PRECISE ATTITUDE ESTIMATION OF SOLAR SAIL SPACECRAFT UTILIZING COUPLING BETWEEN ATTITUDE AND ORBITAL DYNAMICS

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Abstract

Orbit determination is the process to estimate the position and velocity from a given set of observations of spacecraft. For precise orbit determination, the acceleration of spacecraft by solar radiation pressure (SRP) must be taken into account. In order to incorporate the SRP acceleration, the attitude history and the SRP model -- surface shape and optical surface feature of spacecraft -- need to be known beforehand. Especially, in case of solar sail spacecraft, since their SRP effect is strong, coupling between attitude and orbital dynamics is more remarkable than ordinary ballistic missions. Therefore, by positively utilizing this coupling, it is possible to refine the attitude information based on the residual errors of orbit determination. In this study, we assume that the systematic errors in the orbit determination of the spacecraft are caused solely by the errors in attitude information, and propose an attitude error correction method which utilizes the coupling between attitude and orbital dynamics. In order to examine the validity of the method, the analysis of actual data is conducted. As actual data, flight data of IKAROS -- Japanese interplanetary solar sail demonstration spacecraft -- are used. The observations used in the orbit determination of IKAORS are range and range rate. In this paper, an estimator is designed whose input is the residuals of the orbit determination result -- the difference between actual measurements and modelled observations of range and rage rate -- and whose output is the amount of error correction in the attitude history. It is confirmed that orbit and attitude errors are both decreased through this method. This paper explains the theoretical background of this method and shows the result of analysis using the actual data.

軌道運動と姿勢のカップリングを利用したソーラーセイルの精姿勢推定

摘要

本研究では，宇宙機の軌道決定決に含まれる系統誤差を用いて，宇宙機の姿勢決定誤差を推定することを試みる．軌道決定とは一連の観測データをもとに，あるエポックにおける宇宙機の位置・速度（軌道６要素）を推定することである．精密な軌道決定を行うためには，太陽輻射圧（Solar Radiation Pressure, SRP）による加速度を考慮しなければならない．そのためには，宇宙機の姿勢履歴や太陽輻射圧モデル（光学パラメータ，宇宙機形状）を予め組み立てておく必要がある．とりわけソーラーセイルに関しては，太陽輻射圧が極めて大きいため，軌道運動と姿勢のカップリング効果が他の弾道軌道型宇宙機と比べて顕著に現れる．それゆえ，このカップリング効果を利用することで，軌道決定結果に含まれる残差（系統誤差）から宇宙機の姿勢情報を修正することが可能である．本研究では，この姿勢修正法の理論的背景を説明するとともに，ソーラーセイル IKAROS の軌道上データに適用した結果について報告する．

I. INTRODUCTION

This paper describes a method to correct the attitude estimation error of spacecraft based on the systematic error included in the result of orbit determination. Orbit determination is a process to estimate the position and velocity, i.e. six orbital elements, from a given set of observations of spacecraft. The common measurements used in orbit determination are range and range rate. For precise orbit determination, the acceleration of spacecraft by solar radiation pressure (SRP) must be taken into account. In order to incorporate the SRP acceleration, the attitude history and the SRP model -- surface shape and optical surface feature of spacecraft -- need to be known beforehand. Especially, in case of solar sail spacecraft, since their SRP effect is strong, coupling between attitude and orbital dynamics is more remarkable than ordinary ballistic missions. Therefore, by positively utilizing this coupling, it is possible to refine the attitude information based on the residual errors of orbit determination. This paper describes the theoretical background of proposed attitude correction method and the result of actual implementation for IKAROS, -- Japanese interplanetary solar sail demonstration spacecraft.

Orbit determination of IKAROS is conducted by means of Range and Range Rate. Attitude determination of IKARAOS is done using the Sun and Earth angle. The Sun angle is observed by sun sensor on board, while the Earth angle is derived from the spin modulation of RF signal frequency. Therefore, transponder of IKAROS is offset from the spin axis. As for IKAROS, Doppler signal is an important information which is used for both orbit and attitude determination system.

II. THEORY OF ATTITUDE CORRECTION

II.I Range and Range Rate residual

In IKAROS orbit determination system, two-way Range and Rage Rate measurements are used. Range is the instantaneous distance between spacecraft and the ground station on the earth. It is written as

$$R = \left[(\boldsymbol{r} - \boldsymbol{r}_{GS})^T (\boldsymbol{r} - \boldsymbol{r}_{GS}) \right]^{\frac{1}{2}} \quad [1]$$

where $\boldsymbol{r}_{true}$ and $\boldsymbol{r}_{GS}$ expresses the true position of spacecraft and ground station in inertial frame. In this paper, J2000EQ is chosen as inertial frame.

Range rate is the line of sight velocity of the spacecraft viewed form the ground station, and written as follows;

$$V = \rho^T (\boldsymbol{v} - \boldsymbol{v}_{GS}) \quad [2]$$

where $\boldsymbol{\rho}_{true}$ is a unit vector in direction from the ground station toward spacecraft. $\boldsymbol{v}_{true}$ and $\boldsymbol{v}_{GS}$ expresses the true velocity of spacecraft and ground station.

In orbit determination, $\boldsymbol{r}$ and $\boldsymbol{v}$ are estimated by means of the algorithm such as least squares estimation or Kalman filter using R and V as the observation data. If there are any systematic errors in the dynamic model of estimator, the estimated orbital elements also contain systematic error. The relations between true value and estimated value are as follows;

$$\boldsymbol{r}_{true} = \boldsymbol{r}_{est} + \Delta \boldsymbol{r}, \quad \boldsymbol{v}_{true} = \boldsymbol{v}_{est} + \Delta \boldsymbol{v} \quad [3]$$

where $\Delta \boldsymbol{r}, \Delta \boldsymbol{v}$ means systematic error of position and velocity.

One of the good indicators for orbit estimation error is the “Observation Minus Calculation” (OMC) value of Range and Doppler. These values are written as follows;

$$R_{OMC} = \rho^T \Delta \boldsymbol{r} \quad [4]$$

$$V_{OMC} = \boldsymbol{q}^T \Delta \boldsymbol{r} + \rho^T \Delta \boldsymbol{v} \quad [5]$$

where $\boldsymbol{q}$ is expressed as follows;

$$\boldsymbol{q} = \left[\frac{I_{3\times3}}{\left[(\boldsymbol{r} - \boldsymbol{r}_{GS})^T (\boldsymbol{r} - \boldsymbol{r}_{GS}) \right]^{1/2}} - \frac{(\boldsymbol{r} - \boldsymbol{r}_{GS})(\boldsymbol{r} - \boldsymbol{r}_{GS})^T}{\left[(\boldsymbol{r} - \boldsymbol{r}_{GS})^T (\boldsymbol{r} - \boldsymbol{r}_{GS}) \right]^{3/2}} \right]^T (\boldsymbol{v} - \boldsymbol{v}_{GS}) \quad [6]$$

II.II Differential Equation of Position and Velocity error

The systematic error of orbit determination is originated from the modelling error of spacecraft dynamics. In this paper, we assume that the modeming error attributes solely to the attitude error. In case of solar sailor, spacecraft is highly influenced by SRP. So attitude error causes considerable modelling error of SRP. Let us consider how attitude error drives position and velocity error.

Equation of motion of spacecraft is written as

$$\ddot{\boldsymbol{r}} = -\sum_i \mu_i \frac{\boldsymbol{r} - \boldsymbol{r}_i}{\left| \boldsymbol{r} - \boldsymbol{r}_i \right|^3} + \boldsymbol{f}_{srp}(\boldsymbol{r}, \boldsymbol{p}) \quad [7]$$

where μ_i is the gravitational constant of i th planet, $\boldsymbol{r}_i$ is the position of i th planet, and $\boldsymbol{p}$ is the two dimensional vector indicating spacecraft attitude. Because attitude of spinning spacecraft is defined as the direction of spin axis, we need two parameters to define attitude (e.g. right ascension and declination). $\boldsymbol{f}_{srp}$ is the acceleration by SRP expressed as follows:

$$\boldsymbol{f}_{srp} = -\frac{S_0 R_0^2}{c} \frac{S}{m_s} \frac{1}{r^2} \times \left[\begin{array}{l} \left(2\Gamma \cos\theta_s + \frac{2}{3} D \right) \cos\theta_s (\boldsymbol{n} \times \boldsymbol{s}) \times \boldsymbol{s} + \\ (1 - \Gamma - \mathrm{T}) \cos\theta_s + \left(2\Gamma \cos\theta_s + \frac{2}{3} D \right) \cos^2\theta_s \end{array} \right] \quad [8]$$

where θ_s is the sun angle, $\boldsymbol{n}$ is the unit normal vector of the sail, r is heliocentric spacecraft distance, and $\boldsymbol{s}$ is the unit vector in direction from spacecraft to the Sun. Γ, D, T means optical parameters of sail, i.e. specular reflectivity, diffuse reflectivity, and transmittance each other.

By linearizing Eq.[7], we get following equation;

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x} + \boldsymbol{u}(t) \quad [9]$$

where

$$\boldsymbol{x} = \begin{bmatrix} \Delta \boldsymbol{r} \\ \Delta \boldsymbol{v} \end{bmatrix}, \quad \boldsymbol{A}(t) = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ \boldsymbol{A}_{11} & 0_{3\times3} \end{bmatrix}, \quad \boldsymbol{u}(t) = \begin{bmatrix} 0_{3\times1} \\ \frac{\partial \boldsymbol{f}_{srp}}{\partial \boldsymbol{p}} \Delta \boldsymbol{p}(t) \end{bmatrix}$$

$$\boldsymbol{A}_{11}(t) = -\sum_i \frac{\mu_i}{\left| \boldsymbol{r}_{est} - \boldsymbol{r}_i \right|^3} \left(\boldsymbol{I}_{3\times3} - \frac{(\boldsymbol{r}_{est} - \boldsymbol{r}_i)(\boldsymbol{r}_{est} - \boldsymbol{r}_i)^T}{\left| \boldsymbol{r}_{est} - \boldsymbol{r}_i \right|^2} \right) + \frac{\partial \boldsymbol{f}_{srp}}{\partial \boldsymbol{r}}$$

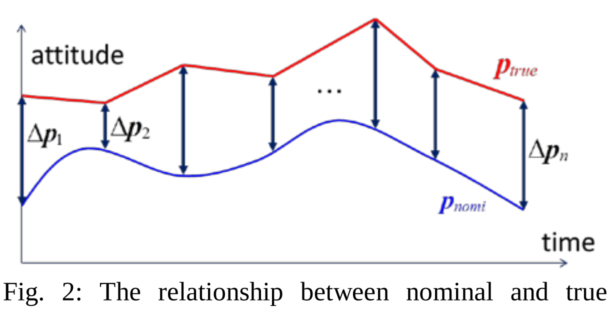

Fig. 2: The relationship between nominal and true attitude

$\Delta\boldsymbol{p}(t)$ is the difference between true attitude and estimated attitude. Eq.[9] shows how the attitude error drives the position and velocity error. Combining this equation with Eq.[4] and Eq.[5], it is possible calculate the Range and Range Rate OMC at arbitrary time.

II.III Solution of Error Equation (Linearization)

Our goal is to determine the attitude error history $\Delta\boldsymbol{p}(t)$ which satisfies Eq.[4] and Eq.[5] for given R_{OMC} and V_{OMC}. In this section, we assume that the true attitude history can be expressed as offset from the given nominal attitude history and linearly interpolated at n attitude nodes as follows;

$$\boldsymbol{p}_{true}(t) = \boldsymbol{p}_{nomi}(t) + a_1(t)\Delta\boldsymbol{p}_1 + \cdots + a_n(t)\Delta\boldsymbol{p}_n \quad [10]$$

Then, $\Delta\boldsymbol{p}(t)$ is written as

$$\Delta\boldsymbol{p}(t) = \boldsymbol{p}_{true}(t) - \boldsymbol{p}_{est}(t) = \boldsymbol{g}(t) + \sum_{i=1}^{n} a_i(t)\Delta\boldsymbol{p}_i \quad [11]$$

where $\boldsymbol{g}(t) = \boldsymbol{p}_{true}(t) - \boldsymbol{p}_{nomi}(t)$.

Eq.[9] can be solved by means of State Transfer Matrix $\Phi(t,0)$ as follows:

$$\boldsymbol{x}(t) = \Phi(t,0)\left[\boldsymbol{x}(0) + \boldsymbol{y}(t)\right]$$
$$\boldsymbol{y}(t) = \int_0^t \Phi(\tau,0)^{-1}\boldsymbol{u}(\tau)d\tau \quad [12]$$

By substituting Eq.[11] into $\boldsymbol{u}(t)$, $\boldsymbol{y}(t)$ can be written as

$$\boldsymbol{y}(t) = \boldsymbol{y}_1(t)\Delta\boldsymbol{p}_1 + \cdots + \boldsymbol{y}_n(t)\Delta\boldsymbol{p}_n + \boldsymbol{y}_g(t) \quad [13]$$

where

$$\boldsymbol{y}_i(t) = \int_0^t \Phi(\tau,0)^{-1}\beta_i(\tau)d\tau, \quad \beta_i(t) = a_i(t)\begin{bmatrix} 0_{3\times2} \\ \dfrac{\partial \boldsymbol{f}_{srp}}{\partial \boldsymbol{p}} \end{bmatrix}$$

$$\boldsymbol{y}_g(t) = \int_0^t \Phi(\tau,0)^{-1}\beta_g(\tau)d\tau, \ \beta_g(t) = \begin{bmatrix} 0_{3\times2} \\ \dfrac{\partial \boldsymbol{f}_{srp}}{\partial \boldsymbol{p}} \end{bmatrix}\boldsymbol{g}(t)$$

$\Phi(t,0)$, $\boldsymbol{y}_i(t)$ can be calculated by solving following ordinary differential equations:

$$\dot{\Phi}(t,0) = \boldsymbol{A}(t)\Phi(t,0) \quad [14]$$
$$\dot{\boldsymbol{y}}_i(t) = \Phi(t,0)^{-1}\beta_i(t)$$
$$\dot{\boldsymbol{y}}_g(t) = \Phi(t,0)^{-1}\beta_g(t)$$

Initial Conditions:

$$\Phi(0,0) = I_{6\times6}, \ \boldsymbol{y}_i(0) = 0_{6\times2}, \ \boldsymbol{y}_i(0) = 0_{6\times1}$$

Therefore, by Eq.[12], $\Delta\boldsymbol{r}$ and $\Delta\boldsymbol{v}$ are expressed as

$$\begin{bmatrix}\Delta\boldsymbol{r}\\ \Delta\boldsymbol{v}\end{bmatrix} = \begin{bmatrix}\varphi_{rr} & \varphi_{rv}\\ \varphi_{vr} & \varphi_{vv}\end{bmatrix}\begin{bmatrix}\Delta\boldsymbol{r}_0\\ \Delta\boldsymbol{v}_0\end{bmatrix} + \begin{bmatrix}\varphi_{rr} & \varphi_{rv}\\ \varphi_{vr} & \varphi_{vv}\end{bmatrix}\sum_{i=1}^{n}\begin{bmatrix}\boldsymbol{y}_{1,i}\\ \boldsymbol{y}_{2,i}\end{bmatrix}\Delta\boldsymbol{p}_i$$
$$+\begin{bmatrix}\varphi_{rr} & \varphi_{rv}\\ \varphi_{vr} & \varphi_{vv}\end{bmatrix}\sum_{i=1}^{n}\begin{bmatrix}\boldsymbol{y}_{1,g}\\ \boldsymbol{y}_{2,g}\end{bmatrix}$$
$$= \begin{bmatrix}\varphi_{rr}\Delta\boldsymbol{r}_0 + \varphi_{rv}\Delta\boldsymbol{v}_0 + \sum C_i\Delta\boldsymbol{p}_i + (\varphi_{rr}y_{1,g} + \varphi_{rv}y_{2,g})\\ \varphi_{vr}\Delta\boldsymbol{r}_0 + \varphi_{vv}\Delta\boldsymbol{v}_0 + \sum D_i\Delta\boldsymbol{p}_i + (\varphi_{rr}y_{1,g} + \varphi_{rv}y_{2,g})\end{bmatrix} \quad [15]$$

where

$$C_i = \varphi_{rr}\boldsymbol{y}_{1,i} + \varphi_{rv}\boldsymbol{y}_{2,i}$$
$$D_i = \varphi_{vr}\boldsymbol{y}_{1,i} + \varphi_{vv}\boldsymbol{y}_{2,i} \quad \boldsymbol{y}_i = \begin{bmatrix}\boldsymbol{y}_{1,i}\\ \boldsymbol{y}_{2,i}\end{bmatrix} \quad [16]$$

Substituting Eq.[15] into Eq.[4] and Eq.[5] yields,

$$R_{OMC} = N_r\Delta r + N_v\Delta v + \sum N_i\Delta p_i + N_c + \varepsilon_R$$
$$V_{OMC} = M_r\Delta r + M_v\Delta v + \sum M_i\Delta p_i + M_c + \varepsilon_V \quad [17]$$

where

$$N_r = \rho_{est}^T\varphi_{rr}, \ N_v = \rho_{est}^T\varphi_{rv}$$
$$M_r = \rho_{est}^T\varphi_{vr} + q_{est}^T\varphi_{rr}, \ M_v = \rho_{est}^T\varphi_{vv} + q_{est}^T\varphi_{rv}$$
$$N_i = \rho_{est}^T C_i, \ M_i = \rho_{est}^T D_i + q_{est}^T C_i$$
$$N_c = \rho_{est}^T(\varphi_{rr}\boldsymbol{y}_{1,g} + \varphi_{rv}\boldsymbol{y}_{2,g})$$
$$M_c = q_{est}^T(\varphi_{rr}\boldsymbol{y}_{1,g} + \varphi_{rv}\boldsymbol{y}_{2,g}) + q_{est}^T(\varphi_{vr}\boldsymbol{y}_{1,g} + \varphi_{vv}\boldsymbol{y}_{2,g}) \quad [18]$$

If the number of measuring point of Range and Range Rate are k and l, the following matrix equations are obtained:

$$Y_R = NX \quad [19]$$
$$Y_V = MX \quad [20]$$

where

$$X = \begin{bmatrix}\Delta\boldsymbol{r}_0\\ \Delta\boldsymbol{v}_0\\ \Delta\boldsymbol{p}_1\\ \vdots\\ \Delta\boldsymbol{p}_n\end{bmatrix}, \ Y_R = \begin{bmatrix}R_{OMC}^{(1)} - N_c^{(1)}\\ \vdots\\ R_{OMC}^{(k)} - N_c^{(k)}\end{bmatrix}, \ Y_V = \begin{bmatrix}V_{OMC}^{(1)} - M_c^{(1)}\\ \vdots\\ V_{OMC}^{(l)} - M_c^{(l)}\end{bmatrix}X$$

$$N = \begin{bmatrix}N_r^{(1)} & N_v^{(1)} & N_1^{(1)} & \cdots & N_n^{(1)}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ N_r^{(k)} & N_v^{(k)} & N_1^{(k)} & \cdots & N_n^{(k)}\end{bmatrix}$$

$$M = \begin{bmatrix}M_r^{(1)} & M_v^{(1)} & M_1^{(1)} & \cdots & M_n^{(1)}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ M_r^{(l)} & M_v^{(l)} & M_1^{(l)} & \cdots & M_n^{(l)}\end{bmatrix}$$

We want to solve Eq.[21] and Eq.[22] about X. The dimension of X is $2n+6$, while the number of equation is $k+l$. If dimension number of X is less than the number of equation, it is possible to solve X in least square fashion as follows;

$$\text{minimize}: \frac{1}{2} X^T W X \qquad \text{subject to}: Y = AX$$

where W^{-1} is an inverse of covariance matrix whose diagonal components are consist of the covariance of $\Delta \boldsymbol{r}_0, \Delta \boldsymbol{v}_0, \Delta \boldsymbol{p}_i$. A and Y can take three values as

$$\begin{aligned} &\text{case1}: (A, Y) = (M, Y_V) \\ &\text{case2}: (A, Y) = (N, Y_R) \\ &\text{case3}: (A, Y) = (\begin{bmatrix} N^T & M^T \end{bmatrix}^T, \begin{bmatrix} Y_R{}^T & Y_V{}^T \end{bmatrix}^T) \end{aligned}$$

Case1 means to solve X using only Range Rate OMC equations, Case2 uses Range OMC only, and Case3 uses both Range and Range Rare OMC equations.

In order to deal with the constraint equation, we introduce the Lagrange multiplier λ. Then, the extended cost function is written as

$$J = \frac{1}{2} X^T W X + \lambda (Y - AX) \qquad [21]$$

The least squares solution is obtained as follows:

$$X = (W^{-1}) A^T (A W^{-1} A^T)^{-1} Y \qquad [22]$$

III. ATTITUDE ERROR EVALUATION

In the previous section, we derived the equation to correct the attitude history from the view of reducing Range and Doppler residual. This method is one way to evaluate the attitude determination error. There is another way to evaluate attitude error; it is to conduct error analysis based on the algorithm of spin axis determination based.

In the process of solving least squares problem, the information about the error covariance of right ascension α and declination δ are needed. Since the attitude of IKAROS is determined based on Sun angle β_s and Earth angel β_e, we need to transform the error covariance of β_s and β_e to that of α and δ. The relations between these angles are written as

$$\begin{aligned} &\cos \beta_s = \boldsymbol{s}^T \boldsymbol{n}, \ \cos \beta_e = \boldsymbol{e}^T \boldsymbol{n} \\ &\boldsymbol{n} = \begin{bmatrix} \cos\alpha\cos\delta & \sin\alpha\cos\delta & \sin\delta \end{bmatrix}^T \end{aligned} \qquad [23]$$

where $\boldsymbol{s}$ is a unit vector in direction from IKAROS to the Sun, $\boldsymbol{e}$ is a unit vector in direction from IKAROS to the Earth, and $\boldsymbol{n}$ is the unit nominal vector of sail, i.e. unit vector in direction to the spin axis of IKAROS. The coordinate is the J2000EQ spacecraft centered inertia.

By differentiating the Eq.[25], following equation is obtained:

$$\begin{bmatrix} -\sin\beta_s & 0 \\ 0 & -\sin\beta_e \end{bmatrix} \begin{bmatrix} \Delta\theta_s \\ \Delta\theta_e \end{bmatrix} = \begin{bmatrix} \boldsymbol{s}^T \\ \boldsymbol{e}^T \end{bmatrix} \frac{\partial \boldsymbol{n}}{\partial \boldsymbol{p}} \Delta \boldsymbol{p}, \ \boldsymbol{p} = \begin{bmatrix} \alpha \\ \delta \end{bmatrix} \qquad [24]$$

Therefore, the transition matrix between $\alpha, \delta, \beta_s, \beta_e$ is written as follows:

$$\begin{aligned} \Delta \boldsymbol{p} &= H \begin{bmatrix} \Delta\theta_s \\ \Delta\theta_e \end{bmatrix} \\ H &= \left(\begin{bmatrix} \boldsymbol{s}^T \\ \boldsymbol{e}^T \end{bmatrix} \frac{\partial \boldsymbol{n}}{\partial \boldsymbol{p}} \right)^{-1} \begin{bmatrix} -\sin\beta_s & 0 \\ 0 & -\sin\beta_e \end{bmatrix} \end{aligned} \qquad [25]$$

Therefore, the covariance matrix of right ascension and declination is obtained as:

$$\Sigma_p = H \Sigma_\theta H^T \qquad [26]$$

where U_s and U_e are the standard deviation of Sun angle and Earth angle. U_s is directory obtained from a specification of the Sun sensor output. Because Earth angle determination is based on the spin modulation of RF signal received at ground station, U_e is derived from the equation about spin modulation and Earth angle. The basic equation is written as [1]

$$\beta_e = \sin^{-1}\left(\frac{A_s}{\gamma \rho r_{ANT} \Omega} \right), \ \rho = \frac{\sqrt{2 - 2\cos(\Omega \Delta T)}}{\Omega \Delta T} \qquad [27]$$

where A_s, γ, r_{ANT}, Ω, ΔT are sinusoidal coefficients of the spin modulation, coherent mode ($\gamma = 2$), an offset distance of antenna from the spin axis, the spin rate of the spacecraft, and the integration time of Doppler measurement.

We assume that Ω, ΔT, and γ are exactly known and that r_{ANT}, and A_s have some error which causes U_e. By taking the derivative of Eq.[30], U_e is derived as follows;

$$\begin{aligned} U_e = \delta\beta_e &= \frac{\partial \beta_e}{\partial A_s} \delta A_s + \frac{\partial \beta_e}{\partial r_{ANT}} \delta r_{ANT} \\ &= \frac{\delta A_s}{\gamma \rho r_{ANT} \Omega \cos\beta_e} - \tan\beta_e \frac{\delta r_{ANT}}{r_{ANT}} \end{aligned} \qquad [28]$$

IV. RESULTS OF IKAROS ATTITUDE CORRECTON

In this section, we use the flight data of IKAROS in order to confirm the validity of the attitude error estimation method above mentioned. The flight data used in this analysis is 2-way range and Range Rate data. The data of six different coasting periods were used. The detail about each coasting period is referred to the table. 1

Fig. 4 to Fig. 10 show example of attitude correction and the associating improvement of orbit determination residual during Period C. Note that in calculation of state transition matrix in Eq.[14], the reference orbit and attitude are needed. As for the reference attitude $\boldsymbol{\rho}_{est}$, the results of real time attitude determination system of IKAROS are used, and as for $\boldsymbol{\rho}_{nomi}$, the fourth order polynomial approximation of $\boldsymbol{\rho}_{est}$ is selected. Reference orbit is the result of orbit determination obtained by using the above reference attitude.

Name	Period	Pass Number
A	07/17 – 07/22, 2010	5
B	07/17 – 07/31, 2010	13
C	09/28 – 10/03, 2010	6
D	10/19 – 10/27, 2010	6
E	06/21 – 06/24, 2011	4
F	07/29 – 08/06, 2011	4

Table. 1: Period and Pass Number of each coasting

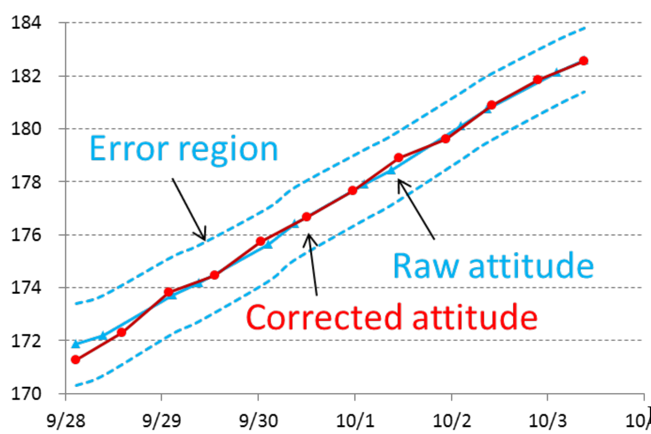

Fig. 4: History of Right Ascension [deg]

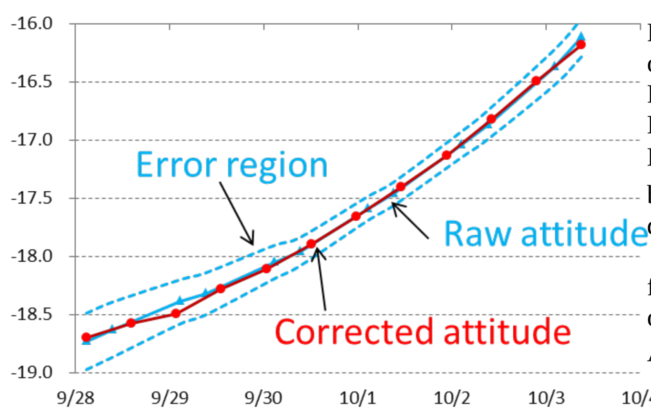

Fig. 5: History of Right Ascension [deg]

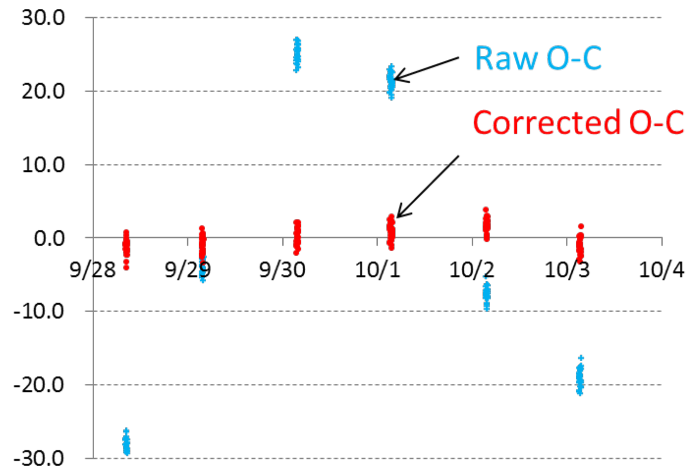

Fig. 6: Plots of Range O-C

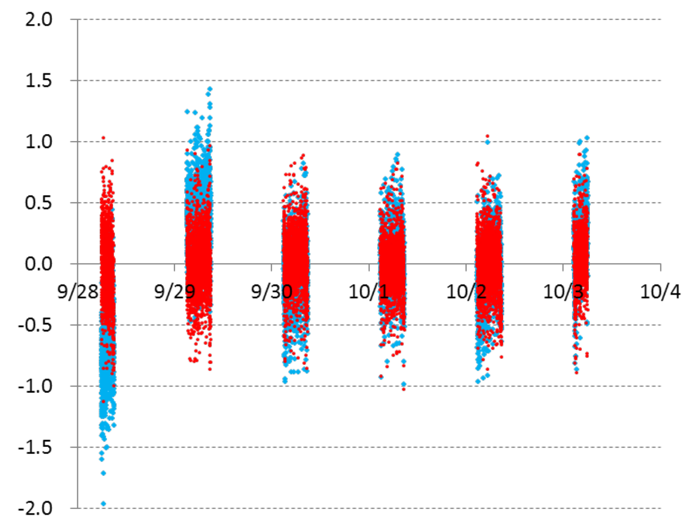

Fig. 7: Plots of Doppler O-C

Fig.4 and Fig.5 indicates corrected attitude history obtained in a trial of removing both Range and Range Rate residuals (Case3). Attitude error obtained from the Eq.[27] is shown in the figures as an 1-sigma error bar. Note that attitude correction $\Delta\boldsymbol{\rho}$ lies within the error bar. From Fig.6 and Fig.7 it can be seen that orbit determination residuals are reduced successfully.

The results of other periods are shown in the following Tables, where Root Meaning Square (RMS) of Residuals is shown for an indicator of error reduction. As for Period A and Period B, $\boldsymbol{\rho}_{nomi}$ is selected as $\rho_{nomi} = \rho_{est}$, while the other period , $\boldsymbol{\rho}_{nomi}$ is the fourth order polynomial approximation of $\boldsymbol{\rho}_{est}$.

A	Range residual RMS [km]	Range Rate residual RMS [km/s]
Raw	1.2e-2	6.1e-7
Case1	9.1e-3	3.3e-7

Table. 2: Residual of period A

B	Range residual RMS [km]	Range Rate residual RMS [km/s]
Raw	2.3e-2	5.3e-7
Case1	1.3e-2	3.9e-7

Table. 3: Residual of period B

C	Range residual RMS [km]	Range Rate residual RMS [km/s]
Raw	2.0e-2	5.6e-7
Case1	9.2e-3	4.1e-7
Case3	1.5e-3	4.1e-7

Table. 4: Residual of period C

D	Range residual RMS [km]	Range Rate residual RMS [km/s]
Raw	9.8e-3	5.1e-7
Case1	4.8e-3	3.7e-7
Case2	6.5e-3	4.1e-7
Case3	1.1e-3	3.7e-7

Table. 5: Residual of period D

E	Range residual RMS [km]	Range Rate residual RMS [km/s]
Raw	4.3e-3	1.3e-6
Case3	4.6e-3	8.0e-7

Table. 6: Residual of period E

F	Range residual RMS [km]	Range Rate residual RMS [km/s]
Raw	7.1e-1	6.3e-7
Case1	6.7e-1	4.7e-7
Case2	8.9e-1	1.3e-6
Case3	3.0e-1	7.6e-6

Table. 7: Residual of period F

As for Period A, B, C, D and E, both Range and Doppler residuals are removed whatever method we use. In particular, the most effective correction seems to be Case3. However, with respect to Period F, the performance of residual remover is not so good as other Periods. This may be because the modelling error of SRP gets bigger. Actually, after June, 2010, the spin rate of IKAROS got smaller than before, and the centrifugal force to keep membrane extended also got smaller. Therefore, it is highly possible that shape of membrane changed due to SRP. It might be one way to construct the SRP model which depends on the spin rate in order to correct the residuals in orbit determination of current coasting period.

V. CONCLUSION

In this paper, we described the method to correct the attitude of spacecraft which is highly affected by solar radiation pressure utilizing the systematic error contained in the result of orbit determination. This method can be a method to examine whether attitude determination is appropriate or not from a view of orbit determination. By applying proposed method to the flight data of IKAROS, we confirmed that the Range and Range Rate residuals are removed by correcting the attitude history. It was shown that the estimated attitude error by this method is within the attitude uncertainty derived from the error analysis of Earth angle determination system based on the spin modulation of RF signal.

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