Analysis for Nutation in One Wheel Control Mode of HAYABUSA2

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Abstract

In order to save the running time of reaction wheels, HAYABUSA2 is stabilized by one-axis bias-momentum wheel which is called One Wheel Control (OWC) mode. In the one of actual operation when the spacecraft is in this control mode, the nutation was diverged gradually. This phenomenon was also seen in the HAYABUSA operation, and was analyzed by means of stability analysis. In this paper, we discuss that the result of stability analysis for HAYABUSA2, and also construct the practical attitude dynamics model in the evaluation with the actual flight data.

はやぶさ２の１軸ホイール制御モードにおけるニューテーション運動の分析

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概要

現在、目的天体へ向けて航行中の「はやぶさ２」では、リアクションホイールの延命のために Z 軸の RW のみによる姿勢制御 (OWC:One Wheel Control) を行っている。この制御方式に移行後の実際の運用において、ニューテーションが発散するような傾向が見られた。先行機である「はやぶさ」においても、同制御方式での運用中に同様の現象が見られており、安定性解析が行われていた。「はやぶさ２」においても同様の安定性解析を行い、ニューテーションの収束・発散の条件を調査すると共に、実データと整合性が取れた運動モデルを構築する。

1 introduction

Last year, the asteroid explorer HAYABUSA2 was launched on December 3rd, 2014. HAYABUSA2 attitude is controlled by 4 reaction wheels. In order to save running time of reaction wheels, HAYABUSA2 is stabilized by one-axis bias-momentum wheel which is called One Wheel Control(OWC) mode, saving other wheels. For the one wheel control mode operation, sun tracking motion is utilized. At one wheel control mode, 3 axes are kept stable because one axis is stabilized through PD control of the angular momentum with the momentum wheel, and the other two axes are stabilized by gyroscopic stiffness from the momentum wheel.

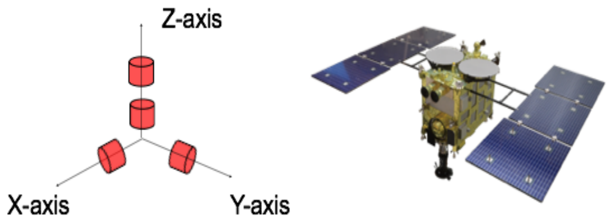


Figure1 HAYABUSA2 reaction wheels

At one wheel control mode operation, the wheel axis sways. It is called nutation. The nutation motion becomes larger, it is called nutation divergence. Nutation divergence happened to the preceding explorer HAYABUSA when HAYABUSA was controlled by only one reaction wheel. Basically, nutation is damped due to fuel sloshing. In the actual one HAYABUSA2 operation, notation did not converge. That is to say, nutation divergence happened gradually. This phenomena is caused by product of inertia. The wheel axis is different from the principal axis. Thus wheel control affects the other two axes dynamics. It causes nutation divergence.

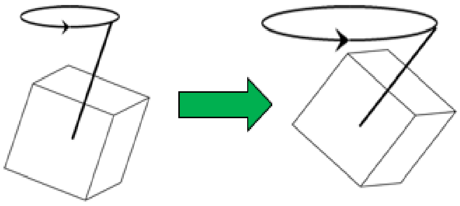


Figure2 Nutation divergence

This paper analyzes the dynamics of HAYABUSA2 and the condition of nutation divergence.

2 Theory

2.1 Cause of nutation

HAYABUSA2 has a little products of inertia. To analyze the dynamics of HAYABUSA2, consider the spacecraft which is controlled by only Z-axis. Consider the condition thant Z-axis angle and an-

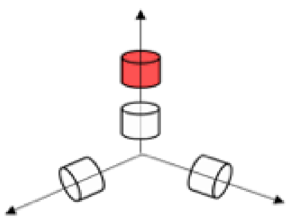


Figure3 Control wheel

gular velocity is used for feedback control of Z-axis wheel, and at initial condition, nutation is happening. We define an inertial reference frame $[a_I]$, and a body fixed wheel-axis reference frame $[a_I]$.

Euler equation in $[a_B]$ is written as

$$\frac{d}{dt}(\boldsymbol{J_B}\cdot\boldsymbol{\omega_{B,I}}+\boldsymbol{H_w})+\boldsymbol{\omega_w}\times(\boldsymbol{J_{B,I}}\cdot\boldsymbol{\omega_B}+\boldsymbol{H_w})=\boldsymbol{T} \tag{1}$$

where $\boldsymbol{J_B}$ is the inertia matrix in $[a_B]$, $\boldsymbol{\omega_{B,I}}$ is angular velocity vector which is described in $[a_B]$ relative to $[a_I]$ and H_w is the angular momentum vector of wheel. From (1), angular velocity vector is written as

$$\dot{\boldsymbol{\omega}}_{\boldsymbol{B.I}}=\boldsymbol{J_B^{-1}}\cdot\{-\dot{\boldsymbol{H}}_{\boldsymbol{w}}-\boldsymbol{\omega_{B.I}}\times(\boldsymbol{J_B}\cdot\boldsymbol{\omega_{B.I}}+\boldsymbol{H_w})+\boldsymbol{T}\} \tag{2}$$

Paying attention to the first term $\boldsymbol{J_B^{-1}}\cdot\dot{\boldsymbol{H}}_{\boldsymbol{w}}$, it is written as

$$\boldsymbol{J_B^{-1}}\cdot\dot{\boldsymbol{H}}_{\boldsymbol{w}}=\frac{1}{\det\boldsymbol{J}}\times \begin{bmatrix} J_{yy}J_{zz}-J_{yz}^2 & J_{yz}J_{xz}-J_{xy}J_{zz} & J_{xy}J_{yz}-J_{yy}J_{xz} \\ J_{yz}J_{xz}-J_{xy}J_{zz} & J_{xx}J_{zz}-J_{xz}^2 & J_{xy}J_{xz}-J_{xx}J_{yz} \\ J_{xy}J_{yz}-J_{yy}J_{xz} & J_{xy}J_{xz}-J_{xx}J_{yz} & J_{xx}J_{yy}-J_{xy}^2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \tau_z \end{bmatrix} \tag{3}$$

where $\boldsymbol{J_B}=\begin{bmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{xy} & J_{yy} & J_{yz} \\ J_{xz} & J_{yx} & J_{zz} \end{bmatrix}$, and $\dot{\boldsymbol{H}}_{\boldsymbol{w}}$ is the control torque from reaction wheel, τ_z is the magnitude of $\dot{\boldsymbol{H}}_{\boldsymbol{w}}$. Seeing this equation, the Z-axis control torque from reaction wheel affects the other two axis angular velocity vector since J_B has the inertia of products. And kinematics equation with quaternion is written as

$$\begin{aligned}\dot{\boldsymbol{q}}_{\boldsymbol{B}}&=\frac{1}{2}\begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \end{bmatrix}\cdot\boldsymbol{\omega}_{B.I} \\ &=\frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{q_B})\cdot\boldsymbol{\omega}_{B.I}\end{aligned} \tag{4}$$

where q_B is the quaternion of $[a_B]$. From this equation, we can see the each angular velocity vector affects the other two axes dynamics. This causes the nutation divergence and convergence.

2.2 Stability analysis

For simplicity, neglecting disturbance torque, euler equation in body-fixed wheel axis frame is written as

$$\boldsymbol{J_B} \cdot \dot{\boldsymbol{\omega}}_{\boldsymbol{B.I}} + \dot{\boldsymbol{H}}_{\boldsymbol{w}} + \boldsymbol{\omega_{B.I}} \times (\boldsymbol{J_B} \cdot \boldsymbol{\omega_{B.I}} + \boldsymbol{H_w}) = 0 \tag{5}$$

And bias-momentum reaction wheel is controlled with PD control using the angular velocity and the quaternion of $[a_B]$. That is to say, feedback value for the PD control are the z-component of the angular velocity and the 3rd component of quaternion in $[a_B]$, or $\omega_{B.Iz}, q_{B3}$. So we can write the magnitude of control torque $\dot{h}$ as

$$\dot{h} = k_d \omega_{B.Iz} + k_p q_{e3} \tag{6}$$

.

To obtain the characteristic equation, we linearize equation (5), (6) by restricting a range of motion. Consider the wheel rotation as near its zero, and attitude is near the target attitude. Namely we suppose

$$\begin{aligned} \boldsymbol{q_B} &= \boldsymbol{q_0} + \boldsymbol{\delta q} \\ \boldsymbol{\omega_{B.I}} &= \boldsymbol{\delta\omega} \\ h &= h_0 + \delta h \\ \boldsymbol{q_0} &= \begin{bmatrix} q_{01} \\ q_{02} \\ q_{03} \end{bmatrix} \end{aligned} \tag{7}$$

.

By substituting these equations (7), (8), (9) and (10) to the equations (5), (6), we get the linear equation

$$\frac{d}{dt}\begin{pmatrix} \boldsymbol{\omega_{B.I}} \\ \boldsymbol{q_B} \end{pmatrix} = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{q_0}) & 0 \end{pmatrix} \tag{8}$$

where $\boldsymbol{A} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k_1 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k_2 \end{pmatrix}$.

Now we get the characteristic equation

$$\det\begin{pmatrix} s\boldsymbol{E} & -\boldsymbol{B} \\ -\frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{q_0}) & s\boldsymbol{E} \end{pmatrix} = 0 \tag{9}$$

2.3 Characteristic equation

Characteristic equation can be written as

$$s^2(s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4) = 0 \tag{10}$$

When s=0, eigenvectors about s=0 are

$$x_1^T = \begin{pmatrix} 0 & 0 & 0 & q_1 & 0 & 0 \end{pmatrix} \tag{11}$$

$$x_2^T = \begin{pmatrix} 0 & 0 & 0 & 0 & q_2 & 0 \end{pmatrix} \tag{12}$$

This means q_1, q_2 are constant with s=0. So s=0 does not affect the stability of dynamics and we can consider the stability of spacecraft by the other 4 eigenvalues.

$$s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 \tag{13}$$

Using Furwitz stability criterion, we get the stability condition

$$cq_{01} + dq_{02} > 0 \tag{14}$$

where c, d is the constant composed of k_d, k_p, h_0, J_{ij}. Using the latest HAYABUSA2 parameter obtained from ground experiment, we get the stability condition as Figure4.

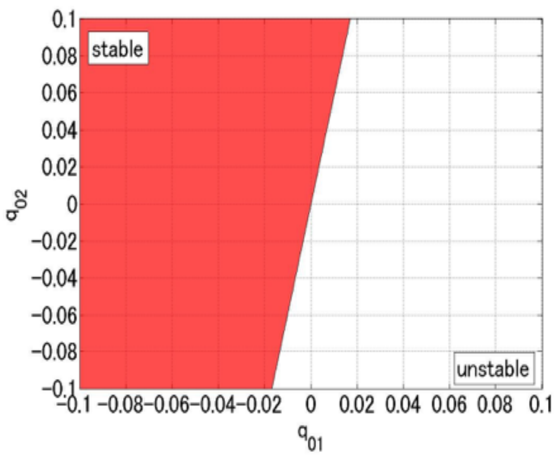


Figure4 Condition of stability at latest ground experiment data of HAYAVUSA2

3 Calculation result

3.1 Stability analysis

We calculate the HAYABUSA2 model dynamics with nutation at initial condition. We set 4 kinds of initial condition of q_1, q_2 around the border in figure 4.

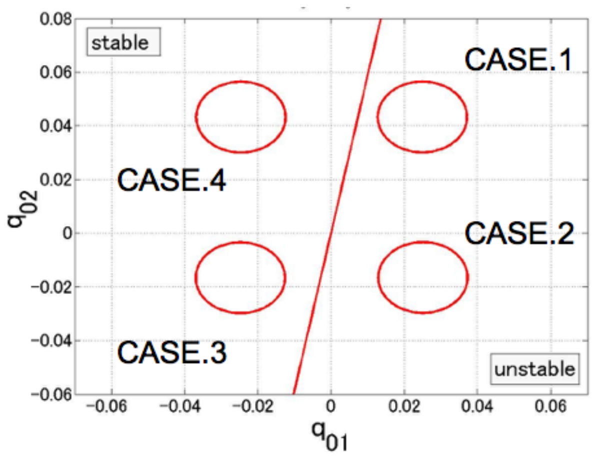
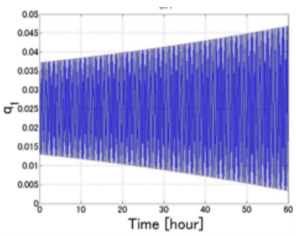
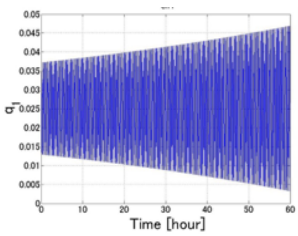


Figure5 Initial conditions

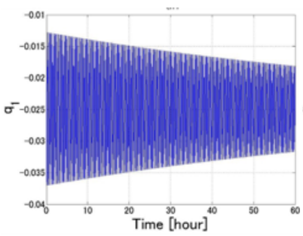
Then, nutation converged at stable area, CASE 3, CASE 4. And CASE1, CASE2, at unstable area, nutation diverged gradually.



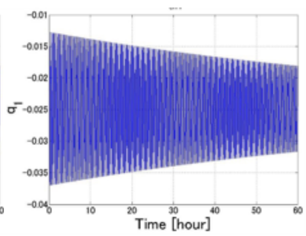
(a) CASE.1



(b) CASE.2



(c) CASE.3



(d) CASE.4

Figure6 Calculation result of q_1

3.2 Moment of inertia

We use the latest parameter of HAYABUSA2, but with actual operations moment of inertia can change. So, we see the sensitivity of moment of inertia and with flight data, we estimate the moment of inertia.

About J_{xx}, we use the latest parameter for the other moment of inertia but J_{xx}. Latest J_{xx} is about 400 $[kg \cdot m^2]$. And change the J_{xx} value from $J_{yz} - 50$ to $J_{yz} + 50$. Similarly, we change the value of J_{yz} from $J_{yz} - 5$ to $J_{yz} + 5$. Latest J_{yz} is about 1 $[kg \cdot m^2]$.

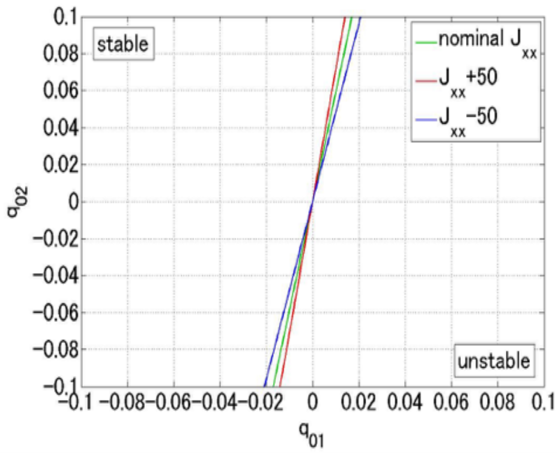


Figure7 Stability condition sensitivity of J_{xx}

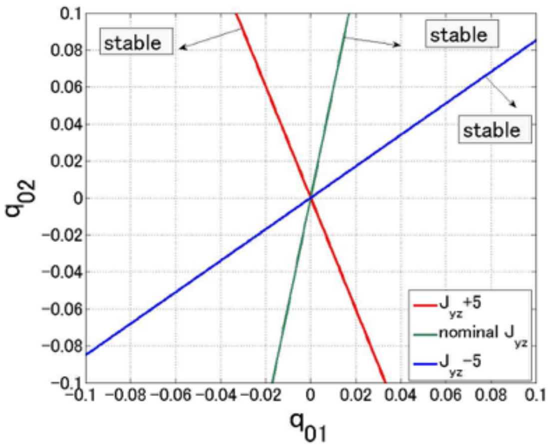


Figure8 Stability condition sensitivity of J_{yz}

Matching the flight data with stability condition about latest parameter, it doesnt suit well. The reason can be expected as the error about the moment of inertia. We tuned the products of inertia and estimated the actual moment of inertia. We set $J_{xy} \pm 0$, $J_{xz} - 8.86$ and $J_{yz} - 0.96$.

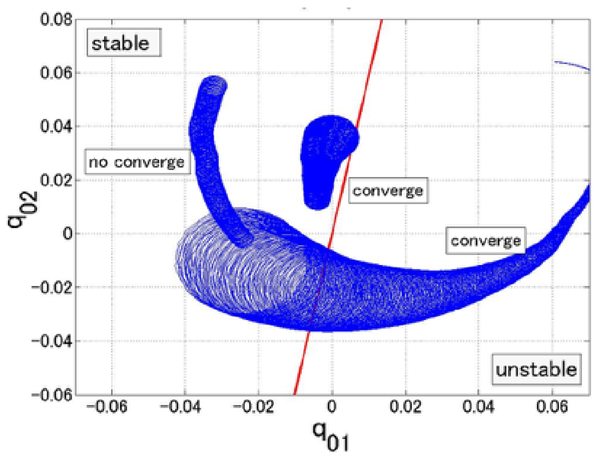


Figure9 Stability condition from latest ground experiment and flight data

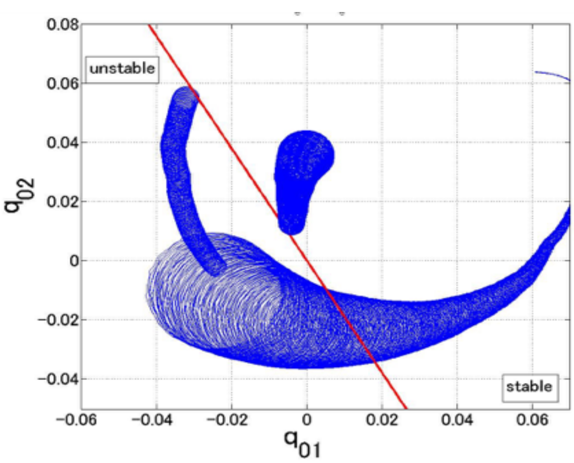


Figure10 Matching stability condition with flight data

4 Conclusion

To analyze the stability of nutation motion, we get the characteristic equation with quaternion. And using Furwitz stability criterion, we get the stability condition. From the stability analysis, we get the stable area for the HAYABUSA2 model obtained from latest data by ground experiment. And by simulation, we confirm the stability area by simulation. But flight data does not suit. Since products of inertia is dominant over moment of inertia coefficient on stability condition, we tune the products of inertia matching flight data.

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