Pareto-Optimal, Low-Energy Transfers to the Moon Using Impulsive Delta-V

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Abstract

The present study globally searches for two-impulse, low-energy transfers to the Moon in the planar bicircular restricted four-body problem with transfer time up to 200 days. A grid search combined with a direct transcription and multiple shooting technique reveals numerous families of optimal low-energy solutions. We investigate a trade-off between transfer time and Δv by exploring Pareto–optimal solutions. Analyzing orbital characteristics based on multi–body dynamics, we show useful perturbations of the Sun, Earth, and Moon to reduce Δv.

1 Introduction

The class of low-energy transfers to the Moon has remarkable benefits such as lower fuel expenditure (Δv) and a wider launch window as compared with the conventional Hohmann transfer [1]. Because of these advantages, HITEN and GRAIL missions adopted low-energy transfers to reach the Moon in spite of long transfer time (Δt).

Recently, Topputo [2] globally explored a Δt–Δv solution plane of optimal two-impulse transfers to the Moon up to 100 days, including low-energy transfers. The solutions cover a wide range of the solution plane and involve many known solutions in previous studies near local minima. Some families of the solutions were novel transfers to the Moon, one of which was later analyzed by Oshima et al. [3].

In the present study, we extend the work in Topputo [2] to compute optimal two-impulse, low-energy transfers to the Moon with longer transfer time up to 200 days. The obtained solutions cover a wide range of the Δt–Δv plane, the large part of which has not been fully explored yet. Some families of solutions result in smaller Δv than known previous solutions. We investigate a trade-off between Δt and Δv and discuss the orbital characteristics of sample Pareto–optimal solutions based on multi–body dynamics.

2 Planar Bicircular Restricted Four-Body Problem

In the present paper, we compute trajectories in the planar bicircular restricted four-body problem (PBRFBP). The PBRFBP models the motion of a massless particle, P_3, under the gravitational influences of three massive bodies, P_0, P_1, P_2 of masses m_0, m_1, m_2, $m_0 > m_1 > m_2$, respectively. The model assumes that P_1 and P_2 revolve in circular orbits around their barycenter, and P_0 revolves in a circular orbit around

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the P_1–P_2 barycenter in the same orbital plane as P_3. In the present study, P_0 is the Sun, P_1 is the Earth, and P_2 is the Moon. The equations of motion are

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t),\ \boldsymbol{x} := \begin{bmatrix} \boldsymbol{r} \\ \boldsymbol{v} \end{bmatrix},\ \boldsymbol{f}(\boldsymbol{x}, t) := \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{g}(\boldsymbol{r}, t) + \boldsymbol{h}(\boldsymbol{v}) \end{bmatrix}, \tag{1}$$

where t is time, $\boldsymbol{r}$ and $\boldsymbol{v}$ are the position (x, y) and velocity $(\dot{x}, \dot{y})$ of a spacecraft, and

$$\boldsymbol{g}(\boldsymbol{r}, t) := \begin{bmatrix} \partial\Omega_4/\partial x \\ \partial\Omega_4/\partial y \end{bmatrix},\ \boldsymbol{h}(\boldsymbol{v}) := \begin{bmatrix} 2\dot{y} \\ -2\dot{x} \end{bmatrix}, \tag{2}$$

where

$$\Omega_4(x, y, t) := \Omega_3(x, y) + \frac{\mu_s}{\sqrt{(x - a_s\cos\theta_s)^2 + (y - a_s\sin\theta_s)^2}} - \frac{\mu_s}{a_s^2}(x\cos\theta_s + y\sin\theta_s), \tag{3}$$

$$\Omega_3(x, y) := \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{\sqrt{(x+\mu)^2 + y^2}} + \frac{\mu}{\sqrt{(x - 1 + \mu)^2 + y^2}} + \frac{1}{2}\mu(1-\mu), \tag{4}$$

where $\mu := m_2/(m_1 + m_2)$; μ_s is the mass of the Sun, a_s is the distance from the Earth–Moon barycenter to the Sun, and the phase angle of the Sun is $\theta_s(t) := \theta_{s,0} + \omega_s t$ for some initial $\theta_{s,0}$ at $t = 0$; ω_s is the relative angular velocity of the Sun. All the physical parameters used in this paper are in accordance with those in Table 3 in Topputo [2].

The state transition matrix (STM) is defined by using the flow of the system given by Eq. (1)

$$\varphi(\boldsymbol{x}_i, t_i, t_f) := \boldsymbol{x}_i + \int_{t_i}^{t_f} \boldsymbol{f}(\boldsymbol{x}, \tau) d\tau \tag{5}$$

as $\Phi(\boldsymbol{x}, t_i, t_f) := \mathrm{d}\varphi(\boldsymbol{x}, t_i, t_f)/\mathrm{d}\boldsymbol{x}$, and its time evolution is

$$\dot{\Phi}(\boldsymbol{x}, t_i, t) = D_{\boldsymbol{x}}\boldsymbol{f}\Phi(\boldsymbol{x}, t_i, t), \quad \Phi(\boldsymbol{x}, t_i, t_i) = I_4, \tag{6}$$

where the subscripts i and f represent initial and final values, respectively, I_4 is a 4×4 identity matrix, and $D_{\boldsymbol{x}}\boldsymbol{f}$ is the Jacobian of the system.

3 Problem Statement

The present study computes two-impulse transfers from an initial circular Earth orbit of altitude $h_i = 167$ km to a final circular Moon orbit of altitude $h_f = 100$ km in the PBRFBP. The first impulse of magnitude Δv_i at the initial time t_i injects the spacecraft into a trans-lunar trajectory, and the second impulse of magnitude Δv_f at the final time t_f inserts it into the final Moon orbit. We set both maneuvers tangential to the local velocities of the initial and final circular orbits. Therefore,

$$\Delta v_i = \sqrt{(\dot{x}_i - y_i)^2 + (\dot{y}_i + x_i + \mu)^2} - \sqrt{\frac{1-\mu}{r_i}}, \tag{7}$$

$$\Delta v_f = \sqrt{(\dot{x}_f - y_f)^2 + (\dot{y}_f + x_f + \mu - 1)^2} - \sqrt{\frac{\mu}{r_f}}, \tag{8}$$

where r is the non-dimensional distance from the center of the celestial body.

In this formulation, the cost of transfer is $\Delta v = \Delta v_i + \Delta v_f$ and the transfer time is $\Delta t = t_f - t_i$, with boundary conditions

$$\boldsymbol{\psi_i} := \begin{bmatrix} (x_i + \mu)^2 + y_i^2 - r_i^2 \\ (x_i + \mu)(\dot{x}_i - y_i) + y_i(\dot{y}_i + x_i + \mu) \end{bmatrix} = \mathbf{0}, \tag{9}$$

$$\boldsymbol{\psi_f} := \begin{bmatrix} (x_f + \mu - 1)^2 + y_f^2 - r_f^2 \\ (x_f + \mu - 1)(\dot{x}_f - y_f) + y_f(\dot{y}_f + x_f + \mu - 1) \end{bmatrix} = \mathbf{0}. \tag{10}$$

4 Methodology

4.1 Generation of Initial Guesses

We compute initial guess solutions by adopting a grid search in the three-dimensional α–β–$\theta_{s,0}$ parameter space; α is an angle between the Earth–Moon line and the line segment from the Earth to the initial position of the spacecraft. β is a proportionality factor of the initial velocity v_i of the spacecraft with respect to the local circular velocity $\sqrt{\frac{1-\mu}{r_i}}$, i.e., $v_i := \beta\sqrt{\frac{1-\mu}{r_i}}$. $\theta_{s,0}$ is the initial phase of the Sun with respect to the Earth–Moon line. Thus, the initial states of the spacecraft in the Earth–Moon rotating frame can be represented as

$$x_i = r_i \cos\alpha - \mu, \quad y_i = r_i \sin\alpha, \quad \dot{x}_i = -(v_i - r_i)\sin\alpha, \quad \dot{y}_i = (v_i - r_i)\cos\alpha. \tag{11}$$

Table 1 summarizes the search set and corresponding numbers of grid points. Note that each set of the three parameters uniquely determines an initial condition of a trans-lunar trajectory in the PBRFBP. We save an initial guess if a trajectory reaches 100 km (IG1) or 10000 km (IG2) altitude from the Moon, and finish propagating the trajectory if IG1 is saved or transfer time becomes longer than 200 days.

Since the present search produces huge amount of initial guesses, we optimize only those satisfying the following conditions of small injection maneuver and high final Jacobi energy, which could result in small total Δv:

- For IG1: $\Delta v_i \leq 3200$ m/s and $C_f \geq 3.1$
- For IG2: $\Delta v_i \leq 3140$ m/s and $C_f \geq 3.05$

Table 1: Parameters for a grid search

Parameter	Minimum	Maximum	Number of grid points
α	0	2π	373
β	1.4	$\sqrt{2}$	500
$\theta_{s,0}$	0	2π	500

4.2 Direct Transcription and Multiple Shooting

The present study optimizes the initial guesses obtained in Section 4.1 by a direct transcription and multiple shooting technique [4, 5, 2], which translates an optimal control problem into a nonlinear programming (NLP) problem. In a spirit of this method, we divide a trajectory into $N - 1$ segments by N meshes of equal time intervals.

We introduce NLP variables

$$\boldsymbol{y} := \{\boldsymbol{x}_j, t_1, t_N\}, \qquad j = 1, \ldots, N, \tag{12}$$

where $t_1 := t_i$ and $t_N := t_f$ are initial and final times, respectively, and $\boldsymbol{x}_j := (x_j, y_j, \dot{x}_j, \dot{y}_j)$ is the state on a j-th mesh at time $t_j = t_1 + \frac{j-1}{N-1}(t_N - t_1)$.

The objective function is given by

$$J(\boldsymbol{y}) := \Delta v_1 + \Delta v_N, \tag{13}$$

where Eqs. (7) and (8) are evaluated on the initial and final meshes, respectively.

The boundary conditions in terms of the distances from the Earth and the Moon, and the tangency of the maneuvers with the local circular velocities (see Section 3) are given by

$$\boldsymbol{\psi_1} = \boldsymbol{0}, \quad \boldsymbol{\psi_N} = \boldsymbol{0}, \tag{14}$$

where Eqs. (9) and (10) are evaluated on the initial and final meshes, respectively.

The state $\boldsymbol{x}_j$ on a j-th mesh ($1 \leq j \leq N-1$) is integrated in the PBRFBP dynamics given by Eq. (1) for the fixed time span $[t_j, t_{j+1}]$. For the continuity of a trajectory, the defect

$$\boldsymbol{\zeta}_j = \boldsymbol{\varphi}(\boldsymbol{x}_j, t_j, t_{j+1}) - \boldsymbol{x}_{j+1}, \qquad j = 1, \ldots, N-1 \tag{15}$$

must vanish.

To avoid impacts to the surfaces of the Earth or the Moon, we impose inequality conditions on each mesh

$$\boldsymbol{\eta}_j := \begin{bmatrix} {R_e}^2 - \{(x_j + \mu)^2 + {y_j}^2)\} \\ {R_m}^2 - \{(x_j + \mu - 1)^2 + {y_j}^2)\} \end{bmatrix} < \boldsymbol{0}, \qquad j = 1, \ldots, N, \tag{16}$$

where R_e and R_m are non-dimensional radii of the Earth and the Moon, respectively. After the convergence, we only save non-impact solutions through their entire trajectories.

For the sake of consistency, an inequality condition $\tau := t_1 - t_N < 0$ is also respected.

4.3 Continuation

Once optimal solutions are computed, a continuous family of solutions of each local optimal solution is constructed for broader exploration of solution structures. To this purpose, we compute optimal solutions of the transfer times $\Delta t' := \Delta t + \delta t$ in each continuation step, where Δt is the transfer time of the optimal solution in the previous step, and δt is the continuation increment ($\delta t > 0$) or decrement ($\delta t < 0$). Therefore, there is another equality constraint for the NLP problem of the continuation

$$\sigma := t_N - t_1 - \Delta t' = 0. \tag{17}$$

Each continuation process is started from the local optimum, and is finished once the optimization fails or Δv$\geq$ 3950 m/s.

5 Results

Figure 1 shows Pareto–optimal solutions (dark) in terms of Δt and Δv extracted from the obtained solutions (light). We only extract those of Δv ≤ 3780 m/s because some Pareto–optimal low-energy solutions were obtained and analyzed in Topputo [2]. The trajectories of the sample Pareto-optimal solutions (i)–(iv) are shown in Figure 2.

Sample (i) closely coincides with the minimum Δv solution in Topputo [2]. In spite of relatively short transfer time among low-energy transfers, it exploits lunar gravity assist and solar perturbation effectively to realize an efficient transfer with a good balance between Δt and Δv.

Sample (ii) looks similar to Sample (i), but having longer Δt and smaller Δv. Unlike ordinary exterior low-energy transfers, this solution exploits the perturbation of Earth at the apolune to decrease the angular momentum with respect to Moon before the lunar capture. Due to the rotational direction of the frame, decreasing the angular momentum with respect to Moon is favorable in terms of insertion Δv at the expense of longer Δt.

Sample (iii) and (iv) exploit high-altitude lunar flybys with multiple revolutions around Earth. The high-altitude lunar flybys replenish the small departure Δv_i by pumping up the semi-major axis at the expense of extra Δt. As a result of increasing the semi-major axis, low-altitude lunar gravity assist occurs and an effective use of solar perturbation outside the lunar orbit is available to finally realize a low-energy capture by the Moon. These solutions exploit dynamics of both inside and outside of the lunar orbit, which are considered to be a new class of transfers to the Moon "interior–exterior transfer".

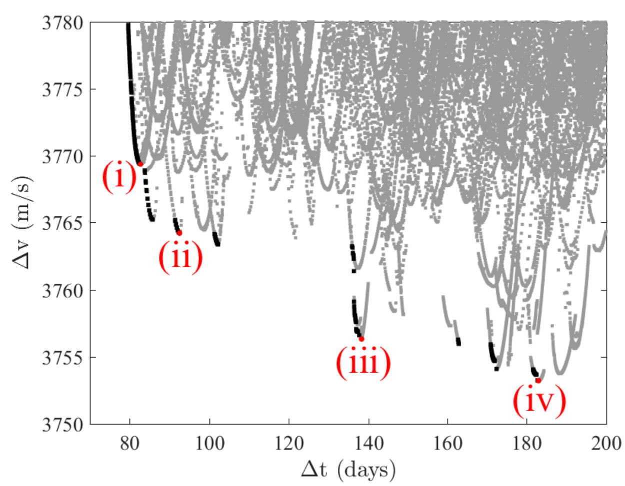

Figure 1: Pareto–optimal solutions (dark) in terms of Δt and Δv extracted from the obtained solutions (light).

6 Conclusions

The present paper globally explored two-impulse, optimal low-energy transfers to the Moon with transfer time up to 200 days in the planar bicircular restricted four-body problem. An extensive grid search with a direct transcription and multiple shooting technique generated optimal solutions in a wide range of the Δt–Δv solution plane. We investigated a trade–off between transfer time and fuel expenditure

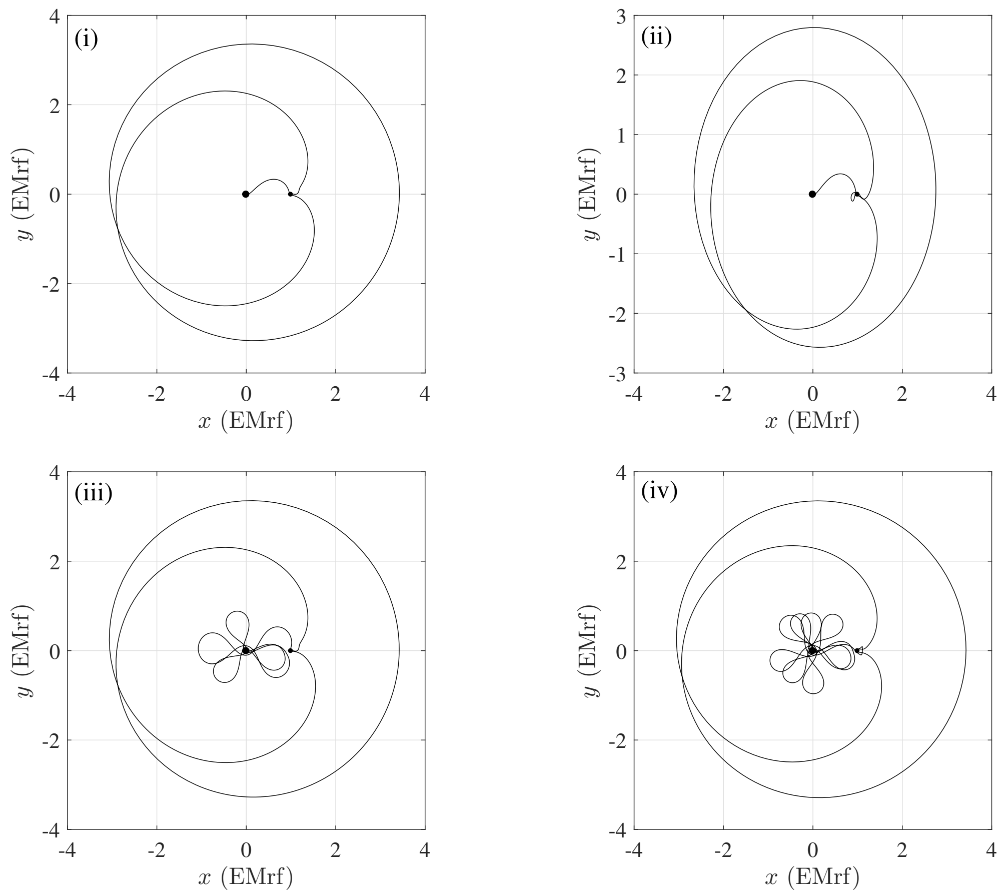

Figure 2: Trajectories of Sample (i)-(iv) in the Earth-Moon rotating frame.

by exploring Pareto–optimal solutions. Analyzing orbital characteristics of sample solutions, we have shown useful perturbations of the Sun, Earth, and Moon to reduce fuel expenditure.

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