics theory, the wheel driving force depends on a wheel slip ratio in steady state of wheel sinkage, and wheel sinkage increases with slip ratio increases. Thus researchers have studied slip ratio control for preventing an increase of slip ratio[7][8].

For success of rover mission, it is important to prevent wheel from getting stuck and the increase of wheel slip. For preventing the increase of wheel...
slip, a wheel dynamics model considering wheel sinkage needs to be formulated. This paper presents the wheel dynamics model based on terramechanics theory. Terramechanics theory considers a wheel travelling motion in static state of wheel sinkage. Thus there are two problems to formulate wheel dynamics model. The authors are formulating the wheel dynamics model considering transient state of wheel sinkage. At first we propose the dynamic normal force model for solving problem of wheel sinkage phenomenon and propose the shear deformation model for solving problem of wheel slip phenomenon in a preliminary consideration. Then we evaluate the proposed wheel model by dynamic simulations.

2. Wheel dynamics model

In this section, the wheel dynamics model based on terramechanics is described. The driving wheel generates forces and a torque as shown Fig.1. Motion equations are described as follows. These equations are a wheel travelling motion equation, a wheel rotational motion equation, and a wheel vertical motion equation.

$$\dot{q} = B u$$

where,

$$q = [\dot{x} \ x \ \dot{\alpha} \ \alpha \ \dot{z} \ z]^T, \quad u = T$$

$$f(q) = \begin{bmatrix}
\frac{1}{m_w}(F_x - F_z) \\
\dot{x} \\
\frac{-1}{I_w}T_R \\
\dot{\alpha} \\
\frac{1}{m_w}(m_w g - F_z) \\
\dot{z}
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}$$

$\dot{x}$ is wheel travelling velocity, $\dot{\alpha}$ is wheel angular velocity, $\dot{z}$ is wheel sinkage velocity, $u$ is an input to this model. The forces and a torque such as $F_x$, $F_R$, $F_z$, and $T_R$ are based on terramechanics.

3. Terramechanics

In this section, the wheel model based on terramechanics is described. Terramechanics is a study of the interaction between wheel and soil. M.G. Bekker created and improved a model for wheel-soil interaction based on experimental data[9]. J.Y. Wong investigated a wheel performance by analyzing wheel-soil stresses[10].

A driving wheel generates forces and a torque as shown Fig.1. A driving force $F_x$ works between wheel and soil in a direction of the wheel travelling. It is calculated by integrating from the wheel entry angle $\theta_f$ to the wheel exit angle $\theta_e$ using the horizontal component of the shear stress $\tau(\theta)$ as follows:

$$F_x = rb \int_{\theta_f}^{\theta_e} \tau(\theta) \cos \theta d\theta$$  \hspace{1cm} (3)

A motion resistance $F_R$ works between wheel and soil in a direction opposite to the wheel travelling. It is calculated by integrating from the wheel entry angle $\theta_f$ to the wheel exit angle $\theta_e$ using the horizontal component of the shear stress $\tau(\theta)$ as follows:

$$F_R = rb \int_{\theta_f}^{\theta_e} \sigma(\theta) \sin \theta d\theta$$  \hspace{1cm} (4)

The drawbars pull $F_{DP}$ is defined using the driving force $F_x$ and the motion resistance $F_R$ as follows:

$$F_{DP} = F_x - F_R = rb \int_{\theta_f}^{\theta_e} \{\tau(\theta) \cos \theta - \sigma(\theta) \sin \theta\} d\theta$$  \hspace{1cm} (5)

A normal force $F_z$ works between wheel and soil in a vertical direction from soil to driving wheel. It is calculated by integrating from the wheel entry angle $\theta_f$ to the wheel exit angle $\theta_e$ using the vertical component of the normal stress $\sigma(\theta)$ and the vertical component of the shear stress $\tau(\theta)$ as follows:

$$F_z = rb \int_{\theta_f}^{\theta_e} \{\tau(\theta) \cos \theta + \sigma(\theta) \sin \theta\} d\theta$$  \hspace{1cm} (6)

A disturbance torque $T_R$ works between wheel and soil in a tangential direction of wheel. It is calculated by integrating from the wheel entry angle $\theta_f$ to the wheel exit angle $\theta_e$ using the shear stress $\tau(\theta)$ as follows:

$$T_R = rb \int_{\theta_f}^{\theta_e} \tau(\theta) d\theta$$  \hspace{1cm} (7)

Wheel sinkage $h$ is described by the following equation.

$$h = h_0 + z$$  \hspace{1cm} (8)
\(h_0\) is an initial wheel sinkage and \(z\) is a displacement of the driving wheel sinkage.

The wheel-soil contact angle can be calculated when the wheel sinkage is obtained. Wheel entry angle \(\theta_f\) is the angle from the vertical angle to the angle where the front part of the wheel makes contact to the soil, and defined by the following equation:

\[
\theta_f = \cos^{-1}\left(1 - \frac{h}{r}\right)
\]  
(9)

### 3.1 Normal stress and shear stress

Wheel forces and torque consist of a normal stress and a shear stress based on terramechanics. The normal stress \(\sigma(\theta)\) works the vertical direction of a wheel surface as shown in Fig.2(a). In this research, the normal stress is modeled based on Bekker’s equation. The normal stress is described by the following equation:

\[
\sigma(\theta) = \begin{cases} 
\left(\frac{k_c}{r} + k_n\right)
& \left[r(\cos\theta - \cos\theta_f)\right]^n
\left(\theta_m \leq \theta < \theta_f\right) \\
\left(\frac{k_c}{r} + k_n\right)
& \left[r(\cos\theta_f - \frac{\theta - \theta_m}{\theta_f - \theta_m})(\theta_f - \theta_m) - \cos\theta_f\right]^n
\left(\theta_f \leq \theta \leq \theta_m\right)
\end{cases}
\]  
(10)

where, \(k_c\) is modulus of cohesion stress, \(k_n\) is modulus of internal friction angle, \(n\) is pressure-sinkage ratio, \(r\) is wheel radius, \(b\) is wheel width, and \(\theta_m\) is maximum stress angle as follows:

\[
\theta_m = (a_0 + a_1 s) \theta_f
\]  
(11)

\(a_0\), \(a_1\) are moduli of maximum stress angle.

The shear stress \(\tau(\theta)\) works the tangential direction of the wheel surface as shown in Fig.2(b). The shear stress \(\tau(\theta)\) initially increases rapidly with an increase in shear displacement, then approaches a constant value with a further increase in shear displacement. This type of shear stress-shear displacement relationship is described by an exponential function. Janosi and Hanamoto modeled the shear stress by shear-displacement curve and the shear stress is described by the following equation[11]:

\[
\tau(\theta) = (c + \sigma(\theta) \tan \phi) \left[1 - e^{-(\theta - j)/k}\right]
\]  
(12)

where, \(c\) is cohesion stress of soil, \(\phi\) is internal friction angle of soil. \(j(\theta)\) is shear displacement of soil and defined as follows:

\[
j(\theta) = r\theta_f - \theta - (1 - s)(\sin\theta_f - \sin\theta)
\]  
(13)

\(k\) is shear deformation modulus. The value of \(k\) is defined by the interaction mechanics between wheel and soil.

\(k_c, k_n, n, a_0, a_1, c, \phi\) are soil parameters. \(s\) is slip ratio. Slip ratio \(s\) is defined as a function a travelling velocity \(\dot{x}\) and a circumference velocity of wheel \(r\hat{\alpha}\) as follows:

\[
s = \begin{cases} 
\frac{r\dot{\alpha} - \dot{x}}{r}\, (> \dot{x} : driving) \\
\frac{r\dot{\alpha} - \dot{x}}{\dot{x}}\, (< \dot{x} : braking)
\end{cases}
\]  
(14)

\(\dot{\alpha}\) is angular velocity of wheel. Slip ratio takes a value between -1.0 and 1.0.

### 4. Wheel sinkage phenomenon

Wheel model based on terramechanics (the conventional wheel model) focus on wheel travelling motion in static state of wheel sinkage. Therefore, the conventional wheel model does not consider wheel sinkage phenomenon. In this section, the problem of wheel sinkage phenomenon and an approach to problem-solving are described.

#### 4.1 Problem of wheel sinkage phenomenon

Fig.3 shows a result of dynamic simulations. A wheel velocity is constant, \(r\dot{\alpha} = 0.05[\text{m/s}]\). The soil parameters and wheel paratemeters used in the simulation listed in Table1. When wheel starts rotating wheel sinkage starts changing repeatedly increasing and decreasing.

![Figure 3. Dynamic simulation result (\(r\dot{\alpha}=0.05[\text{m/s}]\))](image-url)
sinkage, wheel velocity are considered. Then formulate the normal force model that considers the dynamic normal force model is proposed as follows:

\[ F_{zd} = rb \int_{\theta_m}^{\theta_f} \{ \sigma_d(\theta) \cos \theta + \tau_d(\theta) \sin \theta \} d\theta \]  

(15)

\( \sigma_d \) is a dynamic normal stress model and \( \tau_d \) is a dynamic shear stress model as follows:

\[ \sigma_d(\theta) = \begin{cases} 
\frac{k_s}{r} + k_\phi & \left( \theta_m \leq \theta < \theta_f \right) \\
\frac{k_s}{r} + k_\phi & \left( \theta_f \leq \theta \leq \theta_m \right)
\end{cases} \]  

(16)

\[ \tau_d(\theta) = \tau_{max} \left[ 1 - e^{-j(\theta)/k} \right] + \eta (r\dot{\alpha} - \dot{x} \cos \theta) \]  

(17)

where, \( \beta \) and \( \eta \) are defined as soil parameters.

4.2 Approach of sinkage problem-solving

The normal force based on terramechanics considers only the static wheel sinkage. It is necessary to formulate the normal force model that considers the transient state of wheel sinkage. To formulate a dynamic normal force model for the transient state of wheel sinkage, a slip velocity are considered. Then the dynamic normal force model is proposed as follows:

\[ F_{zd} = rb \int_{\theta_m}^{\theta_f} \{ \sigma_d(\theta) \cos \theta + \tau_d(\theta) \sin \theta \} d\theta \]  

(15)

\( \sigma_d \) is a dynamic normal stress model and \( \tau_d \) is a dynamic shear stress model as follows:

\[ \sigma_d(\theta) = \begin{cases} 
\frac{k_s}{r} + k_\phi & \left( \theta_m \leq \theta < \theta_f \right) \\
\frac{k_s}{r} + k_\phi & \left( \theta_f \leq \theta \leq \theta_m \right)
\end{cases} \]  

(16)

\[ \tau_d(\theta) = \tau_{max} \left[ 1 - e^{-j(\theta)/k} \right] + \eta (r\dot{\alpha} - \dot{x} \cos \theta) \]  

(17)

where, \( \beta \) and \( \eta \) are defined as soil parameters.

4.3 Dynamic simulation

The proposed model is evaluated by dynamic simulations. The dynamics model employed the wheel model with the proposed dynamic normal force model. The input of a wheel velocity is constant. Soil parameters, wheel parameters, and parameters of dynamic normal force model list in Table1.

Table 1. Parameters for simulations

<table>
<thead>
<tr>
<th>Soil parameters</th>
<th>Values</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c ) [N/m²]</td>
<td>800</td>
<td>cohesion stress</td>
</tr>
<tr>
<td>( \phi ) [deg]</td>
<td>37.2</td>
<td>internal friction angle</td>
</tr>
<tr>
<td>( k_s ) [N/m²]</td>
<td>1370</td>
<td>cohesion stress modulus</td>
</tr>
<tr>
<td>( k_\phi ) [N/m²]</td>
<td>81400</td>
<td>internal friction angle modulus</td>
</tr>
<tr>
<td>( n ) [-]</td>
<td>1.0</td>
<td>sinkage ratio</td>
</tr>
<tr>
<td>( a_f ) [-]</td>
<td>0.4</td>
<td>maximum stress angle modulus</td>
</tr>
<tr>
<td>( a_m ) [-]</td>
<td>0.15</td>
<td>maximum stress angle modulus</td>
</tr>
<tr>
<td>( k_w ) [m]</td>
<td>0.025</td>
<td>shear deformation modulus</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wheel parameters</th>
<th>Values</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_w ) [kg]</td>
<td>6.0</td>
<td>wheel mass</td>
</tr>
<tr>
<td>( r ) [m]</td>
<td>0.1</td>
<td>wheel radius</td>
</tr>
<tr>
<td>( b ) [m]</td>
<td>0.1</td>
<td>wheel width</td>
</tr>
<tr>
<td>( I_w ) [kgm²]</td>
<td>0.065</td>
<td>wheel inertia</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>Values</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta ) [N/(m/s)]</td>
<td>32.000</td>
<td>soil parameters</td>
</tr>
<tr>
<td>( \eta ) [N/(m/s)]</td>
<td>6.0</td>
<td>soil parameters</td>
</tr>
</tbody>
</table>

5. Wheel slip phenomenon

5.1 Problem of wheel slip phenomenon

Fig.5 shows a result of dynamic simulations with an input of wheel velocity \( r\dot{\alpha}=10.0\) [m/s]. The wheel sinkage increases quickly, and the sinkage reaches a static state of sinkage, \( h=0.025\) [m]. The slip ratio increases and reaches \( s=0.95 \), then decreases to \( s=0.9\) and keeps \( s=0.9 \). Slip ratio starts decreasing 0.5[s] later. It indicates that the wheel does not get stuck with the input of a large wheel velocity. In the fol-
lowing section, the reason of this problem and an approach to problem-solving are described.

5.2 Reason of wheel slip problem

Fig.6 shows relationship between drawbar pull $F_{DP}$ and slip ratio $s$ in the static state of wheel sinkage. The drawbar pull increases with slip ratio increases. The driving force $F_\tau$ increases with slip ratio increases, but the motion resistance $F_R$ decreases with slip ratio increases. It indicates that the wheel accelerates with slip ratio $s=1.0$. Therefore, wheel travelling velocity $\dot{x}$ increases and slip ratio decreases as shown Fig.5.

![Graphs showing relationship between drawbar pull and slip ratio.](image)

**Figure 6.** Simulation for wheel model analysis

5.3 Approach of wheel slip problem solving

Since the driving force consists of the stress model $\tau(\theta)$, the authors focus on the shear stress model to find the reason why the drawbar pull does not become 0.0[N] at $s=1.0$.

The shear stress increases with shear displacement increases. From the equation (13), the shear displacement increases with slip ratio increases. Thus the driving force increases with slip ratio increases. We focus on a shear deformation modulus $k$. The shear deformation modulus is used as a parameter that is a condition between wheel surface and sand. The shear deformation modulus is used as a constant value in conventional research. This is the reason why the drawbar pull does not become 0.0[N] at $s=1.0$.

The shear stress is measured by a direct shear test. In the direct shear test, the sand in the test module is given a certain amount of stress. Then a bottom part of test module is fixed and an upper part of the test module is displaced with uniaxial stress in order to shear between sand in the upper part and sand in the bottom part. Thus sand is sheared between sand and sand. On the other hand, shear phenomenon by the wheel rotation does not always shear between sand and sand. Thus the shear phenomenon by the direct shear test is different from the shear phenomenon by the wheel rotation. If sand does not shear between sand and sand, shear stress does not increase with shear displacement increases.

In the preliminary consideration, we consider a way to use the shear deformation modulus as a variable, and propose a shear deformation model as follows:

$$k = k_1 s^m + (h - k_2)$$  \hspace{1cm} (18)

The shear deformation model equation is formulated as a function of slip ratio and wheel sinkage. The first term of the shear deformation model equation represents a shear phenomenon by wheel slip. If the slip ratio is small, the shear phenomenon occurs between sand and sand. Thus the shear stress under the wheel becomes large. On the other hand, the slip ratio is large, the shear phenomenon occurs between the wheel surface and sand. Thus the shear stress under the wheel becomes small. The second term of the shear deformation model equation represents a shear phenomenon by wheel sinkage. If the wheel sinkage is small, the shear phenomenon occurs between sand and sand due to soft sand. It is easy to shear between sand and sand in soft sand. Thus the shear stress under the wheel becomes large. On the other hand, the wheel sinkage is large, the shear phenomenon occurs between the wheel surface and sand due to hard sand. It is difficult to shear between sand and sand in hard sand. Thus the shear stress under the wheel becomes small.

![Graph showing relationship between drawbar pull and slip ratio.](image)

**Figure 7.** Relation of drawbar pull and slip ratio

Fig.7 shows relationship between drawbar pull and slip ratio with the proposed shear deformation model at various wheel sinkage. Soil parameters is regolith simulant listed in Table1. The parameters of the proposed shear deformation model is listed in Table2. The drawbar pull decreases with slip ratio increases and the driving force decreases with slip ratio increases. The drawbar pull becomes 0.0[N] with $s=1.0$ at the sinkage $h=0.025$[m].

5.4 Dynamic simulation

The proposed wheel model is evaluated by dynamic simulations. Soil parameters of regolith simulant and
Table 2. Parameters of the shear deformation model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$ [m]</td>
<td>0.3071</td>
<td>shear deformation modulus</td>
</tr>
<tr>
<td>$k_2$ [m]</td>
<td>0.0587</td>
<td>shear deformation modulus</td>
</tr>
<tr>
<td>m [-]</td>
<td>5</td>
<td>shear deformation modulus</td>
</tr>
</tbody>
</table>

wheel parameters list in Table1. Moreover, parameters of the proposed shear deformation model list in Table 2. Fig.8 shows a result of the dynamic simulations with an input of wheel velocity $r\alpha=10.0$[m/s]. The wheel sinkage increases quickly, and the sinkage reaches a static state of sinkage 0.25[s] later. The slip ratio increases and reaches $s=1.0$. When the input of the wheel velocity is large, the slip ratio becomes $s=1.0$, when the input of the wheel velocity is large. These proposed wheel models may be available for the dynamic simulation. However, we only show the approach of slip phenomenon problem-solving and the theoretical consideration of this problem is not enough.

In the future work, we will formulate a wheel model for solving problem of slip phenomenon by more theoretical approach.

6. Conclusion

This paper addressed the wheel dynamic model formulation. To formulate the wheel dynamics model, problems of wheel sinkage phenomenon and slip phenomenon using wheel model based on terramchanic were represented. We proposed the dynamic normal force for solving wheel sinkage problem and the shear deformation model for solving wheel slip problem. The dynamics model with the proposed models was evaluated by dynamic simulations. From results of the simulations, the wheel sinkage stopped changing repeatedly increasing and decreasing and reaches a constant wheel sinkage. Moreover, the slip ratio be-

REFERENCES