Transfer trajectories starting from the L2 point of the Earth-Moon system

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Abstract: The libration points of the Earth-Moon(EM) system have recently attracted attention again as a new base for long-duration habitation. They are considered desirable candidate sites for extended ISS operations, which are very important to understand human space capabilities. This paper shows various kinds of transfer trajectories for the use of EM L2.

First half of the paper presents trajectories between the Earth-Moon L1/L2 and Sun-Earth L2. It is shown that small maneuvering can achieve the successful transfer between the points by the effective use of the natural dynamics. Starting from two impulsive trajectories, several transfer plans are investigated clarifying the amount of the total maneuvers, width of the launch window and flight time. This section concludes by comparing results between each transfer strategy in order to propose the most favorable transfer scheme for the situation.

The latter half proposes efficient escape trajectories to the interplanetary space from EM L1 or L2. It begins by a natural escape, where a spacecraft naturally leaves the libration points due to the imbalance of gravity. This type of a trajectory has a merit of being maneuver-less but cannot gain enough energy to visit the outer planets without using some orbital manipulations, i.e., EDVEGA. Next, a technique of the trajectory correction using the solar perturbation is proposed. Controlling the path with the acceleration by the solar gravity leads the targeting at the Earth, achieving the chance of a powered gravity assist and realizing a high improvement of the escape energy. 400 m/s of $\Delta V$ can generate C3 enough to reach the Mars.

1. Part1. Transfers between Earth-Moon L1/L2 and Sun-Earth L1/L2

1.1 Introduction

Problem statement. The problem of transfer between Earth-Moon L1/L2 and Sun-Earth L1/L2 may be interpreted as the transfer between coplanar circular orbits. It is known that the Hohmann transfer is a global minimum of the two-impulsive circle-to-circle transfer. Therefore, the solutions of the transfers between the libration point of the Earth-Moon and Sun-Earth systems are expected to be trajectories similar to the Hohmann solutions.

The major difference between the Hohmann and EML-to-SEL transfer is its gravitational field. The Hohmann assumes that the spacecraft moves in the two-body system where only the gravity of the primary planet is considered. On the other hand, there are three gravitational sources, i.e., the Sun, Earth and Moon, in the EML-to-SEL transfer problem and the influence of perturbations which is neglected in the Hohmann transfer must be taken into account.

Figure 1 shows the schematic diagram of the libration point of the Sun-Earth-Moon system. In the Sun-Earth rotating frame, L1 and L2 of the Sun-Earth system lies on the line intersecting the center of mass of the Sun and the Earth. The libration points of the Earth-Moon system rotate around the its barycenter as the system moves.

Key points. The trajectories are obtained by employing the Sun-Earth-Moon bicircular restricted four-body problem (see Chapter 2) and solving the optimization problem of which objective function is minimizing the total $\Delta V$. The structure of the trajectories is preliminary given expecting a possible solution.

Contribution and Organization. This paper presents the transfer schemes between the libration points of the Earth-Moon (EM) and Sun-Earth (SE) systems: trajectories from EM L2 to SE L2 and from EM L1 to SE L2. It is shown that small maneuvering can achieve the successful transfer by the effective use of the natural dynamics.

We begin by considering two impulsive transfers, which apply impulsive acceleration at the departure and arrival points. The amount of the total $\Delta V$, the width of the launch window and the flight duration are discussed. Next, some extended scheme using the natural dynamics are proposed resulting the improvement of $\Delta V$.

The paper concludes with the comparing investigation between each transfer strategy in order to propose the most favorable transfer scheme to the situation.
1.2 Preliminary preparation

1.2.1 Symmetries of the system

The transfer between Earth-Moon L1/L2 and Sun-Earth L1/L2 basically consists of four types of itineraries as follows,

- Transfers from EM L1/L2 to SE-L1.
- Transfers from EM L1/L2 to SE L2.
- Transfers from SE L1 to EM L1/L2.
- Transfers from SE L2 to EM L1/L2.

Meanwhile, the equations of motion of the secondary-centered three body problem have the following symmetries,

\[ S_1 : (x, y, \hat{x}, \hat{y}, \hat{z}, \hat{t}) \rightarrow (x, -y, \hat{x}, \hat{y}, \hat{z}, -\hat{t}) \]
\[ S_2 : (x, y, \hat{x}, \hat{y}, \hat{z}, \hat{t}) \rightarrow (-x, y, \hat{x}, \hat{y}, -\hat{z}, -\hat{t}) \]
\[ S_3 : (x, y, \hat{x}, \hat{y}, \hat{z}, \hat{t}) \rightarrow (-x, -y, -\hat{x}, -\hat{y}, -\hat{z}, -\hat{t}) \]  

\( S_1, S_2 \) and \( S_3 \) are symmetries about the \( x \)-axis and \( y \)-axis and origin, respectively. Therefore it is not necessary to consider all the four itineraries stated above and one transfer scheme can explain the others by the use of these symmetries. In this research, the second option, the trajectories from EM L1/L2 to SE L2, is discussed.

1.2.2 Moon phase of departure

Both EM L1 and L2 rotate around the barycenter of the EM system synchronizing the Moon motion. Thus the position of L1 and L2 can be represented by that of the Moon, that is defined by the angle measured from the Sun-Earth line \( \psi_f \) as shown in Figure 2. In the following discussion, the term “initial Moon phase (or angle)” shall indicate the position of the Moon when the spacecraft depart from EM L1 or L2.

\[ \psi_f = x_f - x_{SEL} = 0 \]  

(4)

In addition to the boundary equality constraints, the following two inequality constraints also imposed to avoid the collision with the two primaries,

\[ s_1 = r_E - R_E > 0 \]  
\[ s_2 = r_M - R_M > 0 \]  

(5)
(6)

where \( r_E \) and \( r_M \) denotes the distance of the spacecraft to the Earth and the Moon, and \( R_E \) and \( R_M \) are the radius of each planet.

1.3 Departure from Earth-Moon L2

1.3.1 Basic transfer trajectories

Typical transfer trajectories from EM L2 to SE L2 are shown in Figure 3 to 5. Six types can be found which can connect the two points in 200 days. Because the EM L2 point is generally not stable, the spacecraft staying there easily escapes due to some small disturbance. These basic transfers can be obtained by using this unstable characteristic. The spacecraft departs EM L2 without \( \Delta V \) and reaches SE L2. The angle of the Moon at initial time is only the variable parameter and the difference in the velocity at SE L2 is adjusted by an impulsive maneuver.

The Type-1 trajectory shown in Figure 3 is the fastest transfer of the six. The transfer time is 37.8 days and the required \( \Delta V \) at SE L2 is 484 m/s. The trajectory is represented as an nearly elliptical orbit in the inertial frame. Type-2 has an apogee during the flight and therefore takes the longer time, 111 days in this example, than Type-1. The \( \Delta V \) required is 483 m/s and little improvement can be found.

\[ J = \Delta V_1 + \Delta V_2 \]
\[ = ||x_0 - x_{EML}|| + ||x_f - x_{SEL}|| \]  

(2)

where \( x \) donates the position vector. The problem is transcribed into a nonlinear programing problem using a direct collocation. The transfer orbit must connect the prescribed initial point and the prescribed final point, in this case, EM L and SE L. Thus the boundary condition can be written as

\[ \omega_0 = x_0 - x_{EML} = 0 \]  

(3)

Figure 4 shows plots of Type-3 and 4 transfers. These two approach SE L2 from the \( +y \) direction along the halo orbit path. Their \( \Delta V \)s are nearly equal to those of Type-1 and 2 but their flight times are much longer going a longer way round. It can be said that these transfers are better suited for the insertion into the halo orbit around SE L2 rather than SE L2 itself.

Trajectories of Type-5 and 6 in Figure 5 include a deceleration phase due to the solar gravity perturbation. Because the solar perturbation works in the \( -y \) direction near the \( y \)-axis, the spacecraft
going in the +y direction is slowed down resulting one more revolution around the Earth. The time of flight inevitably increases compared to the Type-1, 2. The point that should be noticed is that the launch windows of Type-5 and 3 locate close each other. The sensitivity of the initial Moon phase to the following trajectory near their windows is very high and small displacement of the departing Moon angle may generate a totally different trajectory.

1.3.2 Two impulsive transfer

Each natural escape trajectory mentioned above has a particular departing Moon angle. This narrow launch window can be expanded using an initial departing impulse at EM L2. The following is the calculation results trying to expand the window for the Type-1 trajectory by adding the initial impulsive maneuver.

Figure 6 shows a plot of the two impulse trajectory. The age of the Moon when the spacecraft departs from EM L2 is 90 degrees. Only 2.8 m/s $\Delta V$ at EM L2 can modify the trajectory to be able to target SE L2. The arrival maneuver is 487 m/s, which is at the same level as the original.

Figure 7 and 8 show the magnitude of the required maneuver and the time of flight as a function of the departing Moon phase. The graph insists that $\Delta V$ of several m/s can expand the window up to over 100 degrees in terms of the Moon age.
1.3.3 Two impulsive transfer with initial coast

One of the options to improve the two impulsive trajectory is to add an initial coasting and apply the first impulse at the position closer to the Moon than L2. The spacecraft at EM L2 takes the zero velocity curve heading the Earth and inserted onto the transfer trajectory toward SE L2 at a perilune by applying a tangential maneuver.

Figure 9 shows the one example of this kind of transfer. The spacecraft naturally departs EM L2 and arrives at the perilune after 10.4 days travel. Then adding tangential maneuver of 11.5 m/s enables the spacecraft to target SE L2. The required deceleration at the final is 294 m/s and the total ∆V is 305 m/s. A significant reduction of the total ∆V is observed. Figure 10 is a magnified view of the same trajectory as Figure 9, represented in the Earth-Moon rotating frame. The maneuver is applied in the opposite direction to the Moon motion and works to decrease the energy of the spacecraft with regard to the Earth.

Fig. 9 Initial coast two impulsive transfer from EM L2 to SE L2 in the Sun-Earth rotating frame. Departing Moon phase is 310 degrees.

Fig. 10 Same trajectory as Figure 9 in the vicinity of the Moon in the Earth-Moon rotating frame.

Figures 11 and 12 show the necessary maneuver and the time of flight as a function of the initial Moon phase when the spacecraft departs EM L2. The strategy of including an initial coast succeeds in reducing the total maneuvers less than 400 m/s within the range of the initial Moon angle between -60 to 0 degrees. Meanwhile, some increment can be seen in the flight time because of the longer way passing nearby the Moon compared with the no-coasting strategy.

1.3.4 One impulsive transfer with initial and terminal coast

It may be possible to reduce more ∆V by including a final coast before arriving SE L2. In this case, the spacecraft approaches SE L2 from the direction of the zero velocity curve associated with SE L2.

A plot of one impulsive trajectory is shown in Figure 13. One-time midcourse trajectory correction is applied and its magnitude is about 250 m/s. This value is rather small compared with the ∆V of the previous transfer mode.

Problems of this kind of trajectory, however, is its long flight time and narrow launch window. Because the velocity of the spacecraft moving along the zero velocity curve becomes nearly zero, it consumes much of time to get to the destination.

Another plot is shown in Figure 14. This is a derivation of the Type-5 transfer. It also takes long time to transfer but requires only 290 m/s trajectory correction.

1.3.5 Low thrust transfer

The orbital control by low-level continuous thrust is one of the likely options for the future L-to-L transfers. It is expected that more fuel can be saved because of the high fuel consumption of the low thrust device compared with chemical propulsions.

The following results are obtained by numerically optimizing trajectories with the objective of minimizing the magnitude of the...
acceleration. The constraints on the control are imposed that its magnitude is constant through the transfer and only its direction can be changed.

Figure 15 shows a plot of low thrust transfer between EM L2 and SE L2. The red arrows indicate the direction of the acceleration. In this example, the optimized magnitude of the acceleration is $5.3 \times 10^{-8}\text{km/s}^2$, which corresponds to the acceleration of the spacecraft of 1,000 kg-mass with a 53 mN-thruster. The integral of the acceleration, i.e., $\Delta V$, is 355.8 m/s, the same level as the other transfers.

Figures 16 and 17 indicate the relationship of the magnitude of the acceleration and flight time to the departing Moon age. Just like other transfer modes, the value of the control increases as the Moon age increases. The flight time does vice versa.

1.4 Summary

**What this paper showed.** This paper presents several transfer schemes between L1/L2 of the Earth-Moon and Sun-Earth systems. It began with proposing six types of transfers, which naturally depart from EM L2 as shown in Figure 3 to 5. They look completely different but require a similar magnitude of $\Delta V$. Type-1 provides the fastest transfer among them and Type-2 would be rather suitable for the transfer to halo orbits around SE L2.

The following discussion explained that small control at EM L2 can expand the window of the departure. For example, the day of the launch is uniquely determined in the case of the zero-velocity departure mentioned above but Figure 7 indicated that at least 100 degrees of the window can be ensured. In addition, the magnitude of the first impulse is only several m/s.

Next, the paper presented the indirect transfer, which uses the Moon swing-by. Different from the direct transfer aiming at SE L2...
L2 applying the first impulse at EM L2, indirect transfer efficiently uses the system dynamics and achieves the improvement of the total $\Delta V$. Figure 9 showed one of the examples. Although the flight time becomes longer because of the longer way passing nearby the Moon, the $\Delta V$ at SE L2 reduces by about 100 m/s.

As an additional option, the midcourse impulse transfer was proposed (Figures 13 and 14). It can reduce further $\Delta V$ but at the same time significantly increases the flight time and limits the launch window.

Finally, the transfer with the low continuous thrust was presented as shown in Figure 15. The total $\Delta V$ calculated by the integration of the acceleration is 355.8 m/s, that was at the same level as the impulsive transfer. However, it would require less fuel considering the high fuel consumption of the low thrust devices. Thus, this type of transfer is the most likely option.

As for the transfer starting from EM L1, almost the same conclusion can be obtained. Only one difference is that the spacecraft requires more energy to open up the barrier around EM L2 when it departs from EM L1.

Related researches and my contribution. The related study[2] insisted that EM L1 was an ideal position for the extended human activity beyond LEO. According to their paper, adopting trajectories called interplanetary superhighway system realizes an ultra-low cost transfer and therefore EM L1 can work as an all-round space transportation hub. Their calculation also concluded that the transfer between the EM L1 and SE L2 can be achieved with $\Delta V$ of several m/s. There is much difference in the values of $\Delta V$ between my results and theirs, but this comes from the difference of the boundary condition of the assumed trajectory. My study sets the boundaries at the libration points, while theirs are the halo orbit around it. In general, it requires less cost to send the spacecraft to the halo orbit than the libration point itself ([3]). Their transfer scheme is very attractive but it is also true that another maneuvering is needed to arrive at the vicinity of the libration points. This is the first study that clarifies the necessary $\Delta V$, transfer time and launch window of the transfer trajectories which directly connect the libration point of the EM and SE system.

2. Part2. Escape from Earth-Moon L2 to Interplanetary Space

2.1 Introduction

Problem statement. It is a major topic of astrodynamics how to gain the necessary energy to navigate in the interplanetary space. Over 3.0 km/s of the relative velocity is required in order to reach even the nearest planet, Mars.

In considering the use of EM L2 as a window for the interplanetary transfer, it is absolutely necessary to know the available energy of the spacecraft departing from this point and the acceleration strategies to get prescribed velocity.

Key points. To effectively accelerate the spacecraft and increase its orbital energy, $\Delta V$ should be applied tangentially at where it has a high velocity. It is no question that the spacecraft has highest speed at the perigee when it flies in the Earth gravity field. Thus this paper especially focuses on the trajectories passing nearby the Earth and shows the availability of the powered Earth swing-by.

Contribution and Organization. In this paper, the escape trajectories from the libration point of the Earth-Moon system to the interplanetary space are considered. The paper begins by the direct escape schemes, which are the easiest to achieve. The spacecraft leaves the libration point with impulse and directly arrives at the Earth sphere of influence. Next, the technique of the trajectory correction using the solar perturbation is proposed. Controlling the path with the acceleration by the solar gravity leads the spacecraft to target at the Earth, achieving its powered gravity assist and realizing the high improvement of the escape energy. $\Delta V$ of only 400 m/s can generates $V$-infinity enough to arrive at the Mars.

2.2 Preliminary preparation

2.2.1 Basic escape maneuvers from circular orbits

Three types of basic escape maneuvers from circular orbits are shown in Figure 18. A single impulsive transfer directly escapes from circular orbits and is proven to be the optimum escape mode for the low value of $V$-infinity. Meanwhile, a two or three impulsive transfer get advantage for larger $V$-infinity and their efficiencies depend on the positions of the second and subsequent maneuvers.

![Fig. 18 One-, two-, three-impulsive escape maneuvers from circular orbits.](image)

2.2.2 Earth gravitational sphere of influence

The sphere of influence (SOI) gives an estimation of the range where the gravity of the subject planet is dominant over other
gravitational perturbations. The radius of it generally defined in terms of the ratio between the Keplerian force and the perturbation of the planets and had the form of

\[
r_{SOI} = \left( \frac{\mu}{1 - \mu} \right)^{\frac{1}{4}}
\]

(7)

According to this equation, the radius of the Earth gravitational sphere is calculated as 925,000 km. On an ordinary planning of interplanetary missions, the method of patched conics is employed and it divides the mission into several parts with reference to the distance to the planet. \( V_{\infty} \) is generally defined as the velocity at the crossing of this sphere.

It may be, however, inconvenient to adopt this SOI definition on the escape problem from the collinear libration points because both L1 and L2 originally locate outside of the boundary. Thus, the new definition of a larger region must be introduced in order to evaluate trajectories from L1 or L2. In this paper, the boundaries are set perpendicular to the \( x \)-axis at \( x = x_{\text{Earth}} \pm r_{L2} \), where \( r_{L2} \) indicates the distance between the Earth and SE L2 as shown in Fig. 19. Because the motion of a particle with low energy is restricted by the barrier of energy, the zero velocity curves, there is no need to set boundaries in the \( y \) direction.

![Fig. 19](image)

**Fig. 19** Definition of the boundary of the Earth region.

2.3 Departure from Earth-Moon L2

2.3.1 Natural escape

The trajectories from EM L1 require maneuvers to open up the neck around L2 and reach the sphere of influence, but this is not the case with the trajectories from L2. The energy of EM L2 is higher than that of EM L1 and thus the spacecraft leaving EM L2 can escape the Earth gravity field without any maneuvers. In this section, the trajectories which head for the exterior side from EM L2 with zero-velocity departure are discussed.

Figure 20 shows the escape directions of the trajectories from EM L2 as a function of the departure Moon orientation. The position is expressed as the angle measured from the \(+x\) axis in the Sun-Earth rotating frame. These non-maneuver-type transfers can be divided into two main categories depending on their escape directions. Trajectories of one group escape to around 330 degrees, i.e., in the direction of the anti-Sun as shown in Figure 21 and the others are symmetrical about the Earth as shown in Figure 22.

![Fig. 20](image)

**Fig. 20** Escape direction as a function of the departing Moon phase.

![Fig. 21](image)

**Fig. 21** Family of the escape trajectories heading for the \(+x\) direction. The departing Moon phase is between 40 and 210 degrees.

![Fig. 22](image)

**Fig. 22** Family of the escape trajectories heading for the \(-x\) direction. The departing Moon phase is between -140 and 30 degrees.

Figure 23 shows the relative velocity in the Earth-centered inertial frame at the boundary. It reaches maximum for the departing Moon angle around 40 degrees and 220 degrees. The orbital profile of the trajectory of \( \theta_{M} = 40 \) degrees is shown in Figure 24. This graph shows the path goes through the fourth quadrant in the Sun-Earth rotating frame and escapes. The solar perturbation works to accelerate the orbit along the \( x \)-axis in this area and that is why the orbital velocity increases.
On the other hand, it can be seen that the velocities are nearly constant from 50 to 100 degrees and from 230 to 280 degrees. Within this range, the orbital shape becomes nearly elliptical, the most fundamental form of the natural escape trajectory from EM L2.

![Fig. 23](image)

**Fig. 23** Escape velocity in the Earth-centered inertial frame as a function of the departing Moon phase.

Within this range, the orbital shape becomes nearly elliptical, the most fundamental form of the natural escape trajectory from EM L2.

![Fig. 24](image)

**Fig. 24** Escape trajectories from EM L2 in the Sun-Earth rotating frame. The departing Moon phase is 40 degrees.

### 2.3.2 Direct escape with initial impulse

The next scheme is an extended of the natural escape discussed above. The difference is to add an initial impulse at the EM L2 departure. This is also categorized as one-impulsive trajectory in Figure 18 and can be called as a direct escape. Here, the word "direct" means that the spacecraft leaves the vicinity of the EM system from the start point without encountering either the Earth or Moon.

The initial impulsive maneuver at EM L2 is defined by its magnitude and direction as shown in Figure 25. In a two body problem, a tangential burn can most efficiently increase the orbital energy. On the other hand, it is not always the best for $\Delta V$, especially for small $\Delta V$, to be tangential if a three- or four-body system is considered. In addition, from where to start also affects the results. Therefore, the direct escape trajectory has three independent parameters to select, i.e., the initial Moon phase $\theta_M$, the magnitude $|\Delta V|$ and direction $\theta_{\Delta V}$ of the maneuver.

![Fig. 25](image)

**Fig. 25** Geometry of initial impulse at EM L2.

Here, only the trajectories of $\theta_M=270$ degrees are investigated as an example. Figure 26 shows an available escape velocity as a function of the $\Delta V$ injection angle. Each curve represents the values of the different magnitude of $\Delta V$ varying from 100 to 500 m/s. The velocity bears a proportionate relationship to the magnitude of $\Delta V$, that is, more $\Delta V$ can be available, the more speed the spacecraft can get at the boundary. Figure 26 also shows a desirable departure direction for each $\Delta V$. When the magnitude of $\Delta V$ is large, the resulting velocity monotonically increases as the angle of $\Delta V$ increases reaching the maximum at 90 degrees, which means tangential acceleration. Meanwhile, there is a maximum of the value before the angle reaching 90 degrees in the case of small $\Delta V$. Because the motion with small velocity is governed by the high nonlinearity, it cannot be analytically solved and there is no choice but to depend on numerical calculations.

Figure 27 and 28 show plots of direct escape trajectories in the Sun-Earth fixed rotating frame. The initial Moon phase is 270 degrees measured from the $x$-direction and the magnitude of the first impulse is 100 m/s. The graph indicates the escape position slightly changes depending on the initial $\Delta V$-angle but all the trajectories proceed in the $+x$ direction.

![Fig. 26](image)

**Fig. 26** Available escape velocity at the boundary when the initial Moon phase is 270 degrees.

### 2.3.3 Indirect escape with the perigee impulse

The three impulsive escape mode normally requires three independent maneuvers at the initial, apoapsis and periapsis in the
planet-centered two body problem as shown in Figure 18. The Sun-Earth-Moon four body system, however, can remove the first and second impulse by the effective use of the perturbations of the planets instead. The following discussion begins by designing trajectories which can pass nearby the Earth and then constructs escape modes with the perigee impulsive burn.

**Earth swing-by trajectory.** Plots of the Earth swing-by trajectories from EM L2 are shown in Figures 29 and 30. The spacecraft departs from L2 of the Earth-Moon system in the direction of the zero velocity curve, and is accelerated by the solar perturbation and finally arrives at the Earth and its vicinity.

These trajectories are very sensitive to the initial condition and needs to choose an appropriate departure date. A few degrees difference of the initial Moon age results in different approaches to the Earth. Figures 29 and 30 clearly show the its sensitivity. When $\theta_M$ is 204.21108 degrees, the spacecraft approaches the Earth from the left, and when $\theta_M$ is 206.83193 degrees, it does from the right. They both get a perigee of 1,000 km height from the surface of the Earth and have approximately the same value of the perigee speed, 11.1 km/s. If the spacecraft departs from EM L2 at the time between them, it ends up the collision with the Earth.

**Escape from the Earth accelerated by the Earth powered swing-by.** Applying an impulsive acceleration at the perigee can increase the orbital energy. A plot of the example trajectory with

the Earth powered swing-by is shown in Figure 31. The path from EM L2 to the pre-swing-by is all the same as the trajectory represented before. Applying the tangential impulsive maneuver of different magnitudes at the perigee generates various escape trajectories.

There is clearly a major gap of the escaping speed when one
compares the trajectories of Figures 29 and 31. The results insists that the velocity increment of only 100 m/s at the perigee generates the difference of 1,000 m/s at the boundary.

Figure 32 shows how the escape energy increase with the perigee maneuver. Two lines denote the values under a different swing-by altitude, 1,000km and 10,000km from the Earth surface. When the spacecraft swings by the Earth without acceleration, the gained excess velocity at the boundary is small and the corresponding orbital energy becomes negative. The negative energy in the two-body problem means the spacecraft is captured in an elliptical orbit and cannot escape, but the multi-body system can accept low energy escapes rather than hyperbolic-type orbit. In these figures, the monotonically increasing of the escape velocity with respect to the $\Delta V$ at the perigee can be seen. For the $\Delta V$ of over 400 m/s with 1,000 km-altitude swing-by, the escape velocity exceeds 3.0 km/s and the spacecraft has enough energy to fly to the Mars. However, the larger the swing-by radius is, the less the efficiency of the acceleration becomes.

Figure 33 indicates the time of flight from EM L2 to the crossing of the boundary and it monotonically decreases as $\Delta V$ increases. Figure 34 shows the position of the boundary crossing. It strongly depends on the position of the perigee. The position of the perigee is uniquely determined because the orientation of the Sun, Earth and Moon at the departure must satisfy a certain condition. In this case, the escape directions are nearly in the first quadrant of the Sun-Earth rotating coordinate.

A plot of the Earth powered swing-by trajectory of $\theta_{EM}=206.83193$ degrees is shown in Figure 35, and its escape properties are shown in Figures 36 to 38. The excess velocity and flight time at the crossing of the boundary almost keep the same level of the previous escape mode. The difference between the two lies in the position of escape. Because they have a different approach direction to the Earth, the post-swing-by trajectories proceed in a different direction. In the previous mode, the angle of escape monotonically increases with the perigee $\Delta V$ but in this case the angle reaches a maximum for $\Delta V$ near 30 m/s and gradually decreases. For larger value of $\Delta V$, the escape direction is almost parallel to the x-axis.

2.4 Summary

What this paper showed. In this paper, the escape trajectories from L2 of the Earth-Moon system to the interplanetary field are investigated. The natural escape mode from EM L2 has a merit of requiring no maneuver to arrive at the boundary of the Earth sphere of influence but its available $V$-infinity is small as shown.
the tangential direction to the orbit. On the other hand, when $\Delta V$ is small (around a few hundreds m/s), the optimal launch direction is not always tangential (Figure 26). It should be numerically calculated.

The third option, the quasi-three-impulsive escape mode, has a great advantage of $\Delta V$ efficiency. The term “quasi-three-impulsive” means that this mode has a similar shape to three-impulsive escape of two-body system, but can be achieved with one impulse by utilizing the planets’ gravity. This strategy can generate a large $V$-infinity compared with the former options. Figures 31 and 35 showed the accelerated trajectories which successfully gained large $V$-infinity at the boundary by amplifying the energy of the perigee-kick. Figure 32 indicated that the velocity is accelerated up to 3.0 km/s by applying the 400 m/s impulse at the perigee, i.e., the spacecraft gains the energy enough to arrive at the Mars. Thus, this strategy is recommended for missions aiming at the deep interplanetary space.

**Related researches and my contribution.** As far as I checked, there are few studies on the escape trajectories from EM L2. Hence this study is significant in the sense that it clarified the escape property from EM L2 and proposed effective orbital manipulation method to increase the energy.

**Significance of the results.** The result of this paper supports the possibility of EM L2 as a space port. By adopting quasi-three-impulsive escape mode, a spacecraft can effectively obtain a large energy. They can be also combined with other orbital manipulation strategy, such as EDVEGA.

**Extension.** This study proved that the quasi-three-impulsive escape trajectory can generate large energy enough to reach other planets. However, in fact, it is necessary to consider not just the energy available but the orientation of the Earth, Moon and target body. Thus the next step of this study is the trajectory design including the position investigation.

**References**

