Enhancement of Earth Escape Energy by use of Sequential Lunar Swing-by on a Hyperbolic Orbit

Shuntaro Suda (Graduate school of Engineering, Hokkaido University), Yasuhiro Kawakatsu (JAXA/ISAS), Shujiro Sawai (JAXA/ISAS), Harunori Nagata (Faculty of Engineering, Hokkaido University), Tsuyoshi Totani (Faculty of Engineering, Hokkaido University)

Abstruct: In the modern space development, the opportunities of low-cost space exploration increase by the technical improvement of small scale spacecraft and launch vehicle. Therefore, the design method to construct Earth escape trajectory with high flexibility in the boundary condition such as escape velocity, direction and timing is strongly demanded. In this paper, sequential lunar swing-by on a hyperbolic orbit is conducted to enhance the Earth escape energy and to change escape direction which could lead a spacecraft to further destinations.

連続して月にスイングバイする双曲線軌道を用いた地球脱出エネルギーの増加

須田俊太郎（北大），川勝康弘（JAXA/ISAS），澤井秀次郎（JAXA/ISAS）
永田晴紀（北大），戸谷剛（北大）

概要：近年，小型ロケットや小型宇宙機を用いた高頻度・低コストの深宇宙探査を促進する動向がある．それを背景として，地球影響圏を脱出する際の境界条件（脱出速度・脱出方向・脱出時期）に幅広い自由度を持たせた軌道設計の重要性が増している．本研究では，連続して月スイングバイする双曲線軌道を用いることで，地球脱出エネルギーの増加，脱出方向の変更を行うことができ，幅広い目的地に応じた地球影響圏の脱出軌道が設計できることを示す．

I. Introduction

In the modern space development, there has been a growing interest in demonstration of challenging technologies by using small-scale spacecrafts because they will have potential for successful transition of new technologies into larger mission projects even if there exist several technical risk factors for low-cost and short development time. In addition, small-scale launch vehicles such as Epsilon rocket (JAXA/IHI) have also been developed to reduce launch costs and attain much shorter launch preparation. Owing to these technological revolutions, the realization of small-scale and low-cost deep space mission should be strongly demanded so that it may promote high-frequent and challenging deep space exploration.

Therefore, we need a design method to construct Earth escape trajectory highly flexible in boundary escape condition but still fulfill the strict demands on fuel consumption in various small-scale deep space missions. For this reason, We focus on “Swing-by (Gravity assist)” well known as an effective way to increase or decrease S/C speed and redirect its path using gravity of a planet or a natural satellite without fuel consumption theoretically. In Particular, “Lunar Swing-by” is exceptionally useful to obtain desirable Earth escape conditions such as escape direction, velocity and timing. Actually, multiple lunar gravity assists were implemented in the several missions. For example, the ICE (International Cometary Explorer) mission was one of the first missions to perform multiple lunar flybys to get its energy and finally intercept the comets Giacobini-Zinner and Halley in 1985-86 [1]. Afterwards, the Nozomi (Mars Exploler) mission conducted lunar flybys twice and a powered Earth flyby to boost its energy and departed to Mars [2]. As stated above, lunar swing-by is beneficial for orbital maneuver on Earth escape trajectory and has been used in the several real missions. However, there have been few reports about systematic design method of Earth escape trajectories by use of lunar swing-by. Thus, most of the missions performed lunar gravity assists by looking at the success of past experience in a similar mission.

Previous research casted a spotlight on practical use of solar-perturbed moon-to-moon transfers in CRTBP, characterized and classified them that were stored in a database [3]. The mission designers can explore the solution space easily looking into the database, and generate accurate initial guesses for lunar assisted escape trajectories. Although these are helpful, it is still a big challenge to propose systematic design method to construct lunar assisted escape trajectories as there remains intricacy. Thus, the purpose of this study is to store numerous solutions of lunar assisted trajectories systematically and summarize them for practical use. This paper specifically focuses on moon-to-moon transfer with sequential lunar swing-by because of its drastic energy enhancement, and aims at storing the lunar assisted escape trajectories in a database for practical use.

II. Moon-to-Moon Transfer

In this chapter, we address the design method to construct moon-to-moon trajectory. we determine the motion of a spacecraft interacts only with the Earth, and assume the following to simplify the dynamics.

A) The Moon is in a circular orbit around the Earth:
as the lunar orbital eccentricity is nearly zero.

B) The Moon is massless:
as the Moon's noticeable influence is substantial only for small portion of the transfer. Moreover, this assumption corresponds with zero-patched-conic method that models lunar swing-by as instantaneous change in lunar v_∞ direction (**III**).

C) All motion is planar:
as the lunar orbit inclination with respect to the ecliptic plane is fairly low.

A conventional method is to use a standard two body Lambert solver to solve for a desired moon-to-moon arc (neglecting the gravitational influence of the Sun) with specified time of flight. Fig.1 shows the geometry of the Earth-Moon-S/C system. we fix the 1st lunar encounter position $\boldsymbol{r_{Min}}$ with respect to the Earth. To solve Lambert's problem, we additionally need 2nd lunar encounter position $\boldsymbol{r_{Mout}}$ and the time of flight TOF between two encounter positions.

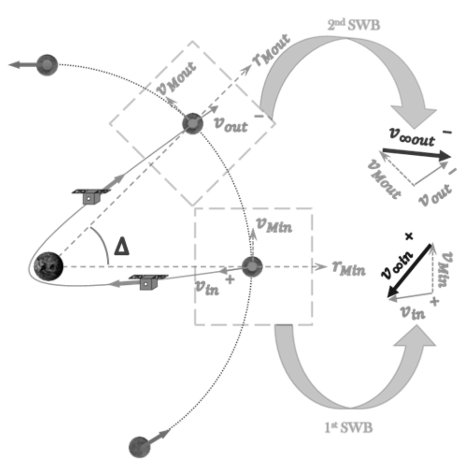

Fig.1 Geometry of the Earth- Moon-S/C system With a Moon-to-Moon Trajectory

The lunar orbital (tangential) velocity v_M with respect to the Earth can be calculated as follows:

$$v_M = \frac{\mu_E}{a_M} = const. \qquad (1)$$

Where μ_E is gravitational parameter of the Earth and a_M is the semi-major axis of the lunar orbit, both of them are constant. In assumed mathematical model, it is obvious that v_M can be regarded as mean motion. Therefore, we can derive TOF only by determining the transfer angle of $\boldsymbol{r_{Mout}}$ for $\boldsymbol{r_{Min}}$ defined as Δ:

$$TOF = \frac{a_M \times \Delta}{v_M} \qquad (2)$$

Once we get $\boldsymbol{r_{Min}}$ $\boldsymbol{r_{Mout}}$ TOF by determining design variable Δ, we solve lambert's problem to obtain S/C velocities $\boldsymbol{v_{in}}^+$ and $\boldsymbol{v_{out}}^-$ with respect to the Earth at each encounter position ("+" indicates "after" lunar swing-by and "−" indicates "before" lunar swing-by). The past literature gives more details on the Lambert solver used in this study [4]. Finally, hyperbolic excess velocities with respect to the Moon can be obtained as follows:

$$\boldsymbol{v_{\infty in}}^+ = \boldsymbol{v_{in}}^+ - \boldsymbol{v_{Min}} \qquad (3)$$

$$\boldsymbol{v_{\infty out}}^- = \boldsymbol{v_{out}}^- - \boldsymbol{v_{Mout}} \qquad (4)$$

Fig.2 shows the moon-to-moon trajectories given by Lambert solver. Δ is ranged from $0°$ to $360°$ and only retrograde trajectories are chosen because prograde ones would be same as the lunar orbit around the Earth. we eliminate the unfeasible Earth collision trajectories of which periapses are less than radius of the Earth. In addition, we get rid of the lastly incoming trajectories because the spacecraft finally has to escape the Earth that is in outgoing direction. In the end, we selected the trajectories shown in Fig.3 as feasible moon-to-moon trajectories in this paper.

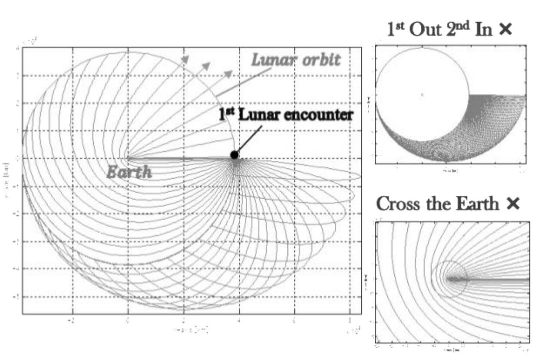

Fig.2 All Moon-to-Moon Trajectories (Left)
Lastly Incoming Trajectories (Upper Right)
Earth Collision Trajectories (Bottom Right)

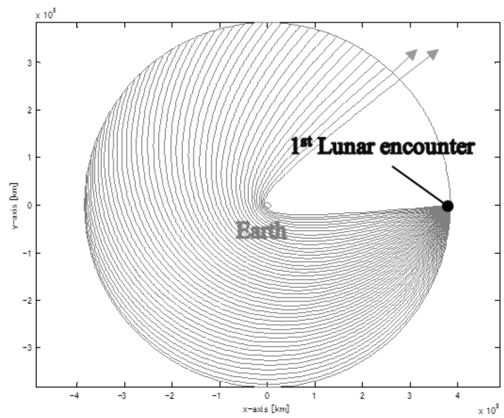

Fig.3 Moon-to-Moon Trajectories (45° $<\Delta<$ 180°)

III. Sequential Lunar Swing-by

In this chapter, we explain the design method to construct sequential lunar swing-by. As stated in Chapter **II**, we can apply zero-patched-conic method to calculate lunar swing-by as instantaneous change in v_∞ direction with respect to the Moon. Therefore, a moon-to-moon trajectory is discontinuously linked with lunar swing-by at two lunar encounter positions respectively. On top of that, a local coordinate system [5] shown in Fig.4 is introduced to discuss on v_∞ direction. The origin of this frame is the position of the Moon at the swing-by, y-axis is in the direction of the Earth, z-axis is in the direction perpendicular to the Moon's orbit plane and x-axis is in the direction of the cross-product of the y- and z-direction. In this paper, the Moon's orbit is assumed as a circle, hence y-direction becomes the same direction as the lunar orbital velocity vector $\boldsymbol{v_M}$. Additionally, v_∞ direction is expressed by the longitude α for which the baseline is located along x-axis.

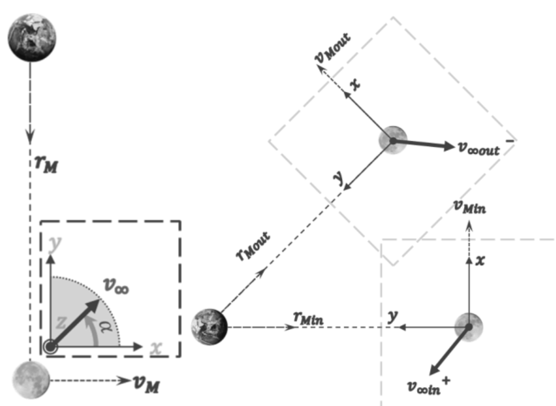

Fig.4 Local Coordinate System

In order to calculate the deflection angle $\emptyset_B$ at lunar swing-by, we use the following relation:

$$\emptyset_B = \sin^{-1}\frac{1}{1 + r_\pi {v_\infty}^2/\mu_M} \tag{5}$$

Where r_π is the swing-by radius, v_∞ is magnitude of $\boldsymbol{v_{\infty in}}^+$ $(\boldsymbol{v_{\infty out}}^-)$, μ_M is gravitational parameter of the Moon. From Eq(5), we can find that $\emptyset_B$ is inversely propotional to r_π. In this paper, r_π is ranged from the radius of the Moon R_M to sphere of influence of the Moon SOI_M. Substituting them into Eq(5), we get:

$$\emptyset_{Bmin} = \sin^{-1}\frac{1}{1 + SOI_M {v_\infty}^2/\mu_M} \tag{6}$$

$$\emptyset_{Bmax} = \sin^{-1}\frac{1}{1 + R_M {v_\infty}^2/\mu_M} \tag{7}$$

For given $\boldsymbol{v_{\infty out}}^-$, a set of $\boldsymbol{v_{\infty out}}^+$ can be defined as those form of $\emptyset_B$ with $\boldsymbol{v_{\infty out}}^-$. Moreover, all motion is assumed as planar so that we can obtain $\boldsymbol{v_{\infty out}}^+$ as dot-product of $\boldsymbol{v_{\infty out}}^-$ and rotation matrix $\boldsymbol{R}(\pm\emptyset_B)$ in 2-dimension. In the same manner, we get $\boldsymbol{v_{\infty in}}^-$ for given $\boldsymbol{v_{\infty in}}^+$.

$$\boldsymbol{R}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \tag{8}$$

$$\boldsymbol{v_{\infty out}}^+ = \boldsymbol{v_{\infty out}}^- * \boldsymbol{R}(\pm\emptyset_B)\Big|_{\emptyset_{Bmin}}^{\emptyset_{Bmax}} \tag{9}$$

$$\boldsymbol{v_{\infty in}}^- = \boldsymbol{v_{\infty in}}^+ * \boldsymbol{R}(\mp\emptyset_B)\Big|_{\emptyset_{Bmin}}^{\emptyset_{Bmax}} \tag{10}$$

Eq(9) and Eq(10) lead to $\boldsymbol{v_{\infty out}}^+$ and $\boldsymbol{v_{\infty in}}^-$. Finally, we can get $\boldsymbol{v_{in}}^-$ and $\boldsymbol{v_{out}}^+$ as follows:

$$\boldsymbol{v_{in}}^- = \boldsymbol{v_{\infty in}}^- + \boldsymbol{v_{Min}} \tag{11}$$

$$\boldsymbol{v_{out}}^+ = \boldsymbol{v_{\infty out}}^+ + \boldsymbol{v_{Mout}} \tag{12}$$

Fig.5 shows the geometry of sequential lunar swing-by. You can find that $\boldsymbol{v_{\infty in}}$ and $\boldsymbol{v_{\infty out}}$ are symmetric about the x-axis in the local coordinate system.

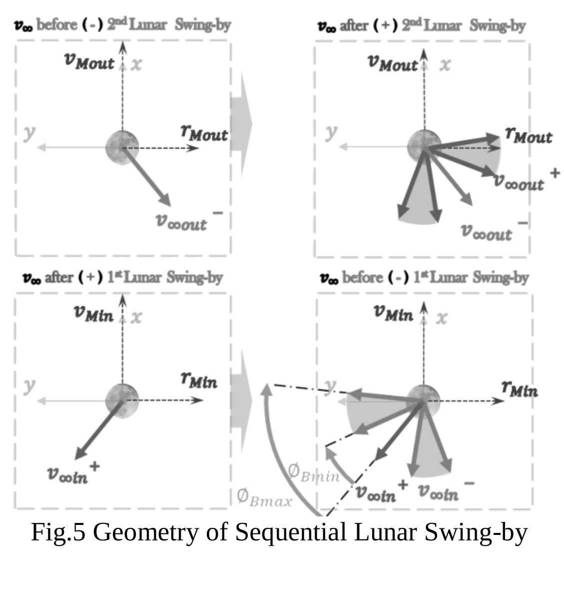

Fig.5 Geometry of Sequential Lunar Swing-by

IV. Earth Escape Trajectory

In the preceeding chapters, we talked about the method to generate moon-to-moon trajectory (**II**) and sequential lunar swing-by (**III**). Further, Earth escape trajectory is construed as a combination of moon-to-moon trajectory and sequential lunar swing-by, so now Earth escape trajectories can be computed.

We underscore the point that a spacecraft boosts its energy drastically by sequential lunar swing-by as described in the title. Thus, it's necessary to estimate its energy difference between before-and-after lunar swing-by sequence with moon-to-moon trajectory. In addition, a spacecraft needs *hyperbolic excess speed* v_∞ on a hyperbolic path when it arrives at infinity so as to escape from the Earth. Or rather, v_∞ can be treated as kinetic energy required to escape from the Earth. With this situation, we introduce the square of v_∞ to evaluate its energy. The concervation of energy for a hyperbolic trajectory gives v_∞ as follows:

$$v_\infty = \sqrt{\|\boldsymbol{v}\|^2 - 2\mu_E/a_M} \tag{13}$$

Where $\boldsymbol{v}$ is spacecraft's velocity at lunar encounter positions. Note that v_∞ and $\boldsymbol{v}$ are with respect to the Earth. Substituting $\boldsymbol{v_{in}}^-$ and $\boldsymbol{v_{out}}^+$ for $\boldsymbol{v}$ in Eq(13), we get:

$$v_\infty{}^- = \sqrt{\|\boldsymbol{v_{in}}^-\|^2 - 2\mu_E/a_M} \tag{14}$$

$$v_\infty{}^+ = \sqrt{\|\boldsymbol{v_{out}}^+\|^2 - 2\mu_E/a_M} \tag{15}$$

The square of v_∞ is denoted C_3, and is known as the *characteristic energy*. C_3 is a measure of the energy required for an interplanetary mission. If C_3 is greater than or equal to zero, we can say that the spacecraft has enough energy to escape from the Earth.

$$C_3{}^- = \|\boldsymbol{v_{in}}^-\|^2 - 2\mu_E/a_M \tag{16}$$

$$C_3{}^+ = \|\boldsymbol{v_{out}}^+\|^2 - 2\mu_E/a_M \tag{17}$$

$C_3{}^-$ and $C_3{}^+$ indicate C_3 before and after 'sequential' lunar swing-by with moon-to-moon trajectory.

Another point to consider is its direction change with respect to the Earth. Therefore, we classify the trajectories with specified time of flight TOF by its direction change between before-and-after sequential lunar swing-by. The longitude α (**II**) in the local coordinate system is of great utility to calculate this turn angle β. It is defined as angle rotated clockwise from $\boldsymbol{v_{in}}^-$direction (α^-) to $\boldsymbol{v_{out}}^+$ direction ($\alpha^+ - \Delta$) as follows:

$$\beta = \alpha^- - \alpha^+ + \Delta \tag{18}$$

V. Analytical conditions

Table.1 shows bulk and orbital parameters used in the analytical calculation. Table.2 shows the design parameters to produce a moon-to-moon transfer and sequential lunar swing-by arbitrarily. we choose 1° as step size of Δ and divide $\emptyset_B$ into 100 equally spaced intervals within the designated range. We would like to mention that the additional parameters determined uniquely by Δ, such as TOF and periapsis, are may be better suit the purpose of this study because we can well imagine the characteristics of a moon-to-moon trajectory.

Table.1 Planet Data (constants)

Parameters	Value
Earth's Radius, R_E	6.38×10^3 km
Moon's Radius, R_M	1.74×10^3 km
Moon's Semimajor axis, a_M	3.84×10^5 km
Moon's SOI, SOI_M	6.61×10^4 km
Earth's standard gravitational parameter, μ_E	3.99×10^5 km^2/s^2
Moon's standard gravitational parameter, μ_M	4.90×10^3 km^2/s^2
Moon's orbital velocity, v_M	1.02 km/s

Table.2 Design parameters

Parameters	Range
Transfer angle, Δ	$45° < \Delta < 180°$
Deflection angle, $\emptyset_B$	$\emptyset_{Bmin} \leq \emptyset_B \leq \emptyset_{Bmax}$

VI. Results

In this chapter, we confirm availability of the lunar assisted escape trajectories from the stand point of energy and direction change. Both of them are purely relevant to the Earth escape conditions. Fig.6 shows whole solution area based on $C_3{}^-$ & $C_3{}^+$ relationship, the vertical line shows $C_3{}^+$ and horizontal one $C_3{}^-$.

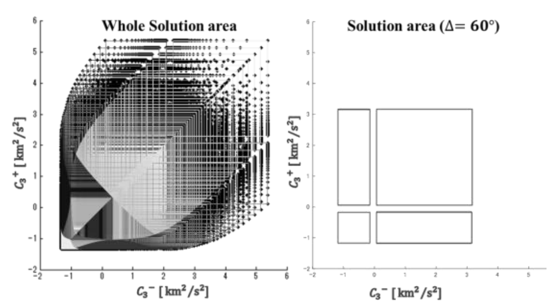

Fig.6 Whole Solution Area (Left)
Solution Area in case that Δ= 60° (Right)

Each dot plot indicates a pair of $C_3{}^-$and $C_3{}^+$ solved under the analytical conditions determined in the previous chapter. Some of these solutions have high sensitivity to the design parameters. That's the reason why it can be seen that the density of dot plots differs locally. For given Δ, the sensitivity-independent solution area comprises the assembly of rectangular solution areas. For your reference, only in case $\Delta = 60°$ is depicted in Fig.6. In addition, you can observe that the dot plots coincide with the assembled frame lines of solution areas. Moreover, the whole solution area is composed of overlapping solution areas. Fig.7 shows the divided regions of whole solution area.

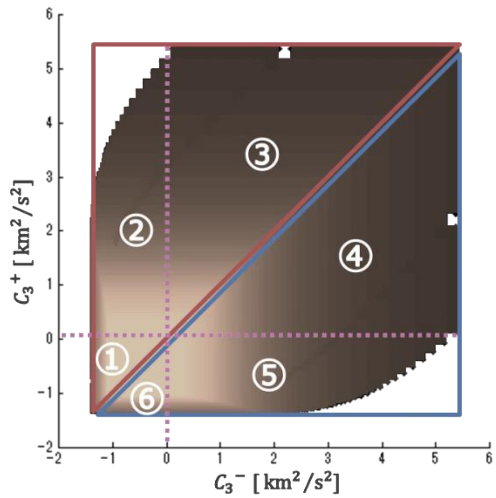

Fig.7 Divided Regions of Whole Solution Area

These six regions respectively have the following relations:

① $C_3^- < 0, C_3^+ < 0, C_3^- < C_3^+$
② $C_3^- < 0, C_3^+ > 0, C_3^- < C_3^+$
③ $C_3^- > 0, C_3^+ > 0, C_3^- < C_3^+$
④ $C_3^- > 0, C_3^+ > 0, C_3^- > C_3^+$
⑤ $C_3^- > 0, C_3^+ < 0, C_3^- > C_3^+$
⑥ $C_3^- < 0, C_3^+ < 0, C_3^- > C_3^+$

In this paper, we particularly put stress on the design of Earth escape trajectories, hence $C_3^+ > 0$ is the absolute condition. Furthermore, $C_3^+ > C_3^-$ is also required relation to enhance spacecraft's energy. As the results, ② and ③, which fulfill these conditions, can be said as the desiable regions to pick up the trajectories. From a quantitative perspective, we can get C_3 maximum increase of about 5.3 $\mathrm{km}^2/\mathrm{s}^2$in ③. Of course you can pick up the other trajectories from this database. For instance, you can seek Earth captured trajectories from deep space in ⑤. However, we can't discuss the direction change and TOF because some solution areas are overlapping in Fig.7.

In order to reveal the relationship between C_3^-, C_3^+ and β for arbitrary TOF, we use additional contour plot shown in Fig.8.

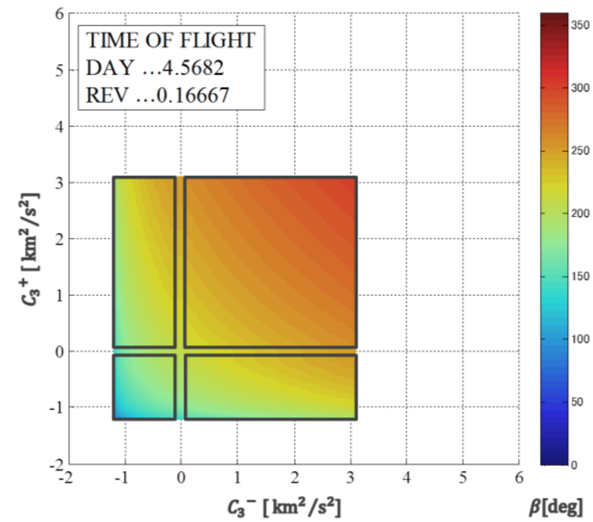

Fig.8 Relationship between C_3^-, $C_3^+$and β in case that $\Delta = 60°$, $TOF \cong 4.57 days$

Fig.8 is an example of graphical representation of the relationship between C_3^-, C_3^+, and β. The contour label indicates the magnitude of β. We can find the distribution of β with respect to C_3^-, C_3^+, hence these parameters in accord with the desired Earth escape conditions can be estimated provided that the mission designer defines the tolerance of TOF.

VII. Conclusion

In this paper, we generate Earth escape trajectory by connecting moon-to-moon transfer and sequential lunar swing-by. As the results, we get the relationship between C_3^-, C_3^+ and β for arbitrary TOF and store them in a database for practical use. The following are the main findings of this study:

1. By use of sequential lunar swing-by, a spacecraft can significantly increase its energy even if it already has enough energy to escape the Earth before sequential lunar swing-by. Therefore, the solved trajectories can be applied not only to a trajectory departuring from the Earth but also an inbound trajectory from deep space.
2. Energy change between before-and-after sequential lunar swing-by ($C_3^+ - C_3^-$) can be selectable in the range from $-5.3\ \mathrm{km}^2/\mathrm{s}^2$ to $5.3\ \mathrm{km}^2/\mathrm{s}^2$.
3. For arbitrary TOF, we can estimate the direction change β in regard to a given pair of C_3^- and C_3^+ by looking into the database.

It remains a challenge for future research to estimate the energy and direction change with respect to the Sun between before-and-after sequential lunar swing-by because we will propose the systematic method to design lunar assisted escape trajectories for small-scale interplanetary missions in the future. In addition, the relationship between ${C_3}^-, {C_3}^+, \beta$ and the orbital elements of Earth escape trajectory including moon-to-moon trajectory also need to be discussed in more detail.

References

[1] Daniel Muhonen, Sylvia Davis, and David Dunham: Alternative gravity-assist sequences for the ISEE-3 escape trajectory, AIAA-84-1977, August 1984

[2] Junichiro Kawaguchi, Hiroshi Yamakawa, Tono Uesugi, and Hiroki Matsuo: On making use of lunar and solar gravity assists in lunar-a, planet-b missions, Acta Astroautica, 35(9):633642, 1995

[3] Gregory Lantoine, and Timothy P. McElrath: Families of Solar-perturbed moon-to-moon transfers, AAS 14-471, 2014

[4] Geza S. Gedeon: A practical note on the use of Lambert's equation, AIAA Journal, Vol. 3, No.1, 1965

[5] Kawakatsu Yasuhiro: V_∞ direction diagram and its application to swingby design. In Proceedings of the 21st International Symposium on Space Flight Dynamics, Vol.1,Toulouse, France, 2009