Model Predictive Motion Control of a Spacecraft in Touchdown Phase to the Asteroid

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Abstract

JAXA is planning the new asteroid exploration mission after "HAYABUSA". The supposed operation of it seems to require much higher guidance accuracy in approach and touchdown phase than previous mission since it is expected to touchdown inside of the newly made crater. For position controller design Model Predict Control (MPC) which can consider limitations in control input and predict future state is applied in this paper. The performance and usefulness of this control algorithm is evaluated through numerical simulation.

Key Words: asteroid probe, model predictive control, touchdown phase, motion control

小惑星へのタッチダウンフェーズでのモデル予測制御による探査機の運動制御

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2010年6月に小惑星探査機「はやぶさ」は地球へと帰還し、小惑星サンプルリターンミッションを終了した. JAXA では「はやぶさ」で得た経験を活かして新たな小惑星探査を計画している. この「はやぶさ2」では、新規のミッションとして何らかの方法で小惑星表面にクレーターを生成し、露出した内部のサンプルの採取を計画しており、太陽系の起源に関するより進んだ解析が期待される。この探査手法を実現するためには生成されたクレーター内、または、その周辺に探査機をタッチダウンさせる必要がある. 確実なサンプルの採取のために、降下・着陸フェーズにおいてより精度の高い制御を行うことが求められている.

本研究では「はやぶさ2」の着陸フェーズにおいて MPC(Model Predictive Control) を用いて探査機の搭載 系による自律運動制御を行うことを提案する. MPC とは主に化学プラントで用いられている予測制御法であ り,その特徴としては、制約を考慮した最適化が行える点や、調整が容易で直観的である点などが挙げられ る.特に制約を考慮出来る点や未来の状態量を予測できる点が、地球からの片道の通信時間約 20 分という小 惑星探査機の制御特有の問題に対して適していると考えられる.本研究では MATLAB/Simulink を用いて数 値シミュレーションを行い、MPC の有用性を検証する.

1. Introduction

In June 2010 the asteroid probe "Hayabusa" returned to the Earth completing asteroid sample return mission. As a follow on mission, JAXA is planning to start a new asteroid exploration mission "Hayabusa2". Hayabusa2 is proposing to make a crater on the surface of an asteroid in some way and to touch down in or in the proximity of the crater for sampling newly exposed material of the asteroid. In order to do this, it is required to control the position of the probe more precisely than the case of previous mission in approach and touchdown phase.

This paper proposes application of MPC (Model Predictive Control) to automatic feed-back position control for Hayabusa2 in touchdown phase. Since MPC can handle the constraints explicitly in the controller design and can predict future states using dynamical model of the plant, it seems to be suitable for the application in the remote place with one way communication delay of approx. 20 minutes from the Earth. Through numerical simulation its feasibility is demonstrated.

2. Approach and touchdown phase to the asteroid

It is assumed that the target asteroid for Hayabusa2 is sphere shape with radius of 500m performing one-axis spin attitude motion. The approach and touchdown phase descending from the place with altitude of 100m to the target site such as newly made crater controlling its lateral velocity to follow to the asteroid surface is dealt with here. The details are as follows.

Through remote control from the Earth, position of Haybusa2 is controlled based on the inertial frame to the place with approx. 100m above the touchdown site. After that it releases TM (Target Marker) which is a ball with reflective surface toward the place for touchdown. Because of the error in direction of release velocity of the TM it is not guaranteed that it lands exactly the desired place for touchdown.

Once the TM is landed on the surface, Hayabusa2 moves

up taking images of both TM and the touchdown site and sending them to the Earth. The ground operator measures relative position between them using images and send this information to the onboard computer.

Again, Hayabusa2 is remote controlled to descend toward the TM within the distance where onboard FLA (Flash LAmp) which is stroboscopic light source can illuminate the TM. Then, the difference image between ONC (Onboard Navigation Camera) images with FLA on and off is calculated onboard and is used for recognition of the place of TM in the image for navigation. In addition to that, LRF (Laser Range Finder) with four laser beams gives distance and attitude to the surface of the asteroid. Using above information, relative position of Hayabusa2 to the touchdown site is calculated and used for position and attitude controller. Figure 1 shows the trajectory in the touchdown phase as is explained in detail below.



Fig. 1 Touchdown trajectory

- 1→2. Descend vertically from altitude 100m to 30m controlling its lateral position to just above the TM and controlling its attitude so that the LOS (Line Of Sight) of ONC will face to TM.
- $2\rightarrow 3$. After descending down to altitude of 30m, change lateral position control target from TM to the touchdown site such as center of the crater utilizing their relative position information given beforehand. The attitude is controlled so that the LOS of ONC will face to the touchdown site.
- 3→4. Descend vertically toward the touchdown site. After descending down to the altitude which is the minimum altitude that TM can be recognized by ONC using FLA (5m) or the altitude that TM is not in the field of view of ONC which is controlled to face orthogonally to the surface of the asteroid, Hayabusa2 loses position information of TM and stops lateral control with vertical velocity control still working until it touches down.

3. Controller design

3.1. Coordinates

Figure 2 shows coordinates defined and used for the controller design.

HP frame : The frame with origin at the centroid of the target

asteroid. Z_{HP} _axis is to the Earth and X_{HP} . Z_{HP} plane contains the Sun. This frame could be regarded as the inertial frame here.

- **B frame :** The probe-fixed frame with origin at the centroid of Hayabusa2.
- **BT frame :** The asteroid-fixed frame with origin at the centroid of the asteroid.



Fig. 2 Coordinates

3.2. Hardware configuration for 6 degrees of freedom control

Hayabusa2 has 12 thrusters for 3 degrees of freedom position control and 4 reaction wheels for 3 axis attitude control. It has ONC for image based navigation particularly measuring the line-of-sight to the TM on the surface of the asteroid and LRF in order to measure distance and relative attitude to the surface. A star tracker and gyro are used with extended Kalman Filter for attitude determination.

3.3. Prior condition for controller design

The assumed condition for controller design is as follows,

- Attitude motion of the asteroid is estimated with sufficient accuracy by before-the-fact observation and the attitude of the asteroid during approach and touchdown phase can be estimated and predicted with sufficient accuracy. Therefore, coordinate transform matrices between three frames (HP frame, BT frame, B frame) are given with sufficient accuracy.
- Relative position between landed TM and the touchdown site in BT frame is given from the ground operator by before-the-fact observation.

3.4. Position control by MPC

MPC is a controller design algorithm which can explicitly take account constraints in states and control inputs. In the case of touchdown position control of Hayabusa2, constraints are maximum thrust force and maximum vertical descent velocity. MPC uses internal dynamical model of translational motion of a spacecraft and it can predict future states using past states and control inputs. Then optimization problem as shown below is solved by QP (Quadratic Programming) problem generating optimal change of control inputs $\Delta \mathbf{u}(k)$ ^[1]

State space internal model for MPC

The state space model for the translational motion of a

spacecraft is

$$y(k+1) = \mathbf{A}x(k) + \mathbf{B}u(k) \tag{0.1}$$

$$V(k) = \mathbf{C}_{y} \mathbf{x}(k) \tag{0.2}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \tag{0.3}$$

$$\mathbf{x}(k) = \begin{bmatrix} \mathbf{r}_c^{m} \\ \mathbf{v}_c^{HP} \end{bmatrix}$$
(0.4)

 r_c^{HP} : spacecraft position (HP frame)

 v_c^{HP} : spacecraft velocity (HP frame)

$$\mathbf{B} = \begin{bmatrix} 0.0009804\\ 0.001961 \end{bmatrix} \tag{0.5}$$

$$u(k)$$
: thrust force [N] (0.6)

$$\mathbf{C}_{y} = \begin{bmatrix} 1 & 0\\ 0 & 1 \end{bmatrix} \tag{0.7}$$

Cost function for MPC

The cost function to minimize in MPC is

$$V(k) = \sum_{i=H_{*}}^{H_{*}} \|\hat{y}(k+i \mid k) - r(k+i \mid k)\|_{\mathbf{Q}(i)}^{2} + \sum_{i=0}^{H_{*}-1} \|\Delta u(k+i \mid k)\|_{\mathbf{R}(i)}^{2} (0.8)$$

 $\Delta u(k+i|k)$: variation of control inputs

 $\hat{y}(k+i|k)$: predicted outputs

r(k+i|k): reference trajectory, the trajectory that the output should follow ideally, given from current observed output y(k) and set-point trajectory s(k)

 H_n : predictive horizon

 H_u : control horizon

"predictive horizon" H_p is the number of steps that MPC predicts future states and "control horizon" H_u is the number of steps that MPC prepares for future control inputs. These are set to be $H_p = H_u = 10$, with sampling interval $T_s = 1[s]$.

$$\mathbf{Q} = diag \Big[\mathcal{Q}(1), \quad \cdots \quad \mathcal{Q}(H_p) \Big] \tag{0.9}$$

Q : weighting matrix for outputs

$$\mathbf{R} = diag[R(1), \cdots R(H_{\mu})] \tag{0.10}$$

R : weighting matrix for control input variation are set to be Q(i) = 1000, R(i) = 1.0.

Above minimization problem eq. (1.8) is solved at every sampling interval and predicted control input variation $\Delta u(k+i|k)$ i=0...H_u-1 is given and $\Delta u(k|k)$ is used as a control inputs for the sampling interval.

Constraints and formulation as a QP problem

Constraint on control inputs and states are expressed below.

$$\mathbf{F}\begin{bmatrix}\mathbf{u}(k)\\1\end{bmatrix} \le 0 \tag{0.11}$$

$$\mathbf{u}(k) = \begin{vmatrix} \hat{u}(k) \\ \vdots \\ \hat{u}(k+H_n) \end{vmatrix}$$
(0.12)

$$\mathbf{G}\begin{bmatrix}\mathbf{y}(k)\\1\end{bmatrix} \le 0 \tag{0.13}$$

$$\mathbf{y}(k) = \begin{bmatrix} \hat{y}(k) \\ \vdots \\ \hat{y}(k+H_p) \end{bmatrix}$$
(0.14)

These constraints are to be expressed as constraint over $\Delta \mathbf{u}(k)$ in order to solve them as a QP problem.

The matrix ${\bf F}$ for constraints on control input (thrust force) can be written as

$$\mathbf{F} = \begin{bmatrix} F_1 & F_2 & \cdots & F_{H_u}, & f \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0, f_{\max} \\ 0 & 1 & \cdots & 0, \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1, f_{\max} \end{bmatrix}$$
(0.15)

$$f_{\rm max}$$
 : maximum thrust force

This **F** is applied for all $\mathbf{u}(k)$ in control horizon.

Using eqs. (1.11), (1.12), (1.15) we get

$$\sum_{i=1}^{H_a} F_i \hat{u}(k+i-1|k) + f_{\max} \le 0 \qquad (0.16)$$

Eq. (1.16) can be rewritten for control input variation $\Delta u(k)$ as follows

$$F_a \Delta u(k) \le -F_1 u(k-1) - f_{\max}$$
 (0.17)

$$F_{a_i} = \sum_{j=1}^{n_u} F_j \qquad F_a = [F_1 \quad F_2 \quad \cdots \quad F_{H_u}]$$
(0.18)

where eq. (0.19) is used as control input constraint formulation for QP problem.

The matrix **G** for constraints on velocity which is a part of states can be written as

$$\mathbf{G}\begin{bmatrix} \Psi x(k) + \gamma u(k-1) + \Theta \Delta u(k) \\ 1 \end{bmatrix} \le 0 \tag{0.20}$$

$$\mathbf{I}' = \begin{bmatrix} \mathbf{A} \\ \vdots \\ \mathbf{A}^{H_p} \end{bmatrix} \tag{0.21}$$

$$\gamma = \begin{bmatrix} B \\ \vdots \\ \sum_{i=0}^{H_{y}-1} \mathbf{A}^{i} \mathbf{B} \\ \vdots \\ \sum_{i=0}^{H_{y}-1} \mathbf{A}^{i} \mathbf{B} \end{bmatrix}$$
(0.22)

$$\Theta = \begin{bmatrix} B & \cdots & 0 \\ AB + B & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{H_{y}-1} A^{i}B & \cdots & B \\ \sum_{i=0}^{H_{y}} A^{i}B & \cdots & AB + B \\ \vdots & \cdots & \vdots \\ \prod_{p=0}^{H_{p}-1} A^{i}B & \cdots & \sum_{i=0}^{H_{p}-H_{y}} A^{i}B \end{bmatrix}$$
(0.23)

where Ψ , γ are matrices for calculating free response using internal model, Θ is matrix for predicting states over predictive horizon using control input variation. **G** is in the form as

 $\mathbf{G} = \begin{bmatrix} \Gamma & g \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0, v_{\max} \\ 0 & 1 & \cdots & 0, & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1, v_{\max} \end{bmatrix}$ (0.24)

 $v_{\rm max}$: maximum velocity constraint

expressing the same constraints at every prediction step. Substituting eq. (1.23) to eq. (1.19) we can get

$$\Gamma\Theta\Delta u(k) \le -\Gamma[\Psi x(k) + \gamma u(k-1)] - g \qquad (0.25)$$

where eq. (1.24) is used as velocity constraint formulation for QP problem.

QP (Quadratic Programming) problem

From eqs. (1.8), (1.17), (1.24), a standard optimization problem known as Quadratic Programming problem is formulated as follows.

$$\min_{\theta} \left[\frac{1}{2} \theta^{T} \Phi \theta + \phi^{T} \theta \right] \qquad subject \quad to \quad \Omega \theta \le \omega$$
$$\theta = \Delta \mathbf{u}(k) \qquad (0.26)$$

$$\Phi = \Theta^T \mathbf{Q} \Theta + \mathbf{R} \tag{0.27}$$

$$\phi = -2\Theta^{T} \mathbf{Q}(\mathbf{r} - \Psi x(k) + \gamma u(k-1))$$
(0.28)

$$\mathbf{r} = \left[r(k+1/k), \quad \cdots \quad r(k+H_p/k) \right]^T \tag{0.29}$$

$$\Omega = \begin{bmatrix} F \\ \Gamma \Theta \end{bmatrix} \tag{0.30}$$

$$\omega = \begin{bmatrix} -u(k-1) - f_{\max} \\ \vdots \\ -u(k-1) - f_{\max} \\ -\Gamma(\Psi x(k) + \gamma u(k-1)) - g \end{bmatrix}$$
(0.31)

As mentioned above, MPC has internal dynamical model and it predicts future states at every control instance. Considering that there is communication delay of approx. 20 min between the Earth and the probe, it is not easy to monitor and remote control the motion of the probe in real time from the Earth. The fact that MPC is always predicting over predictive horizon suggests possibility that it could make judgment onboard for safety of touchdown and in case of emergency it could force the probe to abort.

3.5. On/off thrust model

The control input command from MPC is continuous in the range of $\pm f_{\text{max}}$, whereas actual thrust force is on/off. The conversion between them at every control instance is

$$t_{th} = F_{MPC} \div f_{\max} \tag{0.32}$$

 F_{MPC} : thrust force request from MPC

$$f_{\rm max}$$
 : constant thrust force (35N)

$$t_{th}$$
: thrusting time

When t_{th} is less than the period of minimum impulse 0.1 sec from eq. (1.31), t_{th} is set to be zero.

In order to simulate the error in thrust force random error with $\sigma = 0.1$ [N] is added to the constant thrust force.

3.6. Measurement model

LRF error model

From previous experience, the 1σ error model of LRF for distance measurement to the surface of the asteroid is 3m@100m and 0.1m@10m. These random errors are interpolated by quadratic function and considered.

TM los measurement error model using ONC

Using relative position and attitude of a spacecraft to TM on the surface of the asteroid given from the numerical simulation, the position of TM in ONC (1000x1024 pixels, 60deg x 60deg of FOV) image is simulated and random error of $\sigma = 0.1$ [pixel] is added.

Kalman filter for position estimation

Outputs from LRF and ONC are used as inputs to Kalman filter for estimating relative position and velocity in HP frame.

3.7. Attitude control using line of sight to the TM

Attitude is controlled to point the LOS (Line Of Sight) of ONC to "target". Here "target" means TM in the altitude more than 30m and the touchdown site in the altitude less than 30m.

Error quaternion given from the LOS vector is written as follows. In the numerical simulation, λ and θ_{LOS} are calculated from relative position to the target and LOS vector of ONC following eq. (0.33) and (0.34)

$$q_e = \begin{bmatrix} \mathbf{q}_{e_vector} \\ q_{e_scalar} \end{bmatrix}$$
(0.35)

$$\mathbf{q}_{e_vector} = \boldsymbol{\lambda} \sin\left(\frac{\theta_{LOS}}{2}\right) \tag{0.36}$$

$$q_{e_scalar} = \cos\left(\frac{\theta_{LOS}}{2}\right) \tag{0.37}$$

$$\boldsymbol{\lambda} = -\frac{r_{CT}^{HP} \times r_{LOS}^{HP}}{\left|r_{CT}^{HP} \times r_{LOS}^{HP}\right|} \tag{0.38}$$

$$\theta_{LOS} = \arccos\left(-\frac{r_{CT}^{HP} \bullet r_{LOS}^{HP}}{\left|r_{CT}^{HP}\right|\left|r_{LOS}^{HP}\right|}\right)$$
(0.39)

Using error quaternion q_e and attitude rate of a spacecraft $\mathbf{\omega}^B$, attitude is controlled as

$$\boldsymbol{\tau}^{B} = \mathbf{K}_{r}\boldsymbol{\omega}^{B} + \mathbf{K}_{p}(q_{e \ scalar}q_{e \ vector}) \tag{0.40}$$

- q_e : attitude error quaternion
- \mathbf{q}_{c_vector} : vector part of attitude error quaternion
- $q_{c \ scalar}$: scalar part of attitude error quaternion
 - r_{CT}^{HP} : relative position between the target and a spacecraft (HP frame)
 - r_{LOS}^{HP} : LOS vector of ONC (HP frame)
 - $\theta_{\rm LOS}\,$: LOS angle to the tareget
 - $\boldsymbol{\omega}^{\scriptscriptstyle B}$: attitude rate of a spacecraft (B frame)
 - \mathbf{K}_r : feedback gain (derivative)
 - \mathbf{K}_{p} : feedback gain (proportional)
 - $\mathbf{\tau}^{\scriptscriptstyle B}$: reaction wheel torque command (B frame)

4. Numerical simulation

Initial conditions and final result for numerical simulation are shown below.

Initial condition (BT frame)

initial position of a probe : [10, 10, 600](m)initial velocity of a probe : [0.0, 0.0, 0.0](m/s)target descent velocity : -0.05(m/s)TM position : [2.0, 2.0, 500](m)

touchdown site position : [0.0, 0.0, 500](m)

attitude rate of the asteroid : $[0.0, 2.3136e-004 \quad 0.0](rad / s)$

Final result (BT frame)

final position of a probe : [-0.0061, -0.2464, 500](m) final velocity of a probe :

[-0.0019, -0.0030, -0.0464](m/s)

Figure 3-6 show the result of numerical simulation using conditions shown above. Figure 3 and 4 show position of the probe in asteroid-fixed frame (BT frame). The set-point trajectory for lateral position control during touchdown is set as follows.

- ① Firstly, descend with constant vertical velocity toward TM until the altitude decreases down to 30m
- ⁽²⁾ Secondly, descend with constant vertical velocity toward the touchdown site (center of the crater) while watching at TM for navigation
- ③ When following condition (a) or (b) holds, TM visibility could be lost. Therefore lateral position control is turned off while descending and LOS control is changed from facing to the touchdown site to facing orthogonally to the surface of the asteroid.
 - (a) TM is outside the field of view of ONC



Fig. 3 Trajectory of the probe (X,Y in BT frame)



Fig. 4 Trajectory of the probe (BT frame)



Fig. 5 Velocity of the probe (BT frame)

⁽b) altitude of the probe is less than 5m



Fig. 6 LOS angle of ONC to the target

Both in figure 3 and 4, the red circle (radius:3m) shows the assumed crater size created and the magenta circle (radius: 6m) shows the area where debris spread during the process of creating the crater. The probe has to touchdown inside of the magenta circle for successful sampling. Factors which cause closed loop position control error would be measurement error (LRF, ONC and Kalman Filter), thrust force error and control interval. Since the touchdown position is [-0.0061, -0.2464] m in BT frame, it is regarded that the position controller by MPC algorithm successfully achieved this purpose under the constraint of maximum thrust force.

Figure 5 shows velocity of the probe in BT frame. The descent velocity at the instance of touchdown is required to be around -0.05 m/s from successful sampling. The result of the touchdown velocity from numerical simulation is -0.0524m/s and satisfies the requirement. The absolute value of the lateral velocity at the touchdown is desirable to be less than 0.04 m/s and from the result of numerical simulation this requirement is also satisfied.

Figure 6 shows LOS of ONC relative to the target during touchdown representing attitude control error. The abrupt changes at around 1500 sec and 2000 sec are supposed to be caused by switching the pointing target from TM to the crater and from the crater to the line which is orthogonal to the asteroid surface.

5. Monte Carlo simulation

Monte Carlo simulation is tried using the same initial condition as the simulation in 4. Random errors are considered in LRF output (relative position to the asteroid surface), LOS of ONC to the target and three degrees of freedom thruster output following error model shown in 3.5 and 3.6. The result from 100 times trial is shown in figures 7-10. Figure 7 shows touchdown points and Figure 8-10 show touchdown velocity in each direction. As shown in figure 7, touchdown points are within the circle with radius of 1m and this accuracy is enough for sampling inside the crater. Lateral touchdown velocities shown in figure 8 and 9 are quite small and vertical touchdown velocity is around -0.05m/s which are also suitable for successful sampling.

The statistical results are shown below. Touchdown point error (BT frame)

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$\mathrm{X}\left(1\mathbf{\sigma} ight)$:	0.2574	(m)
$Y(1\sigma)$:	0.2959	(m)

Touchdown velo	city error	(BT)	frame)	
$\mathrm{X}\left(1\pmb{\sigma} ight)$:	0.0026	(m/s)		
Y(1 σ):	0.0030	(m/s)		
$Z(1\sigma)$:	0.0014	(m/s),	average :	-0.0480 (m)

10			1	1	
8					
6		/			
4	/				
2	-/	1			
VBT 0			•)		
-2				/	



from Monte Carlo simulation



Fig. 8 Touchdown velocity (X in BT frame) from Monte Carlo simulation





from Monte Carlo simulation

3. Concluding remarks

The position controller of Hayabusa2 for touchdown to the asteroid is designed applying Model Predictive Control with constraints such as maximum thrust force and maximum descent velocity. The attitude controller to point the LOS (Line Of Sight) of ONC (Onboard Navigation Camera) to the target such as TM (Target Marker) or newly created crater is designed by quaternion feedback PD (Proportional and Derivative) control.

The performance of these controllers is validated by numerical simulation including Monte Carlo simulation for the probe to safely follow and touch down to the surface of the spinning asteroid satisfying relative position and velocity requirement.

It is assumed that attitude motion of the asteroid is given with sufficient accuracy in this simulation. However the error is expected in relative attitude between estimated BT frame and HP frame. The controller strategy to tackle this problem considering actual measurement constraints could be the next goal of this research.

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