Dynamics Simulation of Flexible Solar Power Satellite Using Geomagnetic Control

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Abstract

This research is concerned with a dynamics simulator of Solar Power Satellites (SPS’s) in orbit using geomagnetic control. The SPS simulator calculates the orbital motion, the attitude motion, and the structural flexibility simultaneously. For this purpose, the numerical simulator contains the dynamics of flexible multibody systems and the external force acting on the system, i.e. gravity, \( J_2 \) term perturbation, solar radiation pressure, atmospheric force, and geomagnetic force. This research tries controlling the attitude of USEF SPS using geomagnetic force, which has the configuration of gravity gradient stabilization. Controlling the magnitude of electric current in power generation and transmission panels, we apply torques to the satellite by the interaction with geomagnetism and try stabilizing the attitude. Furthermore, we remove the tethers and buses, cause of reliability decreasing, and try stabilizing the attitude of a SPS which is not stabilized by gravity gradient using geomagnetic control.

1 Introduction

In 1968, Peter E. Glaser of the United States suggested Solar Power Satellite (SPS). Since then, researches about SPS have been continued all over the world including Japan. ‘06 USEF SPS model [1] was suggested in Japan and SPS-ALPHA [2] in the United States recently. However, existing models do not have sufficient feasibility of system from the viewpoint of dynamics on orbit and control. So we examine new system designs with high feasibility.

There are two major problems for realization.

- Multi-physics simulator does not exist which analyses rigid or flexible bodies’ coupled motion of orbit dynamics and attitude dynamics including thermal deformation.
- Existing SPS models have some problems from the viewpoint of structure and control so that we have to create new feasible SPS models.

In this research, we develop multi-physics simulator and examine the engineering problem about structure and control.

First, the dynamic analysis by numerical simulations is necessary to research and develop SPS’s. For this reason, taking over the previous research [3, 4, 5, 6], we have developed the SPS simulator based on mechanism analysis software ADAMS. A SPS is modeled by a flexible multibody system in the simulator, and external forces are applied, e.g. gravity, the gravity perturbation term which called \( J_2 \) term, solar radiation pressure, air pressure, geomagnetic force, and so on. Second, in this research, we examine highly feasible satellite based on USEF SPS model. USEF SPS model’s configuration has passive gravity gradient stabilization so that this satellite’s attitude is highly stable. However, once oscillating motion is generated, it never gets smaller naturally. Then we try controlling the attitude magnetically in order to reduce the oscillating motion. In addition, USEF SPS model is tether system, which has reliability problem. We try controlling a SPS model magnetically that does not have tethers and buses.

2 Summary of the SPS simulator

Until now, we have constructed the SPS simulator described below. As shown in Fig. 1, we model a flexible satellite system by a multibody system composed of rigid and flexible bodies connected by springs and joints. As external forces to the satellite, we consider gravity, \( J_2 \), solar radiation pressure, air pressure, and the geomagnetic force.
If we take $\Sigma_I$ whose origin is at the center of the earth as the ADAMS reference coordinate-frame, we cannot run an accurate simulation due to numerical errors. Then we separate the behavior of the system into the behavior of the orbital coordinate-frame $\Sigma_O$ observed from $\Sigma_I$ and the behavior of the multibody system observed from $\Sigma_O$. The former is obtained by solving orbital dynamics of the mass center of the system. For the latter, we transform the equations of motion observed from the inertial coordinate-frame $\Sigma_I$, into those observed from $\Sigma_O$.

Because ADAMS solves the equations of motion observed from the inertial coordinate-frame, we can get the motion of the system observed from $\Sigma_O$ by applying the pseudo external forces that appear in the equations of motion observed from the orbital coordinate-frame $\Sigma_O$.

3 Summary of the analysis

3.1 Analysis objects

The analyzed USEF '06 model consists of 25 units where each unit is shown in Fig. 2.

In this analysis, we show the analysis result of 1 unit. Next, removing tethers and buses, we create a new model consisting of only one panel and show its analysis result.

The satellite is built on the geostationary orbit, and its configuration has the passive gravity gradient stabilization effect. The bus is a rigid body of $5.6 \times 10^4$ [kg] mass. A tether made of Kevlar fiber is modeled by a spring element that generates tensile force only when it is stretched. The size of a power generation and transmission panel is $500 \times 475 \times 0.02$ [m], its mass is $1.0 \times 10^6$ [kg], and is modeled by a uniform rigid plate. Its surface for transmission always faces to the earth. The standard distance between the bus and the panel is $5 \times 10^3$ [m]. In case of a Panel SPS model consisting of only the panel without bus and tethers, this satellite's configuration does not have gravity gradient stabilization effect because of its inertial moments.

3.2 Calculation of geomagnetic field

\textbf{Geomagnetic dipole} The earth has geomagnetic field and it affects magnet elements and electric current in the satellite so that geomagnetic torques and forces are generated. In this simulator, we assume that the earth is geomagnetic dipole and calculate its geomagnetic field.

Geomagnetic dipole's north and south axis is not agree with earth's rotating axis. Furthermore, the dipole's direction is always changing against the rotating axis. However, north geomagnetic pole moves only $1^\circ$ during 20 years. So we assume its position does not change in this simulation for examine the behavior of satellite's attitude. In 2014, the value of dipole moment is $7.73 \text{[T} \cdot \text{m]}$, north geomagnetic pole lies at $80.2^\circ$ north latitude and $72.5^\circ$ west longitude [7]. We use these values in this simulation.

\textbf{Geomagnetic field on orbit} Suppose that the value of dipole moment is $M_e$, geomagnetic field’s center of the earth direction’s component and tangent direction’s component are respectively $B_r$ and $B_t$ at the point away from center of the earth $r_0$ and angle $\lambda$ from south magnetic pole (Fig. 3). Then $B_r$ and $B_t$ are expressed as follows:

$$B_r = \frac{2M_e}{r_0^3} \cos \lambda \quad (1)$$

$$B_t = -\frac{M_e}{r_0^3} \sin \lambda \quad (2)$$

\textbf{Electromagnetic force} When electric current $i$ flows in a straight wire in geomagnetic field, this wire is applied electromagnetic force as follows:

$$F = i(L \times B_{\text{mag}}) \quad (3)$$

where $L$ is a wire-length vector whose magnitude is same with current-flow direction, and $B_{\text{mag}}$ is a geomagnetic field vector which passes the wire.
3.3 Equation of a rigid body on circular orbit

Equation of a rigid body is expressed as follows, whose angular velocity against the inertia reference frame is \( \omega \),

\[
I \cdot \frac{d\omega}{dt} + \omega \times (I \cdot \omega) = \tau
\]

where \( I \) is inertia tensor of the rigid body and \( \tau \) is the force applied to it.

We define the body-fixed that is right-handed coordinate frame fixed to the rigid body. At the initial attitude, its first axis is in the direction of travel and third axis is in the direction to center of the earth. Consider a rigid body whose moments of inertia are \( I_1, I_2, \) and \( I_3 \) with respect to the each axes of body-fixed frame. We assume that the rigid body’s rotation angle is very small and its attitude changes very slowly. When the rotation angles of roll, pitch, and yaw are \( \theta_1, \theta_2, \) and \( \theta_3 \) respectively, linearized euler equation of a rigid body is as follows from equation (4), where the orbital angular velocity is \( \omega_{O} \) to the earth.

\[
\begin{align*}
I_1(\dot{\theta}_1 - \omega_{O}\theta_3) + (I_2 - I_3)(4\omega_{O}^2\theta_1 + \omega_{O}\theta_3) = 0 \\
I_2\dot{\theta}_2 + 3\omega_{O}^2(I_1 - I_3)\theta_2 = 0 \\
I_3(\dot{\theta}_3 + \omega_{O}\theta_1) + (I_2 - I_1)(\omega_{O}^2\theta_3 - \omega_{O}\theta_1) = 0
\end{align*}
\]

(5)

We design control system regarding ’06 USEF SPS model as one rigid body. Then, we add geomagnetical torque to the panel and try attenuating oscillating motion of the satellite.

3.4 Optimal feedback control

State vector is taken as \( x = [\theta_1 \ \dot{\theta}_1 \ \theta_2 \ \dot{\theta}_2 \ \theta_3 \ \dot{\theta}_3]^T \). Considering only gravity of the earth as external disturbance force, state equation is as follows.

\[
\dot{x} = Ax
\]

\[
A = \begin{bmatrix} -C & -K \\ I_{3\times3} & 0 \end{bmatrix}
\]

\[
= \begin{bmatrix}
0 & 0 & \frac{l_1 - l_3 + l_2}{t_1} \omega_{O} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\frac{l_2 - l_3 - l_1}{t_3} \omega_{O} & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
\frac{4(l_3 - l_1)}{t_1} \omega_{O}^2 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{3(l_1 - l_3)}{t_2} \omega_{O}^2 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{(l_2 - l_1)}{t_2} \omega_{O}^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(7)

From the above equation, roll motion and yaw motion are coupled. Pitch motion is independent of other rotational motion. Furthermore, matrix \( C \) has no components about pitch axis. Therefore, once oscillating motion is generated, the motion does not get smaller naturally.

We try attenuating this oscillating motion using a geomagnetical control. As in Fig. 4, we build two electric wires along panel’s surfaces and \( i_1 \) and \( i_2 \) means the current which flow in these wires respectively. Now input vector is \( u = [i_1 \ i_2]^T \). Using geomagnetic torque caused by interaction between these electric current and geomagnetism, we try controlling the attitude of the satellite.

Each length of panel sides along axes of body-fixed reference frame are taken as \( L_1, L_2, \) and \( L_3 \). And components of geomagnetism \( B_{mag} \) in the body-fixed frame are taken as \( B_1, B_2, \) and \( B_3 \). In this analysis, initial position of the satellite is as in Fig. 5. However, geostationary orbit’s angular velocity is the same as the angular velocity of the earth. In the result, the relation between north magnetic pole, geographical north pole and SPS are shown in Fig. 5. Therefore, the geomagnetism passing SPS is always constant so that \( B_1 \) and \( B_2 \) take some non-zero constant value and \( B_3 \) equals to 0.

From the above, the state equation against the system including input vector is as follows.

\[
\dot{x} = Ax + Bu
\]

(8)

where

\[
B = \begin{bmatrix} B_1L_1L_2 & 0 \\ B_1L_1L_2 & 0 \\ 0 & B_2L_2L_3 \end{bmatrix}
\]

(9)

The output equation is as follows:

\[
y = Cx + Du
\]

(10)
where \( C \) is a unit matrix and \( D \) is 0.

Next, we design an optimal regulator for the state equation. For the design, we minimize below evaluation function \( J \) and determine control law.

\[
J = \int_0^\infty [y^T Q y + u^T R u] \, dt \tag{11}
\]

Input vector \( u \) is determined uniquely as follows

\[
u(t) = -R^{-1} B^T P_\infty x(t) \tag{12}
\]

where \( P_\infty \) equals to \( P \) which satisfies following Riccati equation.

\[
A^T P + PA - PBR^{-1} B^T P + C^T QC = 0 \tag{13}
\]

From the above, we can determine the control law for linearized system. Finally, we apply the control law to the SPS simulator of a nonlinear system and analyze its response of control system.

4 Analysis result

4.1 USEF SPS

We analyze the motion of '06 USEF SPS model which is rotated about roll axis by 1°. The result with no geomagnetical control is as in Fig. 6 and with geomagnetical control is as in Fig. 7. The horizontal axis represents time and the vertical axis represents rotational angle.

In the case with no geomagnetical control, the satellite keeps oscillating about the roll axis with almost constant amplitude 1°, which is given as the initial rotation angle. Because the roll motion and the yaw motion is coupled, the satellite keeps oscillating about the yaw axis with about 4° amplitude. The result suggests that these amplitudes are constant for long time analysis.

In the case with the geomagnetical control law determined by optimal regulator design, these amplitude can be attenuated gradually and the satellite becomes in standard attitude. The \( Q \) and \( R \) are as follows:

\[
Q = \text{diag} \begin{bmatrix} 1 \times 10^{19} & 1 \times 10^{19} & 1 \times 10^{19} \\ 1 \times 10^{10} & 1 \times 10^{10} & 1 \times 10^{10} \end{bmatrix} \tag{14}
\]

\[
R = \text{diag} \begin{bmatrix} 1 \times 10^2 & 1 \times 10^2 \end{bmatrix} \tag{15}
\]

Then \( i_1 \) and \( i_2 \) are as in Fig. 8. This figure suggests that sufficient current for controlling needs 400 [A]. The wire is made of copper whose density is \( 8.96 \times 10^3 \) [kg/m³] and electric resistivity is \( 2.23 \times 10^8 \) [Ωm] at 100 [°C] [8]. Cross sectional area of the wire is taken for 1.0 \times 10^{-6} [m²]. With respect to \( i_1 \), winding up its wire along panels surface about 400 times like a coil, we can reduce the value to about 1 [A]. Then mass of the wire is about \( 7.0 \times 10^3 \) [kg] which is 0.7 % of the panel. This does not matter for design of the satellite. Electric power generation of this SPS is about 80 [MW] and above electric current needs 0.7 [MW], so generated power is sufficient for controlling.

4.2 Panel SPS

We discuss about the Panel SPS. This configuration has advantage which needs no tethers. However, for inertia moment of only this panel, restoring torque by gravity gradient does not work. Therefore, more geomagnetical torque is needed for stabilising attitude so that more electric current is needed. Then, we put an upper limit of current. We use the control designed by the optimal regulator. The result is as in Figs. 9 - 11. The upper limit is \( 2.0 \times 10^7 \) [A]. If a smaller limit is used, the attitude of the satellite is derivated largely. The \( Q \) and \( R \) for the optimal regulator is as follows:

\[
Q = \text{diag} \begin{bmatrix} 5 \times 10^{27} & 5 \times 10^{27} & 5 \times 10^{27} \\ 1 \times 10^{18} & 1 \times 10^{18} & 1 \times 10^{18} \end{bmatrix} \tag{16}
\]

\[
R = \text{diag} \begin{bmatrix} 1 \times 10^{18} & 1 \times 10^9 \end{bmatrix} \tag{17}
\]
where $R$ is determined in order to increase contribution of $i_2$ against $i_1$. Thickness of the panel is only 2 [cm] because this base model is USEF SPS. Therefore, this tends to increase the controlling current for the torque around yaw axis. Solving this problem, we consider to stretch the length $L_3$ in Fig. 4 which contributes to $i_2$. The result suggests that $i_2$ needs $2.0 \times 10^7$ [A]. We should reduce the value to $2.0 \times 10^5$ [A] for controlling only with the generated electric power. This needs $L_3$ length about 200 [m]. If the panel becomes this thickness, it increases inertia moment greatly. So only the wire is built apart from the panel, hanged by some light-weight structure. In order to decrease the value of electric current to 10 [A], we need winding up the wire $1 \times 10^3$ times and its mass is about 0.87 % of the panel.

5 Conclusion

In this research, we have developed SPS dynamics analysis simulator that analyzes the behavior of large satellites in orbit. Using this simulator, we have analyzed the 1 unit model of ’06 USEF SPS as a specific example and examine the control system geomagnetically. First, we have shown that we can attenuate the oscillation of USEF SPS magnetically which has gravity gradient stabilization effect. USEF SPS employs tether system which brings some problems about system reliability under construction and operation. Removing tethers and bus, the Panel SPS no longer has gravity gradient stabilisation effect. Second, we have shown that we can control the attitude of the Panel SPS, which does not have gravity gradient stabilisation effect.

Reference