Design of Position and Vibration Controller

Which is Based on ANCF Model for Flexible Structures

Yoshiki SUGAWARA¹, Nobuyuki KOBAYASHI²

¹Akita University, 1-1, Tegataakuen-machi, Akita-shi, Akita-ken, Japan
²Aoyama Gakuin University, 5-10-1, Fuchinobe, Chuo-ku, Sagamihara-shi, Kanagawa-ken, Japan

Abstract In order to achieve attitude and vibration control of extremely flexible structures which are used for space applications, we propose a controller design procedure for position and vibration control of simple flexible structures, i.e. two-dimensional and three-dimensional beams. In controller design, the feature of mathematical expression is utilized, which is derived by the use of Absolute Nodal Coordinate Formulation (ANCF). Numerical example is demonstrated for the validation of the proposed method and application to actual flexible space structure is discussed.

1. Introduction

In last decade, deployment mechanisms with extremely flexible material are often employed for spacecraft in order to achieve large or long structure in orbit. For example, a solar power sail IKAROS [1] was stowed inside of payload fairing during launch, which diameter was less than 1.6 [m] and height is less than 1.0 [m]. Then, IKAROS was deployed into membrane structure in orbit, which had a diagonal distance of 20 [m] and a thickness of 0.0075 [mm]. S310-36 project [2] demonstrated a deployment of a large mesh antenna which had a triangle shape with the one side's length of over 10 [m] and was made of thin strings of less than 1 [mm] diameter. It was also stowed in the fairing of rocket with diameter of about 30 [cm] during launch and deployed in orbit. Except for above two examples, there were several spacecraft which used extremely flexible structure, i.e. Kukai [3], S-520-25 [4], and so on. Generally, it is difficult to demonstrate such deployment behaviors on the ground before their operation in orbit. Therefore, a lot of studies about numerical analysis have been done from the past. Furthermore, the controls of such a structure are also quite important for practical use of the spacecraft with extremely flexible structure, and assured control method is required strongly.

Absolute Nodal Coordinate Formulation (ANCF) [5], which is a kind of nonlinear finite element method, has been studied and developed for the flexible multibody systems with large deformation and large rotation in the past decade. In that formulation, the deformation displacements and slopes are described with respect to global coordinate system and such a treatment enables the method to represent the rigid body motions precisely though the conventional finite element methods show incorrectness of rigid body motion.

A lot of researchers have studied various topics on ANCF. However, almost all studies for ANCF are dedicated to the improvement of analytical ability of flexible multibody system's behavior [6] [7], and there are few studies which extract controllers from the mathematical expressions derived by ANCF nor utilize the advantage of the mathematical expression of ANCF in order to control multibody flexible systems.

On the other hand, authors have proposed a controller design procedure which utilizes the structure of the mathematical expression derived by ANCF [8]. The proposed procedure utilizes the structure of L1-T1 model [9] which is the one of ANCF method and based on continuum mechanics. Some assumptions and manipulation on the mathematical expression converts it into a linear system with several uncertainties. Therefore, it is easy to apply μ -synthesis, which is one of the robust control design framework, to the obtained equation. In the author’s study, two-dimensional and three-dimensional flexible beams are employed to investigate and study the
proposed controller design procedure. Then, numerical analyses have demonstrated the validity of the proposed procedure. Furthermore, original procedure has problem on the effectiveness of controller design procedure and authors have also proposed a dimension reduction method for the proposed controller design procedure in order to solve the problem of inefficiency. The validity of the proposed dimension reduction method has been also shown by numerical analyses. The proposed method including controller design procedure and dimension reduction can be applied to a system of which mathematical expression has same structure with that of aforementioned two-dimensional and three-dimensional flexible beams, then main aim of this study is to discuss the applicability of the proposed method to other multibody flexible system for space use.

This paper is organized as follows. In Section 2, overview of ANCF and proposed method is introduced briefly. In section 3, a two-dimensional beam with tip mass is employed as controlled object in order to discuss the applicability of the proposed method. In section 4, further investigation is carried out about the applicability of our proposed method. Finally, conclusions are described in Section 5.

2. Overview of Proposed Controller Design Method Based on the Mathematical Model Derived by ANCF

In order to introduce the overview of ANCF and proposed method, two-dimensional beam is employed in this chapter as an example.

2.1 ANCF (Absolute Nodal Coordinate Formulation)

As Fig.1 shows, ANCF employs nodal coordinates which are defined in global coordinate system, and that coordinates consist of displacements and slopes of their nodes. In the case of elements of two-dimensional beam, nodal coordinate vector is described as follows.

\[ \mathbf{e}_i = [e_{i1}, e_{i3}, e_{i4}, e_{i5}, e_{i6}, e_{i8}]^T \]  \hspace{1cm} (1)

\[ e_{i4} = \frac{\partial X}{\partial X_{k_{i-1}}} \bigg|_{k_{i-1}} \hspace{1cm} e_{i5} = \frac{\partial Y}{\partial X_{k_{i-1}}} \bigg|_{k_{i-1}} \hspace{1cm} e_{i6} = \frac{\partial X}{\partial X_{k_{i-1}}} \bigg|_{k_{i-1}} \hspace{1cm} e_{i8} = \frac{\partial Y}{\partial X_{k_{i-1}}} \bigg|_{k_{i-1}} \]  \hspace{1cm} (2)

Introducing a shape function \( S_i \) which consists of polynomials, global coordinate \( r_j \) of a point on the element is given as

\[ r_j = S_i \mathbf{e}_i . \]  \hspace{1cm} (3)

Using Eq. (3), kinetic energy and potential energy are derived. Furthermore, applying Lagrange equation to the derived kinetic and potential energy, and considering constraints between elements, mathematical expression of two-dimensional flexible beam is given by DAE form as

\[ \begin{bmatrix} M & C_q \left[ \frac{\partial}{\partial t} \right] \mathbf{q} \\ C_q & 0 \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q} \\ \gamma \end{bmatrix} \]  \hspace{1cm} (4)

where \( \mathbf{q} = [e_1, e_2, \cdots, e_8]^T \), \( C_q \) is constraint equations, \( C_q = \frac{\partial C}{\partial \mathbf{q}} \), \( \lambda \) is Lagrange multiplier, \( M \) is inertia matrix, \( \mathbf{Q} \) is stiffness and external force term, \( \gamma \) is the term related to the second time derivative of \( C \) and \( N \) is the number of element. Due to the special feature of ANCF, \( M \) becomes constant matrix and stiffness term has strong nonlinearity.

2.2 Controller design procedure

As mentioned before, two-dimensional beam is employed for introduction of the proposed method, then control objective as shown in Fig.2 is introduced.

The control object consists of two-dimensional flexible beam and one end is fixed by pinned-support and control torque is applied to that end. Furthermore, control objective is to move the flexible beam from an initial position to a target position and to suppress residual vibrations of the flexible beam, and for simplicity target state is defined on the positive X-axis as Fig. 2 shows.

![Fig. 1 Two dimensional beam (Upper figure) and its i-th element and nodal coordinate by ANCF (Lower figure)](image)

![Fig. 2 Controlled object](image)
Employing L1-T1 model for derivation of Eq.(4) and eliminating $\lambda$ from Eq.(4) by velocity transformation matrix [10], the mathematical expression of the system is given by

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}_r \mathbf{q} + \sum_{i=1}^{N} \delta_i(\mathbf{e}_i) \mathbf{K}_L \mathbf{q} = \mathbf{Q}$$  \hspace{1cm} (5)

where $\mathbf{q}$ is coordinate vector after elimination of $\lambda$ from Eq.(4) and consists of independent coordinates, $\mathbf{M}$ is inertia matrix, $\mathbf{K}_r$ is transverse stiffness matrix, $\delta_i(\mathbf{e}_i) \mathbf{K}_L$ is longitudinal stiffness matrix, $\delta_i$ is longitudinal strain and $\mathbf{Q}$ is external force term including control input. Note that elimination of $\lambda$ corresponds to the elimination of dependent coordinate, that is, Eq.(5) has only independent coordinate.

Extracting the equation about $Y$ dynamics from Eq.(5) and supposing the range of longitudinal strain, i.e. $|\delta_i(\mathbf{e}_i)| < \beta$, following equation is obtained:

$$\mathbf{M}_Y \ddot{\mathbf{q}}_Y + \mathbf{K}_{Yr} \mathbf{q}_Y + \sum_{i=1}^{N} \Delta_i \beta_i \mathbf{K}_{YL} \mathbf{q}_Y = \mathbf{Q}_Y$$  \hspace{1cm} (6)

where suffix $Y$ means that the parameter is related to the dynamics of $Y$ direction and $\Delta_i$ is the parameter which satisfies $|\Delta_i| < 1$. Then, we can assume that Eq.(6) is linear system with $N$ uncertainties. Therefore, controller design framework of $\mu$ synthesis [11] can be applicable to the system. Detail of the procedure is omitted due to space limitation and their details are given in the reference [8].

### 2.3 Dimension reduction

Moving the third term in the left hand side of Eq.(6) to the other side and assuming obtained right hand side to be external force, following expression is obtained

$$\mathbf{M}_Y \ddot{\mathbf{q}}_Y + \mathbf{K}_{Yr} \mathbf{q}_Y = \tilde{\mathbf{Q}}$$  \hspace{1cm} (7)

where $\tilde{\mathbf{Q}} = \mathbf{Q}_Y - \sum_{i=1}^{N} \Delta_i \beta_i \mathbf{K}_{YL} \mathbf{q}_Y$. Eq.(7) can be assumed linear system with external force and it is possible to apply dimension reduction method, and Component Mode Synthesis (CMS) is employed for the dimension reduction of Eq.(7). Some additional manipulations are required before the reduction is applied to Eq.(7) so that the reduction becomes effective. Procedures of the manipulation are omitted due to space limitation, and details of those are given in the reference [12]. Applying the same procedure described in Section 2.2 after the dimension reduction of the model, controller is derived more effectively than the controller derived from the model without dimension reduction.

### 3. An Example of Expansion of the Proposed Method

The applicability and validity of the proposed method has been confirmed in the case of two and three dimensional flexible beam. However, such a simple structure is not useful for space applications from a practical point of view. In general, spacecraft which has flexible components consists of several rigid bodies and flexible bodies.

Therefore, a system shown in Fig. 3 is employed as a typical object in order to investigate the applicability of the proposed method to practical application for space use. The system in Fig.4 has two rigid bodies and they are attached to the both ends of one two-dimensional flexible beam. One rigid body is fixed by pinned support and control torque is applied around that fixed point. Spacecraft does not have fixed point in orbit, therefore such an assumption on fixed point is contradiction. However, translational dynamics can be ignored as long as the speed of motion is small and then aforementioned assumption can be introduced without loss of generality when the applicability of the proposed method is discussed.

![Fig. 3 Controlled object with rigid and flexible bodies](image)

The control objective is to move the system from an initial position to a target position and to suppress residual vibrations of the flexible beam, and for simplicity target state is defined on the positive X-axis as Fig. 3 shows.

First of all, mathematical expression i.e. L1-T1 model, of the system is derived for controller design by the proposed method. In order to describe the rigid bodies attached to the both sides of flexible beam, one element is used for each rigid body and they are connected to flexible beam by constrained equation. Supposing the number of element for the system is $N$, and element number $i=1$ and $i=N$ are assigned to rigid bodies. In the following, mathematical expression for each body is shown.

- **Rigid body fixed by pinned support ($i=1$)**
  
  Because we assume that the rigid body is a kind of flexible beam, ANCF for two-dimensional flexible beam can be applied for the derivation of mathematical expression. As well as Section 2.2, L1-T1 model is used to describe the rigid body and it is given by
  
  $$\mathbf{M}_1 \ddot{\mathbf{e}}_1 + \mathbf{K}_{1r} \mathbf{e}_1 + \delta_1(\mathbf{e}_1) \mathbf{K}_{1L} \mathbf{e}_1 = \tilde{\mathbf{Q}}_1.$$  \hspace{1cm} (8)

- **Flexible beam ($i=2, 3, \cdots, N-1$)**
  
  As well as Section 2.2, The mathematical expression for each element of flexible beam is given by
Rigid body fixed at the tip of flexible beam \((i=N)\)  
As well as the rigid body fixed by pinned support, the mathematical expression of the rigid body fixed at the tip of flexible beam is given by

\[
\text{M}_i \ddot{\mathbf{e}}_i + \mathbf{K}_i \mathbf{e}_i + \mathbf{Q}_i \mathbf{e}_i = 0 \quad \text{for} \quad (i = 2, 3, \cdots, N-1).
\] (9)

\[
\text{M}_i \ddot{\mathbf{e}}_i + \mathbf{K}_n \mathbf{e}_i + \mathbf{Q}_i \mathbf{e}_i = 0. \quad \text{(10)}
\]

In the above expressions, each parameters are defined for the \(i\)-th element. \(\mathbf{e}_i\) is nodal coordinate, \(\text{M}_i\) is inertia matrix, \(\mathbf{K}_i\) is transverse stiffness matrix, \(\mathbf{Q}_i\) is external force term including control input for the first element.

Considering constraints between each element, mathematical expression is derived by DAE as Eq. (4), and applying velocity transformation matrix to the obtained DAE, mathematical expression about independent coordinate variable is derived as following equations.

\[
\mathbf{M}_i \ddot{\mathbf{q}}_i + \mathbf{K}_i \mathbf{q}_i + \mathbf{Q}_i = 0 \quad \text{for} \quad (i = 2, 3, \cdots, N-1)
\] (11)

where \(\mathbf{q}_i\) consists of independent coordinates, \(\mathbf{M}_i\) is inertia matrix, \(\mathbf{K}_i\) is transverse stiffness matrix, \(\mathbf{Q}_i\) is external force term including control input. It is clear that Eq. (5) and Eq. (11) have the same structure, therefore the proposed method can be applied and the control objective of the system shown in Fig. 3 can be satisfied.

In order to validate the aforementioned discussion, numerical analysis is performed about the closed loop system with the controller derived by aforementioned process. Parameters given in Table 1. Fig. 4 shows the time history of the tip rigid body. As the results shows, the system is stabilized at target position, i.e. \(+X\) axis and the validity of the derived controller is shown, and it also shows that the applicability of the proposed method to more practical object for space use. Note that the performance of the controller seems to be no-excellent and there is possibility to improve the performance. But the purpose of the paper is to discuss the applicability, so higher control performance is not pursued here.

<table>
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<tr>
<th>Table 1. Parameters for numerical analysis</th>
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<tr>
<td>Parameter</td>
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<tr>
<td>Length of the beam</td>
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<td>Height of the beam</td>
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<td>Thickness of the beam</td>
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<td>Material of the beam</td>
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<tr>
<td>Mass of rigid body</td>
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<tr>
<td>Inertia moment of pinned rigid body</td>
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4. Study on the extension of the proposed method to more complex structure

As long as an expression which has same structure with Eq. (5) is obtained, proposed method can be applied to other system. Mathematical expressions for any systems derived by multibody dynamics are given by DAE as Eq. (4). Therefore, condition of the applicability of the proposed method to other system with flexible element, e.g. Fig. 5, depends on the possibility of conversion of DAE to a mathematical expression which has same structure with that of Eq. (5). Hereinafter, the condition is called “Conversion Condition”. In order to investigate the Conversion Condition, conversion of DAE by the use of velocity transformation matrix is shown in the following.

The equation after elimination of dependent coordinates is given by

\[
\mathbf{B}^{\top} \mathbf{M} \ddot{\mathbf{q}}_i - \mathbf{B}^{\top} \mathbf{Q} - \mathbf{B}^{\top} \mathbf{M} \ddot{\mathbf{q}}_i = 0 \quad \text{(12)}
\]

where \(\mathbf{q}_i\) is the independent coordinate vector and

\[
\mathbf{B}(\mathbf{q}_i, \dot{\mathbf{q}}_i, t) = \begin{bmatrix} 1 & -C_{q_i}^{-1}C_{\dot{q}_i} \end{bmatrix}
\] (13)

\[
\mathbf{H}(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i) = \begin{bmatrix} 0 & \mathbf{C}_{\dot{q}_i}^{\top} + \mathbf{C}_{\ddot{q}_i}^{\top} \end{bmatrix} \mathbf{q}_i
\] (14)

\[
\mathbf{C}_{\dot{q}_i} = \frac{\partial \mathbf{C}}{\partial \mathbf{q}_i}
\] (15)
\[ C_{q_p} = \partial C / \partial q_{dp} \]  \hspace{1cm} (16)
\[ C_i = \partial C / \partial t \]  \hspace{1cm} (17)

As Eq. (12) indicates, the structure of the system depends strongly on the constraint equation \( C \). Conversion Condition includes the condition that Eq. (12) can be converted to a expression which has same structure with Eq. (5). It is quite difficult to find a necessary and sufficient condition to achieve such a conversion. However, it is not so difficult to find some necessary conditions and following condition is focused on and discussed here in order to study the applicability of the proposed method.
\[ C = Aq = 0 \]  \hspace{1cm} (18)

As Eq. (18) shows, the condition means that the constraint equation consists of linear sum of generalized coordinates. As long as Eq. (18) is satisfied, mathematical expression as Eq. (5) is obtained. Constraints given by Eq. (18) mean simple connections of several elements, for example, pin joint, simple support and so on. If such a connection is achieved, some complex and large or long structure which consists of flexible beam can be configured, and consequently it becomes possible to control the attitude and vibration of such a structure. Mesh-like membrane as Fig. 6 is an example of achieved structure by Eq. (18). As Fig. 6 shows, it is possible to configure mesh-like structure which consists of flexible beams connected by pinned support. Therefore, there are possibilities to apply our proposed method to more complex structure like membrane. However, more complex structure results in the difficulty in the iterative calculation during controller design, even if dimension reduction method for effective iteration calculation is proposed. Furthermore, assignment of actuators is also important issue and to be investigated in order to achieve an efficient deployment structure and control of them. Consequently, development of more powerful dimension reduction method and detail study of the actuator assignment are future works.

5. Conclusion

In this paper, a controller design procedure and dimension reduction method for the procedure were focused on, which utilized the mathematical expression derived by ANCF. At first, overview of the proposed method was shown and feature of the method was discussed. Introducing an object which was assumed to be a spacecraft with flexible body for space use, applicability of the proposed method was studied and numerical analysis showed the validity of them. Furthermore, focusing on the mathematical expression of general system with flexible bodies, applicability of the proposed method was discussed and the problem of them was made clear.

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References


