Study of the Generation Method of the Quasi-halo Orbit and the Application to the Transfer

準ハロー軌道の生成法とその遷移軌道の解析

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This paper investigates transfers from the Earth to the quasi periodic orbit in the vicinity of the libration point (Quasi-halo orbit) of the Sun-Earth system in order to expand the launch window. For the mission requirement of the first Japanese Lagrange point mission, SPICA (SPace Infrared telescope for Cosmology and Astrophysics), the constraint of the nominal trajectory is loose. Therefore, we discuss the transfer to the Quasi-halo orbits instead of to the Halo orbits in this study. First, the orbital conditions such as the altitude of perigee and the inclination at the given launch site for the transfers to the Quasi-halo orbits by using the characteristic of the dynamical system like stable manifolds, which are converge to the Quasi-halo orbits naturally, are shown. Next, the way of the expanding the launch window for the lift-off to Quasi-halo orbits is discussed.

1. Introduction

Sun-Earth libration points are located where the gravity of the Sun and Earth and centrifugal force acting on spacecraft are balanced. In particular, the position of L2, which lies on the line connecting the Sun and Earth, is an ideal place for the astronomical satellite because the radiative cooling could be effective and long observable area could be obtained due to the stable geometrical condition with respect to the Sun and Earth. In fact, several astronomical satellites such as WMAP, Herschel and Plank have already utilized around the L2 points (e.g., Halo/Lissajous orbits) 1. From now, the large astronomical observatories like JWST and SPICA will likely be located near the Sun-Earth L2 point [2, 3].

For future libration mission, we study the transfer trajectories to Halo orbits using the stable manifold [4-9], considering launch conditions in this study. Stable manifolds are a dynamical characteristic of three-body model and converge to the Halo orbit naturally. By using the stable manifold, we could not only reduce the propellant but also avoid some critical operations such as the attitude control and insertion impulse maneuver to insert the Halo orbit.

First, the transfer from the Earth to the Halo orbit using the stable manifold is discussed, investigating the characteristic of the stable manifold. Second, the way of the expansion of the launch window is shown by using the quasi halo orbit.

2. Circular Restricted Three－Body Problem

2.1. Equations of motion

The physical system considered in this study is the circular restricted Sun-Earth three-body problem. (CR3BP), which describes the dynamics of a massless particle attracted by two point masses revolving around each other in a circular obit (see Fig. 1). The equations of motion for spacecraft in this model are given by10,

$$\ddot{x} - 2\dot{y} + x = \frac{\partial U}{\partial x} \tag{1}$$

$$\ddot{y} + 2\dot{x} - y = \frac{\partial U}{\partial y} \tag{2}$$

$$\ddot{z} = \frac{\partial U}{\partial z} \tag{3}$$

where $U = \frac{1-\mu}{d_S} + \frac{\mu}{d_E} + \frac{1}{2}(x^2 + y^2)$ is the Pseudo potential, $\mu = m_E/(m_S + m_E)$ is the gravitational parameter (m_S, m_E are the masses of the Sun and Earth, respectively), and d_S and d_E are distance from the Sun and Earth to the spacecraft (S/C).

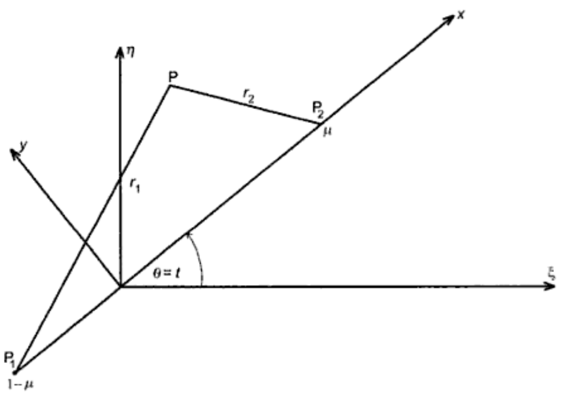

Fig. 1: Geometry of the Circular Restricted Three-Body Model

2.2 Libration Points, Halo orbit, stable manifold

In the circular restricted Three-body problem model there are five equilibrium points where the gravity of the Sun and Earth and centrifugal force acting on S/C are balanced, which are called libration points (see Fig. 2). In the vicinity of the libration point, there are three-dimensional periodic orbits called Halo orbits Furthermore, there exist invariant structures associated with the Quasi-halo orbits, called stable manifolds [3-5] (see Fig. 3). These are trajectories that wind onto the Halo orbit automatically. We exploit the stable manifolds for the transfer to the Halo orbit.

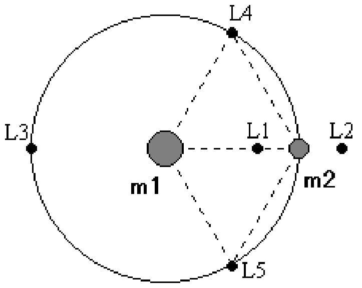

Fig. 2: Libration Points.

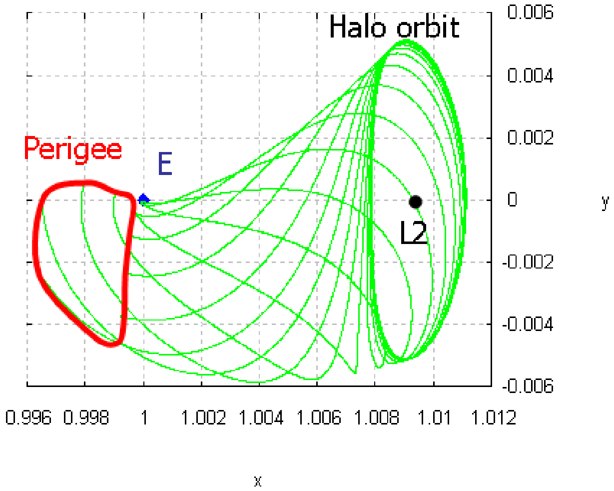

Fig. 3: Quasi Stable Manifold (Until first Perigee)

3. Transfer between the Earth and the Halo orbit using the stable manifold

In this paper, we assume that the S/C is placed into the stable manifold from the low earth orbit. Subsequently, S/C is inserted to the Halo orbit with a nearly zero velocity correction. Since the stable manifold wind onto the halo orbit naturally, we can avoid critical operations such as a large velocity correction at the insertion.

3.1 Characteristics of the perigee of the stable manifold

First, we investigate the first perigee passage points of stable manifolds, where an impulsive maneuver may be performed to place the S/C into the stable manifold, propagated from points on the Halo orbit in backward (see Fig. 3). Fig. 4 shows the relation between the perigee distance from the center of the Earth and the inclination to the ecliptic plane of the perigee points of the stable manifold for the Halo orbit of the *z*-amplitude (*Az*) = 0.4 million km. It was found that there are two solutions to connect the stable manifold with the 300 km low earth orbit (LEO). Upper and lower solutions are called the slow and fast transfer, respectively, and Fig. 5 shows these trajectories.

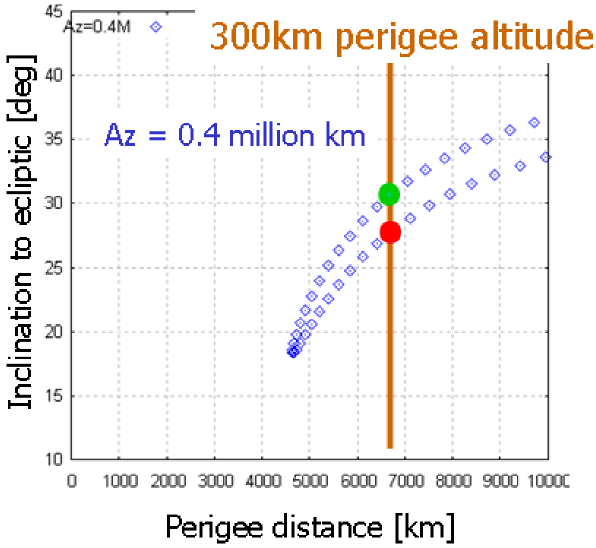

Fig. 4: Perigee Distance and Inclination of the First Perigee of the Stable Manifold

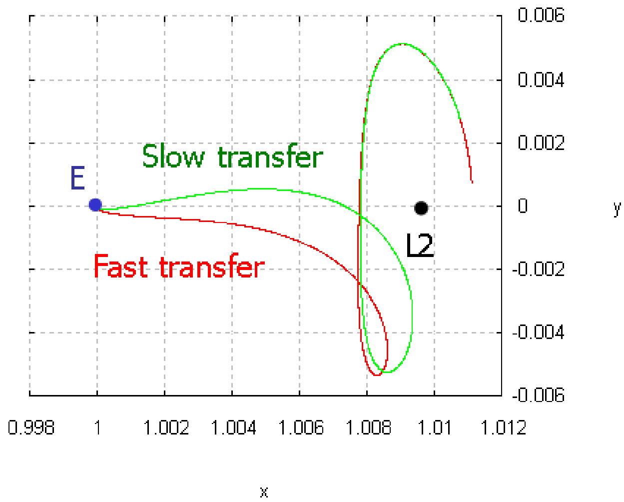

Fig. 5: Fast and Slow Transfer

3.2 Possibility to connect the LEO with stable manifold

Next, to study a possibility of the transfer to the Halo orbit using the stable manifold for the given launch conditions, we investigate the inclination to the equatorial plane of the perigee of the stable manifold at the 300 km altitude, based on the results in Fig. 4. Fig. 6 shows the inclination to the equatorial plane as a function of the number of days from January 1st, assuming the lift-off from the Japanese launch site (about 30 degrees north latitude). Thick and thin lines indicate results for the slow and fast transfers, respectively. We can say that there are four occasions to be launched at the north latitude of 30 degrees in a year even if using both the fast and slow transfers.

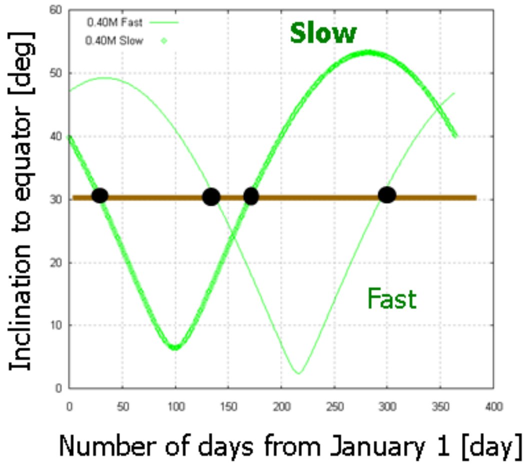

Fig. 6: Launch Opportunities in a Year

4. Expansion of the Launch Window

From the result of the previous section, we only have four opportunities in a year to put the S/C into the stable manifold from, for instance, the Japanese launch site for the transfer to the Halo orbit. For the practical launch, we need a few weeks for the launch window at a launch. Nakamiya reported that the launch window to the Halo orbit can expand by changing the size of the Halo orbit [11]. Instead, in this study, we discuss the expansion of the launch window to utilize the Quasi-halo orbit instead of the Halo orbit.

4.1 Quasi-halo orbit and the method to produce Quasi-halo orbit

The quasi-halo orbit is the quasi periodic orbit in the vicinity of the libration point, and it is a torus around the Halo orbit (see Fig. 7). Using the stable manifold of the quasi halo orbit could expand the possibility of the launch window because its periapsis points such as in Fig. 3 offer greater flexibility.

In the past, there have been a number of studies to generate the quasi-halo orbit [12-14]. In this study, we utilize the method of Kolemen and Kasdin [15]. This method produces the Quasi-halo orbit by iterating to align the section of the quasi-halo orbit in the x-y plane and the return section integrating until crossing the x-y plane again, using the initial condition from the Monodormy matrix as shown in Fig. 8.

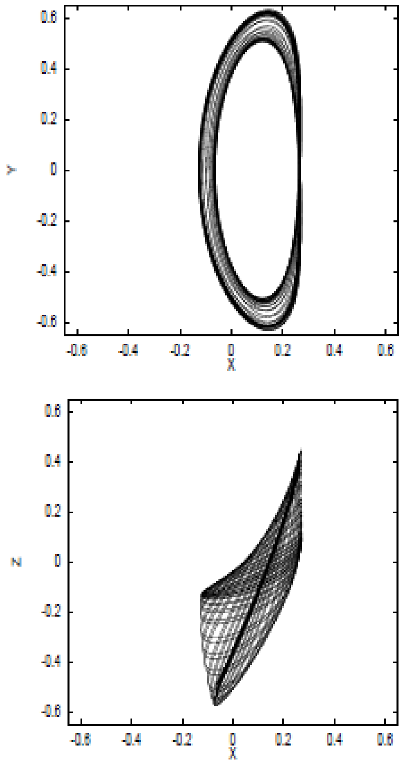

Fig. 7: Quasi-halo Orbit [14].

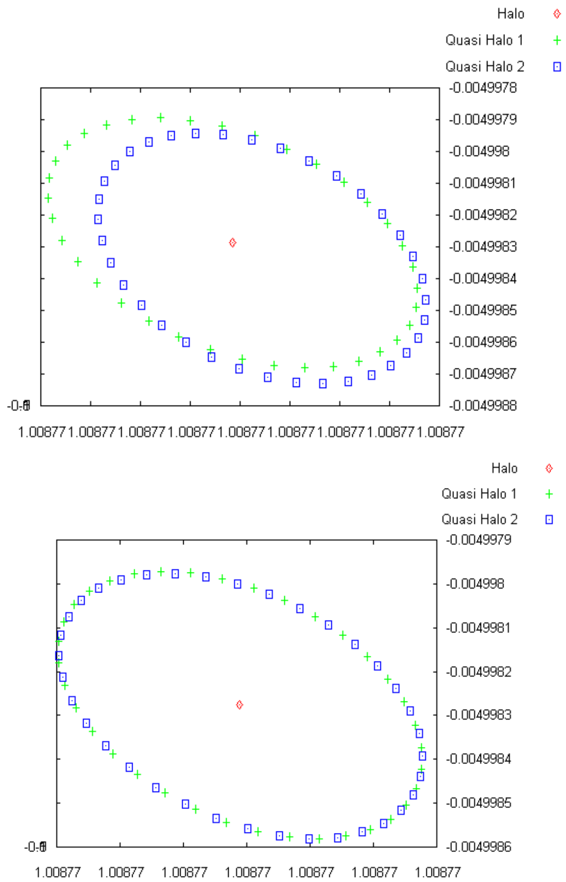

Fig. 8: Section of the Quasi-halo Orbit in the x-y plane.

4.2 By utilizing the Quasi-halo orbit

We expect that the larger quasi halo orbit (the thicker torus) is used, the longer the launch window become because of the great flexible periapsis of the manifold. However, the only thin quasi halo orbit can be generated so far (The radius of the torus is less than1500 km; Fig. 9). Thus, the expanding the launch window by using the quasi halo orbit is not progressing as much as we had hopes.

At the symposium, we are going to present a better deal.

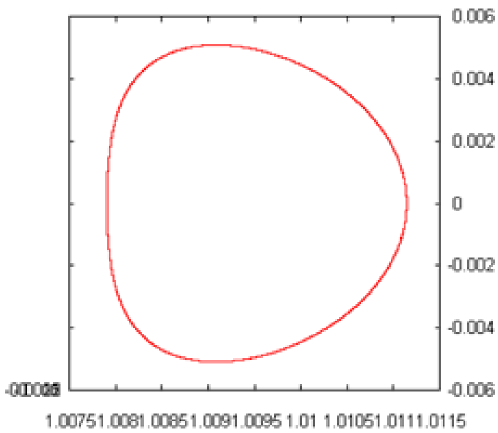

Fig. 9: Generated Quasi-Halo Orbits (Radius of the Torus is about 1500 km)

Conclusion

This paper presents the transfer trajectories to Quasi-halo orbits using stable manifolds considering the expansion of the launch conditions. Currently, the thick quasi-halo orbit is not produced. Therefore, the launch window is not expanded enough. At the symposium, we are going to present a better deal.

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