TM-Diagram and its Application to Translunar Phasing Orbit

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Abstract

On the lunar transfer, there is a sequence named “phasing orbit” which is different from the widely used direct transfer sequence. In this method, the spacecraft is not directly injected into the translunar orbit, but stays on a long elliptical phasing orbit for revolutions. A major merit of this method is that, by using the phasing orbit as a buffer, the translunar orbit can be fixed for acceptable width of launch window. Discussed in the paper is a design method of the phasing orbit using a chart developed by the author named "TM-diagram". It mainly targets on solving "phase shifting problem" in practical aspect using graphic method.

TM ダイアグラムとその月遷移フェージング軌道への応用

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摘要

月遷移軌道の一方式として、打上後、地球を廻る長楕円軌道（フェージング軌道と呼ばれる）を数周回した後に月に到る方法がある。フェージング軌道を用いる主たる理由は、充分な打上ウィンドウを確保しながら、月接近条件を固定できる点にある。本発表では、実用的なフェージング軌道設計における「位相調整問題」を検討するために考案した「TM ダイアグラム」を紹介し、その使用例を示す。

1. Introduction

With the opening of the new century, the moon is attracting attention again as a target of the space exploration. SMART-1 launched by ESA is the lead-off visitor to the moon of the new century. Japan launched KAGUYA in 2007, which was followed by Chinese CHANG’E-1 and Indian CHANDRAYAAN-1. U.S. launched LRO in 2009 to reconnaissance the moon for our re-visit, and as the latest news, China successfully launched their second lunar orbiter CHANG’E-2 in October, 2010.

To focus on lunar transfer sequence, most of lunar explorers were injected directly into a translunar orbit. In this case, it takes only five days from the launch to the moon arrival, which leads to a simple straightforward lunar transfer sequence.

On the other hand, there is another lunar transfer sequence named “phasing orbit” which was developed in 1980’s (Figure 1). In this method, a spacecraft is not directly injected into a translunar orbit, but stays on a long elliptical phasing orbit for a number of revolutions. A major merit of this method is that, by using the phasing orbit as a buffer to adjust the duration from the launch to the translunar orbit injection, the translunar orbit can be fixed for acceptable width of launch window. This merit of phasing orbit was firstly made use of by the missions with complicated lunar swingby sequences, such as HITEN, GEOTAIL of ISAS in 1990s[1), 2)]. The unique first lunar encounter condition as the start of its complicated swingby sequence requires the adoption of this method.

The merit of applying the phasing orbit to the lunar mission (the sequence terminates at the moon) was pointed out by the author[3)]. Even in the simple transfer to the moon, the property of the orbits change by the launch date. And there is a merit of selecting “the best” translunar orbit irrespective of the launch date. Additionally, the opportunities of the perigee passages (which are not given in the case of direct transfer) can be utilized for the efficient correction of the launch injection error. The opportunities of perigee passages also enable to treat the launch energy as one of the design parameter (which is fixed in the case of direct transfer). These merits are quantitatively evaluated in the paper using the case of KAGUYA as an example.

A typical outline of the phasing orbit design is described as follows (Fig. 1). Initially, a unique translunar orbit is determined based on the mission requirements. Then, a series of launch dates are set approximately a few months prior to the translunar orbit injection. The launch trajectories and the post-launch initial orbits are designed for each launch date so that the perigees of injection orbits coincide with that of the translunar orbit. The phasing orbit injection maneuver at the first perigee passage is set to adjust the period of the phasing orbit to pass the perigee at the time of translunar orbit injection after a number of revolutions. Finally, translunar orbit injection maneuver is set to inject the spacecraft into the translunar orbit.

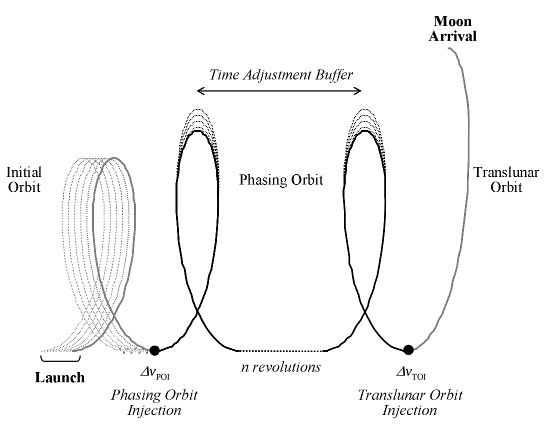

Fig. 1. Schematics of Phasing Orbit.

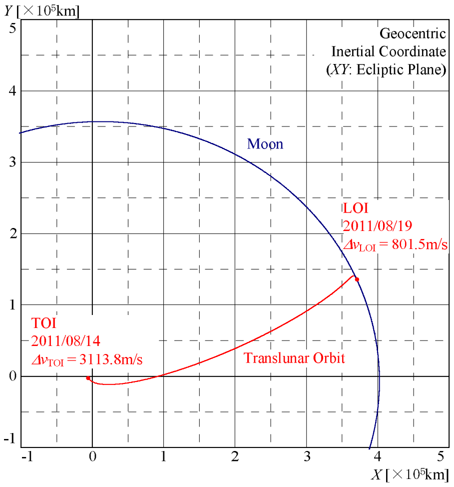

Fig. 2. Nominal Translunar Orbit.

Discussed in the paper is the design method of the phasing orbit under the practical conditions. In the following Section 2, the design process of phasing orbit is overviewed using a concrete design example. Section 3 introduces a chart developed by the author named “TM diagram” with its usage in the phasing orbit design process. Finally, the paper is concluded in Section 4.

2. Phasing Orbit Design

2.1 Definition of Design Problem

Introduced in this section is the definition of the phasing orbit design problem used in this paper. A concrete example is described as well, which is used in the following sections.

Firstly defined is a unique “translunar orbit”, which composes the last part of the sequence. Assumed in this paper is a hypothetical mission to a lunar polar orbit. As discussed by the author[4], in spatial lunar transfer problem, the velocity increment (Δv) required for the lunar orbit injection (LOI) varies by the date. Accordingly, the translunar orbit which provides the least LOI Δv in the month is selected as the unique “nominal translunar orbit”. Assuming the moon arrival in August, 2011, and the eastward launch from the Japanese launch site (at the latitude 30.4deg.N), a concrete example of the nominal translunar orbit is constructed as is shown in Fig. 2.

The launch window and the launch condition are defined in the following way. The launch window is assumed to be set two months before the translunar orbit injection (TOI). Consecutive 10 days starting from June 15 are set as the launch window. For all the launch dates, the position at the initial orbit injection (IOI) in inertial frame is set to coincide with that at TOI. The velocity direction at IOI is set to be horizontal, which is the same with that at TOI. Consequently, IOI is the perigee of the initial orbit. To meet with these conditions, the launch sequence of the date k days prior to TOI is set by shifting the sequence used to construct the nominal translunar orbit by exactly k “sidereal” days. Δv at IOI from the parking orbit ($\Delta v_{\rm IOI}$) is assumed to be 3100m/s in tangential direction, which results in the orbital period of the initial orbit ($P_{\rm IO}$) to be approximately 8.70day.

2.2 Sequence design under two body model

Given the boundary condition of the phasing orbit in the previous section, a simple method of the phasing orbit design is introduced in this section. This method uses two assumptions to simplify the problem.

Firstly, the method assumes Keplerian motion of the spacecraft during the phasing loops. That is to say, the spacecraft comes back exactly to the same position after orbits. As defined in the previous section, the position at IOI is set to coincide with that at TOI. Hence, as far as the orbit maneuvers are performed only at the position at IOI (and TOI), the connection of the orbits is guaranteed from the initial orbit to the translunar orbit. As a result, with a small constraint on the location of the maneuvers, we are freed from the orbit connection problem, and can concentrate on the phasing problem.

The second assumption is related to the orbit maneuvers. The position of the maneuvers, which is the position of IOI and TOI as well, is assumed to be the perigee of the orbits, and the direction of the maneuvers is assumed to be tangential to the orbits. Apparently, this assumption supposes that the orbit maneuver is performed in the way that the orbital period is changed most efficiently.

Under this simplified model, a phasing orbit sequence can be constructed in the following manner. Two numbers are required to be specified in advance, the number of revolution on the initial orbit ($N_{\rm IO}$) and that on the phasing orbit ($N_{\rm PO}$). Once these numbers are specified, the time at the phasing orbit injection ($T_{\rm POI}$) and the orbit period of the phasing orbit ($P_{\rm PO}$) are calculated as

$$T_{\rm POI} = T_{\rm IOI} + N_{\rm IO} P_{\rm IO} \qquad (1)$$

$$P_{\rm PO} = \left(T_{\rm TOI} - T_{\rm POI}\right) / N_{\rm PO} \qquad (2)$$

where $T_{\rm IOI}$ and $T_{\rm TOI}$ are the time at IOI and TOI respectively, which are defined in the launch sequence. By use of $P_{\rm PO}$ and the assumption of the fixed perigee, the velocity at the perigee of the phasing orbit ($v_{peri\ \rm PO}$) is calculated. By comparing $v_{peri\ \rm PO}$ with the velocity at the perigee of the initial orbit and that of the translunar orbit, Δv required at POI ($\Delta v_{\rm POI}$) and at TOI ($\Delta v_{\rm TOI}$) are obtained respectively.

By use of the procedure above, phasing orbit sequences are constructed for all the launch dates in the launch window. The result is shown in Table 1. Ten cases represent the sequences for each launch date. $N_{\rm IO}$ is assumed to be one, which results in $T_{\rm POI}$ 8.70day after $T_{\rm IOI}$. Since the unique nominal translunar orbit is used for all cases, $T_{\rm TOI}$ is the same. Two values of $N_{\rm PO}$, four and five, are assumed and used in Δv calculation. The sum of $\Delta v_{\rm POI}$ and $\Delta v_{\rm TOI}$ ($\Delta v_{\rm total}$) of the two results are compared, and the inferior (the larger) one is shadowed for each cases. For the cases 1 to 8, $N_{\rm PO}$ of four gives superior results (i.e. smaller $\Delta v_{\rm total}$), while $N_{\rm PO}$ of five does for the cases 9 to 10. That is to say, the optimum

Table 1. Phasing Orbit Sequences Designed under Simplified Model.

Case	Time (UTC)			Δv [m/s] ($N_{PO}=4$)			Δv [m/s] ($N_{PO}=5$)		
	T_{IOI}	T_{POI}	T_{TOI}	Δv_{POI}	Δv_{TOI}	Δv_{total}	Δv_{POI}	Δv_{TOI}	Δv_{total}
1	6/15 05:14:24	6/23 21:58:23	8/14 01:18:29	23.3	9.5	32.8	10.5	3.2	13.8
2	6/16 05:10:28	6/24 21:54:27	8/14 01:18:29	22.3	8.5	30.7	9.3	4.4	13.8
3	6/17 05:06:32	6/25 21:50:31	8/14 01:18:29	21.2	7.4	28.6	8.1	5.7	13.8
4	6/18 05:02:36	6/26 21:46:35	8/14 01:18:29	20.0	6.3	26.3	6.8	7.0	13.8
5	6/19 04:58:40	6/27 21:42:39	8/14 01:18:29	18.9	5.1	24.0	5.4	8.4	13.8
6	6/20 04:54:44	6/28 21:38:43	8/14 01:18:29	17.7	3.9	21.6	4.0	9.8	13.8
7	6/21 04:50:48	6/29 21:34:47	8/14 01:18:29	16.4	2.7	19.1	2.6	11.2	13.8
8	6/22 04:46:53	6/30 21:30:51	8/14 01:18:29	15.1	1.4	16.5	1.0	12.7	13.8
9	6/23 04:42:57	7/01 21:26:55	8/14 01:18:29	13.8	0.0	13.8	0.5	14.3	14.8
10	6/24 04:39:01	7/02 21:22:59	8/14 01:18:29	12.4	1.4	13.8	2.1	15.9	18.1

N_{PO} depends on the launch date. Note that Δv_{total} of 13.8m/s obtained for all the cases is the minimum required Δv to transfer from the initial orbit to the trans-lunar orbit (Δv_{min}). Δv_{min} is defined as the difference between v_{peri} of the initial orbit ($v_{peri\ IO}$) and the translunar orbit ($v_{peri\ TO}$).

2.3 Discussion on the Design Method

The method introduced in the previous section is simple and straightforward. Moreover, it provides intrinsic information, such as Δv_{total}, of the phasing orbit under the given condition.

However, it must be pointed out that the result in the previous section shows only one of the possible sequences. As is mentioned, there are two design parameters, N_{IO} and N_{PO}, whose other combination yields different sequence. Besides, orbit maneuvers can be added at the other perigee passages, which change the framework of the sequence. And in these other sequences, there are ones whose Δv_{total} is equal to Δv_{min}.

If the only interest is an example of the sequence, the result in the previous section is sufficient. However, if you are to know the whole set of the possible sequences, or if you are to know the structure of the design solution space, another approach is necessary.

3. TM-Diagram for Phasing Orbit Design

3.1 TM-Diagram

In this section, a chart is introduced to discuss phasing orbit design. Though it is a simple chart plotting the time profile (t) of mean anomaly (M), it has some advantages in the analysis of phasing orbit sequence. Hence, the chart is given a specific name, "TM-diagram", which is used hereafter.

Fig. 3 shows an example of TM-diagram. The horizontal axis represents the relative time (t) whose origin is set at T_{TOI}. The vertical axis, ΔM, represents the deviation of the spacecraft's mean anomaly (M) from that of the reference orbit. The orbit osculating with the translunar orbit at TOI is used as the reference orbit. Consequently, ΔM at t is defined as

$$\Delta M(t) = M(t) - M_{TO}(t) \tag{3}$$

where M_{TO} is the mean anomaly of the reference orbit.

In this chart, a spacecraft's position on the orbit at a moment is expressed as a point. There are some points on the chart to be noted individually. First, the origin of the chart (i.e., the point $(0,0)$) signifies the state that the spacecraft is at the perigee (i.e., $M = M_{TO} = 0$) at T_{TOI}. This state coincides with the boundary condition to connect with the translunar orbit at TOI. Second, the orange lines in the chart represent the contours of M whose values are the multiples of 2π, which means the perigee of the orbit. The event which occurs at perigee must be placed some-where on these orange lines.

A spacecraft's motion on the orbit is expressed as a line in the chart. The green line in Fig. 3 is an example, which represents the profile of the case 1 ($N_{PO}=5$) in Table 1. The profile starts from IOI whose relative time t_{IOI} is

$$t_{IOI} = T_{IOI} - T_{TOI} \tag{4}$$

It is approximately 60 days before TOI. M_{IOI} is determined from two design parameters N_{IO} and N_{PO} by way of

$$M_{IOI} = -2\pi \times (N_{IO} + N_{PO}) \tag{5}$$

In case of this example, $N_{IO}=1$ and $N_{PO}=5$ which result in M_{IOI} of -12π. The next event is POI whose relative time t_{POI} and mean anomaly M_{POI} is calculated by

$$t_{POI} = T_{POI} - T_{TOI} \tag{6}$$

$$M_{POI} = -2\pi \times N_{PO} \tag{7}$$

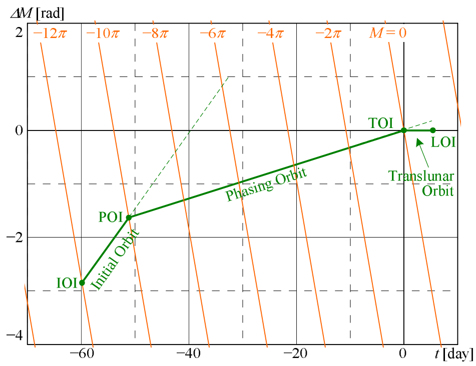

Fig. 3. TM-Diagram.

The line connecting IOI and POI represents the profile of ΔM during the initial orbit.

In general, the slope of the line means the time derivative of ΔM. If we assume Keplerian motion of the spacecraft, the time derivative of M is constant, which is known as mean motion (n). Assuming the same for the translunar orbit, the slope of the line (k) is expressed as

$$k = \frac{d(\Delta M)}{dt} = \frac{dM}{dt} - \frac{dM_{\mathrm{TO}}}{dt} = n - n_{\mathrm{TO}} \tag{8}$$

where n_{TO} is mean motion of the reference orbit. Eq. (8) means that k signifies the deviation of n from n_{TO}, and it is constant. In summary, assuming Keplerian motion of the spacecraft, the profile is expressed as a straight line in the chart whose slope is

$$k = n - n_{\mathrm{TO}} = \Delta n \tag{9}$$

Back to the green line in Fig. 3, and the next event is TOI. As is mentioned, TOI is placed at the origin so as to connect with the translunar orbit. The straight line connecting POI and TOI represents the profile of ΔM during the phasing orbit. The slope of the line (k_{PO}) signifies Δn of the phasing orbit (Δn_{PO}), from which n of the phasing orbit (n_{PO}) and P_{PO} is easily derived.

After TOI, the spacecraft is injected in to the translunar orbit. In this chart, Keplerian motion of the spacecraft is assumed during the translunar orbit, and ΔM is constantly zero from its definition. The profile end with LOI whose relative time t_{LOI} is

$$t_{\mathrm{LOI}} = \frac{1}{2} P_{\mathrm{TO}} \tag{10}$$

where P_{TO} is the period of the translunar orbit. Eq. (10) implies that LOI is assumed to be the apogee of the translunar orbit, and the mean anomaly at LOI (M_{LOI}) is π.

The slope of the line changes at POI and TOI. The change of slope means the change of Δn (equivalent to the change of n), which is caused by an orbit maneuver. is changed most efficiently by the orbit maneuver at the perigee of the orbit in the tangential direction. Assuming that deviation of the orbits from the reference orbit are small, the relation between the change of slope (Δk) and Δv at the perigee (Δv_{peri}) is expressed as

$$\Delta v_{peri} = -\frac{a_{\mathrm{TO}}}{3}\sqrt{\frac{1-e_{\mathrm{TO}}}{1+e_{\mathrm{TO}}}}\Delta k \tag{11}$$

where a_{TO} and e_{TO} are semi-major axis and eccentricity of the reference orbit respectively.

In Fig. 3, the slope of the line changes at POI and TOI. Both points are on the orange line, which means that both events occur at the perigee of the orbit. Hence, we can apply Eq. (11) to relate Δv_{POI} and Δv_{TOI} with the change of slope at POI (Δk_{POI}) and TOI (Δk_{TOI}) which yields

$$\Delta v_{\mathrm{POI}} = -\frac{a_{\mathrm{TO}}}{3}\sqrt{\frac{1-e_{\mathrm{TO}}}{1+e_{\mathrm{TO}}}}\Delta k_{\mathrm{POI}} \tag{12}$$

$$\Delta v_{\mathrm{TOI}} = -\frac{a_{\mathrm{TO}}}{3}\sqrt{\frac{1-e_{\mathrm{TO}}}{1+e_{\mathrm{TO}}}}\Delta k_{\mathrm{TOI}} \tag{13}$$

The total Δv required through the profile is

$$\begin{aligned}\Delta v_{\mathrm{total}} &= |\Delta v_{\mathrm{POI}}| + |\Delta v_{\mathrm{TOI}}| \\ &= \frac{a_{\mathrm{TO}}}{3}\sqrt{\frac{1-e_{\mathrm{TO}}}{1+e_{\mathrm{TO}}}}\left(|\Delta k_{\mathrm{POI}}| + |\Delta k_{\mathrm{TOI}}|\right)\end{aligned} \tag{14}$$

Paying attention to their sign, $|\Delta k_{\mathrm{POI}}|$ and $|\Delta k_{\mathrm{TOI}}|$ are expanded into

$$|\Delta k_{\mathrm{POI}}| = k_{\mathrm{IO}} - k_{\mathrm{PO}} \tag{15}$$

$$|\Delta k_{\mathrm{TOI}}| = k_{\mathrm{PO}} - k_{\mathrm{TO}} \tag{16}$$

where k_{IO}, k_{PO} and k_{TO} are the slope of the lines of the initial orbit, the phasing orbit and the translunar orbit respectively. Substitution of (15) and (16) into (14) yields

$$\Delta v_{\mathrm{total}} = \frac{a_{\mathrm{TO}}}{3}\sqrt{\frac{1-e_{\mathrm{TO}}}{1+e_{\mathrm{TO}}}}\left(k_{\mathrm{IO}} - k_{\mathrm{TO}}\right) \tag{17}$$

The right hand side of Eq. (17) is equal to Δv supposing the direct transfer from the initial orbit to the translunar orbit. This Δv is once defined as Δv_{min} in Section 2.2. That is to say, $\Delta v_{\mathrm{total}}$ of the profile in Fig. 3 is equal to Δv_{min}, which result coincides with the data of the case 1 ($N_{\mathrm{PO}} = 5$) in Table 1.

Generally speaking, if the sign of Δk_{POI} and Δk_{TOI} are the same, $|\Delta k_{\mathrm{POI}}| + |\Delta k_{\mathrm{TOI}}|$ in (14) is reduced to $|k_{\mathrm{IO}} - k_{\mathrm{TO}}|$, which results in $\Delta v_{\mathrm{total}}$ being equal to Δv_{min}. Based on this fact, you can evaluate from TM-diagram if a given phasing orbit sequence can be achieved with $\Delta v_{\mathrm{total}}$ equal to Δv_{min}. First, draw the line of the sequence in TM-diagram. The only parameters to be given are T_{IOI}, P_{IO}, N_{IO}, N_{PO} and T_{TOI}. Then, check the changes of slope at POI and TOI. If the directions of change are the same, the sequence can be achieved with $\Delta v_{\mathrm{total}}$ equal to Δv_{min}. The phasing orbit sequence with this property is given a specific name, "minimum Δv phasing orbit", which is used hereafter. Let me show an example.

Fig. 4 is a TM-diagram on which the lines of the sequence listed in Table 1 are drawn. Note that the lines can be drawn without the knowledge of Δv_{POI} and Δv_{TOI}. The blue lines represent the sequences with $N_{\mathrm{PO}} = 4$, whereas the green lines represent those with $N_{\mathrm{PO}} = 5$. The case number is noted in the left side of each line. For each line, the change of slope at POI and TOI is checked. If the directions of change are the same (in this case, the slope of the line decrease at both points), the sequence is evaluated as a "minimum Δv phasing orbit" and expressed in a solid line. On the other hand, if the directions of change are different, $\Delta v_{\mathrm{total}}$ of the sequence is evaluated to be larger than Δv_{min} and drawn in a broken line. You can see that the minimum Δv phasing orbits found in the TM-diagram (the case 9 and 10 of $N_{\mathrm{PO}} = 4$, and the case 1 to 8 of $N_{\mathrm{PO}} = 5$) coincide with those identified in Table 1 based on Δv cal-

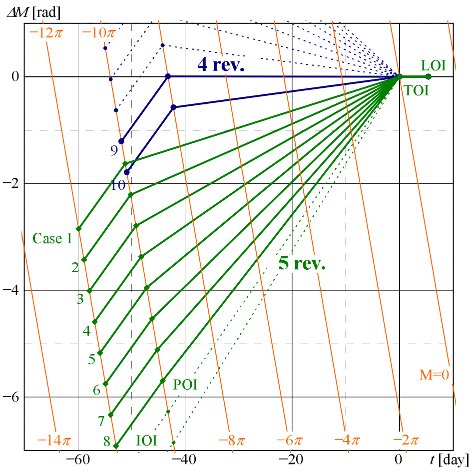

Fig. 4. TM-Diagram of Various Sequences.

culation.

3.2 Design space on TM-Diagram

For a given combination of N_{IO} and N_{PO}, there are perigee passages during the sequence (except IOI). Assuming Keplerian motion of the spacecraft, an orbit maneuver at any perigee in the tangential direction changes P (or n) efficiently in the same way. In the previous section, the method is introduced to evaluate "minimum Δv phasing orbit" on TM-diagram in case of two orbit maneuvers (at POI and TOI). The method is naturally extended to the case where the number of orbit maneuvers is larger than two.

When you draw a phasing orbit sequence as a broken line on TM-diagram, check the changes of slope at all vertices. If the directions of change are all the same, the sequence is a "minimum Δv phasing orbit". In other words, irrespective of the number of orbit maneuvers, if the slope of the line monotonically decreases (or increases), the sequence represented by the line is a "minimum Δv phasing orbit". Note that this rule holds under the condition that the orbit maneuvers are performed at the perigee of the orbit. It means that the vertices of the orbit maneuvers are placed on the orange lines in TM-diagram.

Based on the rule above, we can define the design space of minimum Δv phasing orbit on TM-diagram. Fig. 5 shows an example, where the settings of T_{IOI}, P_{IO}, N_{IO} and N_{PO} are the same with those of the case 1 in Table 1. Under the given presumption, the line connecting IOI and the next perigee passage (V_1) as well as the line after TOI (to LOI) are determined. The sequence left to be designed is that between V_1 and TOI.

From the rule in the previous paragraph, the slope of the line should monotonically decrease (note that k_{IO} is larger than k_{TO}) so as the sequence to be a "minimum Δv phasing orbit". If a line of the sequence between V_1 and TOI lies in the part below the line straightly connecting the two points, the line necessarily contains at least one vertex at which the slope of the line increases. Similarly, if a line of the sequence between V_1 and TOI

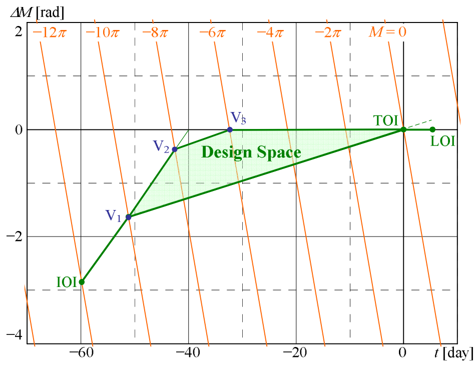

Fig. 5. Design Space of Minimum Δv Phasing Orbit.

lies in the part above the line connecting V_1, V_2, V_3 and TOI, the line necessarily contains at least one vertex at which the slope of the line increases (note that the vertices must be on the orange line so as to be the orbit maneuvers at the perigee, which is the reason that the slope of the line changes at V_2 and V_3). Consequently, for the sequence represented by the line to be a minimum phasing orbit, the line between V_1 and TOI needs to lie in the region painted green in the figure. Note that this is just a necessary condition. Even if the line lies in the region, if it has a zigzag shape, it is no longer a minimum Δv phasing orbit. However, bearing the caution in mind, the graphical expression of the design space of minimum Δv phasing orbit on TM-diagram is easy to understand and useful in practical phasing orbit design.

Prior to show an example of its usage, I want to introduce practical situation where the design space of minimum Δv phasing orbit is investigated. As is mentioned, lines of the sequence in the design space are all minimum Δv phasing orbits. Accordingly, there is no difference in Δv_{total} between the sequences. The difference between the sequences is in their time profile of the orbital phase (or M), or equivalently, the time of events whose occurrence is defined by geometrical or geographical conditions. Hence, the design space is meaningful when we want to change the time of event while holding Δv_{total} to be equal to Δv_{min}. Let me show an example.

As is mentioned in Section 2.2, the basic concept of the phasing orbit assumes Keplerian motion of the spacecraft during the phasing loops. In practical, a number of perturbation sources effect on the orbit, which should be considered and be coped with in detailed orbit design. In most cases, the effects are small, which can be coped with in the range of tuning or adjustment. However, in case that the spacecraft closely approaches to the moon during the phasing loop at its apogee, the orbit is largely disturbed. The effect of the disturbance is so large that large Δv is needed in order to connect to the nominal translunar orbit. Hence, the spacecraft's close approach to the moon should be avoided in the phasing orbit design.

Fig. 6 shows a phasing orbit design process on TM-diagram to avoid the spacecraft's close approach to

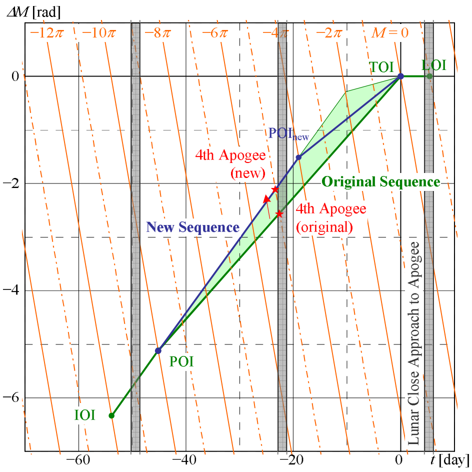

Fig. 6. Avoidance of Close Approach to the Moon.

the moon. The case 8 in Table 1 is used as an example. The spacecraft's close approach to the moon occurs when two conditions are satisfied simultaneously. That is, the moon approaches to the apogee of the phasing orbit, at the same time, the spacecraft is at the apogee of the phasing orbit.

The time that the moon approaches to the apogee of the phasing orbit is calculated from the ephemerides of the moon. The apogee direction of the phasing orbit is basically identical with that of the nominal translunar orbit, which is constant irrespective of intermediate phasing sequence. The duration while the apogee direction of the phasing orbit passes through the sphere of influence of the moon is painted in gray in Fig. 6. The green line connecting IOI, POI, TOI and LOI represents the profile of the case 8 ($N_{PO} = 5$) in Table 1, which is named as "original sequence". The orange chain lines in the chart represent the contours of M whose values are $2k+1$ where k is an integer. That is, the point on the lines is at the apogee of the orbit. The apogee passages of the original sequence are found as the intersections between the green line and the orange chain lines. As is expected, LOI is at the apogee of the orbit (i.e. the point is on the orange chain line), at the same time, it is in the neighborhood of the moon (i.e. the point is in the gray band). Attention should be focused on the forth apogee of the orbit, which is marked with a red star on the original sequence. The point is in the gray band, which signifies that the spacecraft closely approaches to the moon at the forth apogee.

Next, the way is sought to avoid the close approach to the moon without the increase of Δv_{total}. In other words, the sequence without the close approach to the moon is sought in the design space of minimum Δv phasing orbit. The design space is defined in the same way as Fig. 5, which is painted in green in Fig. 6. In the design space, you can find another line of sequence ("new sequence") which is drawn in blue and connects IOI, POI_{new}, TOI, and LOI. The difference between the new sequence and the original sequence is in the allocation of N_{IO} and N_{PO}. $(N_{IO}, N_{PO}) = (1, 5)$ in the original sequence, while $(N_{IO}, N_{PO}) = (4, 2)$ in the new sequence. It results in the placement of POI_{new} at the fourth perigee passage. To focus on the fourth apogee passage of the new sequence, which is marked with a red star on the new sequence, it is placed at the outside of the gray band, which means that the close approach to the moon is avoided in the new sequence.

Thus, TM-diagram and the design space of minimum Δv phasing orbit defined on it is useful in finding alternative sequences which has more preferable phasing condition.

5. Conclusion

Discussed in the paper is the design method of the phasing orbit. The design process of phasing orbit is overviewed using a concrete example. The chart "TM diagram" is introduced with its usefulness in the phasing orbit design process.

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