Study of the Essential Properties about the Halo-Orbit around L2

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Abstract: This study investigates the essential properties of the Halo-orbit in the dynamical Circular Restricted 3-Body Problem (CR3BP). Halo-orbit around Lagrange point (L-point), especially at L2 point, is recently in the spotlight because of its periodicity, suitable location for deep-space astronomical observations, and stable thermal environment. The spacecraft Space Infrared telescope for Cosmology and Astrophysics (SPICA) is being designed to utilize the Halo-orbit of L2 as a first Japanese Lagrange point mission. Therefore, for the specific interest for the Halo-orbit, the detailed study about the orbit's essential properties is important and it is useful to understand the exact spacecraft motion in the orbit. As major properties, the orbit divergence, convergence, and rotation angle are mainly discussed in this paper.

L2 点周りのハロー軌道に対する主要特性に対する研究

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摘要: 力学系円制限 3 体問題を用いて，ハロー軌道の持つ主要特性についての研究を行った．円制限 3 体問題下における力の平衡点はラグランジュ点（L 点）呼ばれ，特に L2 点周りに形成されるハロー軌道は，周期性・深宇宙観測への広い視野・安定した温度環境などという利点を持ち, 注目されている. 現在, 宇宙航空開発機構（JAXA）では，日本で初めてのラグランジュ点近傍へ投入する次世代赤外線天文衛星（SPICA）を計画しており，SPICA は L2 点周りのハロー軌道を運用起動と想定している．したがって，実用的な問題としてもハロー軌道に対して詳細な宇宙機の軌道運動および特性解析が重要であると考える．本研究では，主要な特性として回転角と収束・発散性についての報告を行っている．

1. Introduction

This study investigates the essential properties of the Halo-orbits [1)] in the dynamical Circular Restricted 3-Body Problem (CR3BP). [2)] Halo-orbits around Lagrange point (L-point), especially at L2 point, are recently in the spotlight because of its periodicity, suitable location for deep-space astronomical observations, and stable thermal environment. In near future, several missions like technical demonstration and deep-space observation missions will be launched to utilize Halo-orbits.

Japan Aerospace Exploration Agency (JAXA) is currently planning the first Japanese Lagrange point mission named Space Infrared telescope for Cosmology and Astrophysics (SPICA). SPICA mission is designed to explore the universe with a large infrared telescope. The spacecraft is injected into a Halo-orbit around L2 point of the Sun-Earth system. The infrared observation requires having low temperature for the telescope in order to obtain very accurate infrared imaging thus the environment nearby the L2 point is very attractive. In addition, Halo-orbits are periodic and, therefore, the orbits are desirable for the mission operation.

Therefore, for the specific interest for the Halo-orbit, the detailed study about the orbit's essential properties is important and it is useful to understand the exact spacecraft motion in the orbit. As major properties, the orbit divergence, convergence, and rotation angle, which are found by using the eigen-structure of the state transition matrix of the reference orbit, are mainly discussed in this paper. The divergence and convergence give the information about stability for the orbit and the rotation angle is a useful parameter to know the orbit recurrence.

2. CR3BP

The dynamical system theory which used in this study is Circular Restricted 3-Body Problem (CR3BP) of Sun-Earth system. In this theory, the Earth rotates around the Sun without eccentricity and inclination. And this theory deals a motion of infinitesimal mass particle. The massless particle does not affect the Sun and Earth motions. [3]

When consider a non-inertial, co-moving rotational frame of reference xyz whose origin lies as the center of mass O of the Sun-Earth system, with the x-axis directed towards the Earth, the equations of motion for the CR3BP are [4]:

$$\ddot{x} - 2\dot{y} + x = {\partial U}/{\partial x} \tag{1}$$

$$\ddot{y} + 2\dot{x} - y = {\partial U}/{\partial y} \tag{2}$$

$$\ddot{z} = {\partial U}/{\partial z} \tag{3}$$

where, U is the pseudo potential given in terms of mass ratio, $\mu = m_{earth}/(m_{sun} + m_{earth})$, solar distance, d_{sun}, and earth distance, d_{earth}, as:

$$U = {(1-\mu)}/{d_{sun}} + {\mu}/{d_{earth}} + {(x^2 + y^2)}/{2} \tag{4}$$

At the equilibrium point, the particle has no acceleration and no velocity. Therefore, the location of equilibrium points in the CR3BP can be solved by applying these conditions:

$$\ddot{x} = \ddot{y} = \ddot{z} = 0$$
$$\dot{x} = \dot{y} = \dot{z} = 0$$

There are five equilibrium points called L-points in this model and a particle placed at an L-point, it will presumably stay there.

Fig. 1 illustrates the coordinate and the cross symbols show L-points. Our specific interest is of the L2 and, therefore, the L2 is shown in red colored.

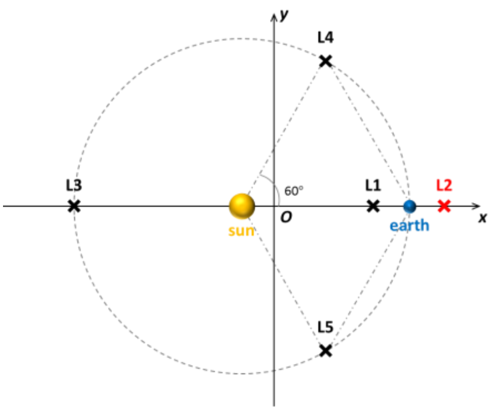
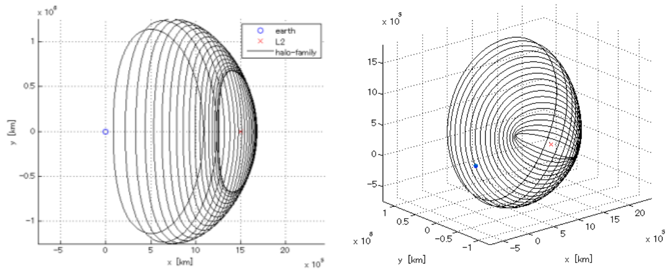


Fig. 1: Illustration of the Coordinate and L-Points

3. Properties of the Halo-Family

The Halo-orbit related to a particular L-point can take various shapes. [5] Shown in Fig. 2 are the several size of the Halo-orbits (Halo-family) related to the L2. The black curves in Fig. 2 exhibit the motion of the Halo-family around the L2. The red cross and blue circular symbols are the positions of the L2 and the Earth position, respectively. In both Fig. 2 (a) and (b), the frame center is fixed to the Earth position.



(a) in-plane motion (b) 3-D motion

Fig. 2: Halo-Family around L2 in Earth Fixed Coordinate

Depending on the size of Halo-orbit, the orbit properties are differed. The major properties of the Halo-orbit are divergence and recurrence. Convergence is also a major property, however, it can be argued as reversible phenomena as the divergence. Hence, the orbit convergence is discussed in same logic as the divergence. In this paper, the orbit size definitions are represented by A_z, which is the maximum position in z-axis.

3.1. Diverging Magnitude as function of Orbit Size

The orbit divergence gives the knowledge of how sensitive the orbit is to the deviation from the reference orbit. It is obvious that when the orbit has large magnitude of diverging, a particle motion may be easily departed from the orbit when the particle is putted into the diverging direction. In contrast, when the orbit has a large diverging magnitude, an orbit convergence becomes greater. This is because, the magnitude of converging is a reciprocal number of the diverging magnitude.

A primary eigenvalue of the monodromy matrix, which is the State Transition Matrix during one orbital period, bring the diverging magnitude of the orbit. [6] Fig. 3 shows the variation of diverging magnitude in the range $A_z =$ 0.0 to 18.0×10^5 km. The diverging magnitude loses its number according to the growth of the Halo-orbit size. Consequently, a large size of the Halo-orbit is easy to maintain the motion in the orbit but hard to converge into the orbit.

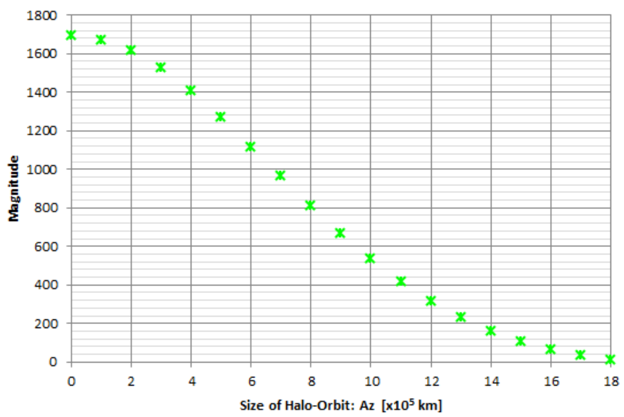


Fig. 3: Variation of the Diverging Magnitude vs Orbit Size

3.2. Rotation Angle as function of Orbit Size

The Halo-orbit is a periodic orbit around the collinear Lagrange points. Furthermore, there are quasi periodic orbits Quasi-Halo which collaborate to every Halo-orbit. The Quasi-Halo is not closed during one revolution but return with twisted state about the reference Halo-orbit. The calculated rotation angle for each Halo-orbit predicts how much angle the state twisted after one revolution.

From the calculated monodromy matrix of a Halo-orbit, we obtain pair of eigenvalues and there is complex conjugate pair in there. The eigenvectors for the complex conjugate pair are related to the rotation angle of the orbit. (see also in Ref. 4) Shown in Fig. 4 is the result of the calculation of rotation angle correlated to specific size of the Halo-orbit.

From the result, we can find the relationship between the rotation angle and orbit size. The angle is getting bigger when the orbit increases its size and even it can achieve 180°.

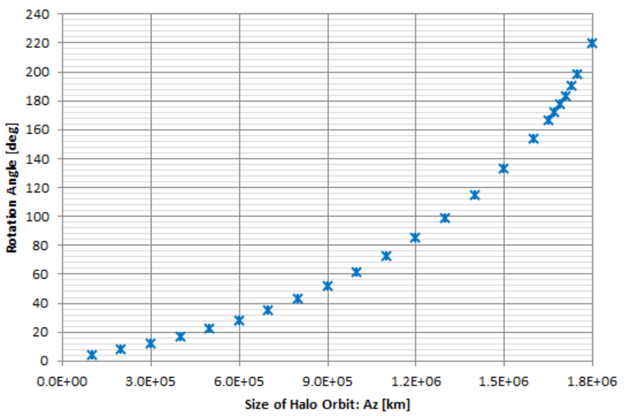


Fig. 4: Variation of the Rotation Angle vs Orbit Size

The rational number of 360° has a particular meaning for the rotation angle. For instance, if the rotation angle becomes 180° the Quasi-Halo will realize the exact same state after second revolution. Therefore, the rational number, which equals to 360 divided by integer *n*, proposes us to have *n* times recurrence of Quasi-Halo.

Table. 1 summarizes the rotation angle with respect to the orbit size, A_z. The table also presents the revolution times required to realize the initial state.

Table. 1: Summary of Rotation Angle as function of A_z

A_z [$\times 10^5$ km]	6.3	8.3	9.8	14.3	17.0
Angle [°]	30	45	60	120	180
revolutions	12	8	6	4	2

The rotation angle nearby the reason of 180° shown in Fig. 5, however, shows the irregular variation. The calculation process is based on the solution from the complex conjugate and nearby 180° the pair of the complex conjugate eigenvalue is solved as repeated root. Hence, it is impossible to get a straightforward result.

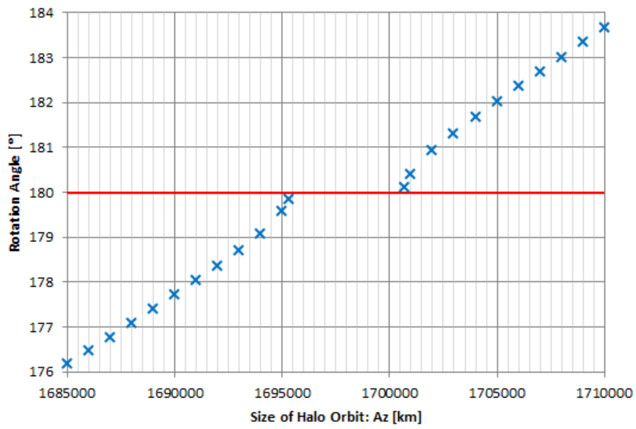


Fig. 5: Close View nearby 180° of Rotation Angle

4. Diverging and Converging Directions

This chapter describes the direction of orbit divergence. In the previous chapter, we show the results of Halo-orbit diverging magnitude and rotation angle for the Halo-orbits as function of the orbit size in terms of A_z. For the orbit divergence, diverging (unstable) direction could become another key factor. The small displacement into the unstable direction will rapidly grow its magnitude depending on the diverging magnitude. Hence, a spacecraft, which is following the orbit, put into unstable direction will naturally depart from the orbit with minimum energy and constitute unstable manifold.

4.1. Local Method

The unstable direction is obtained from a diverging eigenvector of monodromy matrix which related to primary eigenvalue. It differs at a particular position of the Halo-orbit. We call the directions calculated by monodromy matrix at each epoch the "local" directions corresponding to the local information. This method is defined as the Local method. This method is intuitive to obtain the direction and to construct the manifold but, when we need to search a plenty of manifolds for the Halo-orbit, it requires the multiple monodromy matrix calculations.

4.2. Scaling Method

The unstable directions, in contrast, can be also computed from a single unstable direction and the growth of the diverging magnitude from a specific starting point. STM will generate the growth of the diverging magnitude from the point and we can utilize the growth for scale factors which relative to any desirable points. The Scaling method which utilizes a direction and acquired scale factors is especially beneficial to construct the multiple manifolds without monodromy matrix calculations.

The scaling method is also effective to calculate the converging (stable) directions and, similar to the unstable manifolds, the stable manifolds are constructed from the knowledge of stable direction. If we put a small displacement in a stable direction and execute a backward propagation, the trajectory constructs a stable manifold.

The stable manifolds are very beneficial trajectories in the view point of transfer into the Halo-orbit. This is because the orbit injection can be performed by ideally zero velocity change if a spacecraft flies on the stable manifolds. Fig. 6 shows the stable manifolds of $A_z = 4.0 \times 10^5$ km size of Halo-orbit in Earth centered rotational frame which are designed by the normal Local method.

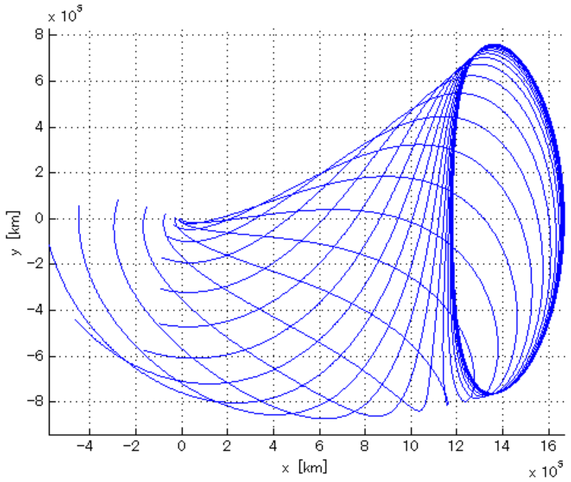


Fig. 6: Stable Manifolds Designed by Local Method

Every stable manifolds show a different trajectory but all the trajectories are smoothly converged into the Halo-orbit in the end. However, the calculation requires multiple matrix computations as we mentioned. The scaling method reduces this computation load. In addition, the scaling method can construct a lot of manifolds nearby the interested reason, intensively.

Fig. 7 shows an example of the scaling method results. The blue trajectories are designed to realize the manifolds by local method and the red trajectories are designed the manifolds intensively nearby the Earth.

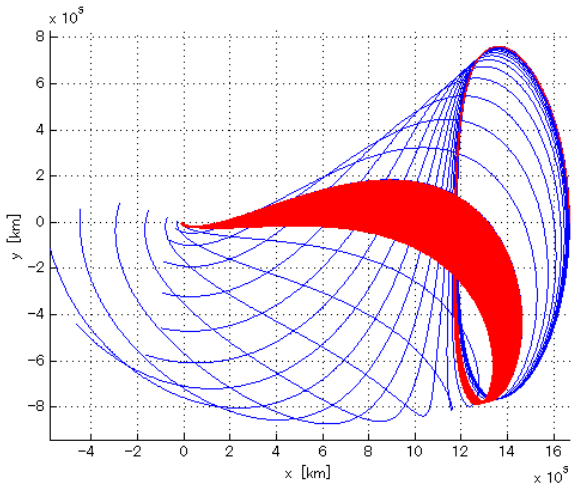


Fig. 7: Stable Manifolds Designed by Scaling Method

5. Conclusion

This study presents the essential properties of the Halo-orbit around L2. The divergence and rotation angle of the orbit are particularly interested in this paper.

We show the relationship between the orbit size and those properties. The diverging magnitude loses its number according to the growth of the Halo-orbit size. The rotation angle is getting bigger when the orbit increases its size. The directions of the diverging and also converging are exploited by the scaling method which method can construct optional manifolds on the occasion.

6. Reference

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