Sun Synchronous Orbits with Gravitation of the Moon

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Abstract

Sun synchronous orbits are used by earth observation satellites because they can keep the solar attitude of the observation points and heat input from the sun throughout the year. However, their altitude are limited to a few thousand kilometers because the synchronousness is caused by J2-term of the earth. In this paper, as solutions of 4-body (sun, moon, earth and satellite) problem, sun synchronous orbits whose altitude are several hundreds of thousand kilometers are reported.

月重力を利用した太陽同期軌道

概要

地球周回太陽同期軌道は同じ太陽高度で地球観測を行えることや入熱が一年を通して一定となるなどの利点から地球観測衛星などで利用されているが，地球の J2 項によるため数千km程度の高度でしか利用できない．本研究では太陽，月，地球，衛星の 4 体問題の解として得られる高度数十万 km の太陽同期軌道を報告する．

1. Introduction

Today, the usefulness of sun synchronous orbits (SSOs) for some earth observation mission is widely accepted. This is because SSOs enable us to observe points on the earth at the same solar altitude. Another advantage of SSOs is that the heat input to a satellite on a SSO can be kept constant throughout a year. However, since SSOs result from the J2-term of the geopotential, their altitude is restricted up to several thousand kilometers (Figure 1) [1] [2].

From the view point of heat input, however, higher altitude is preferable because it lessen the variation of heat input from the reflection and the radiation of the earth. In this study, we found a new class of sun synchronous orbits using the gravity assist of the moon and the tidal force of the sun. The altitude of these orbits is on the order of magnitude of a million kilometers.

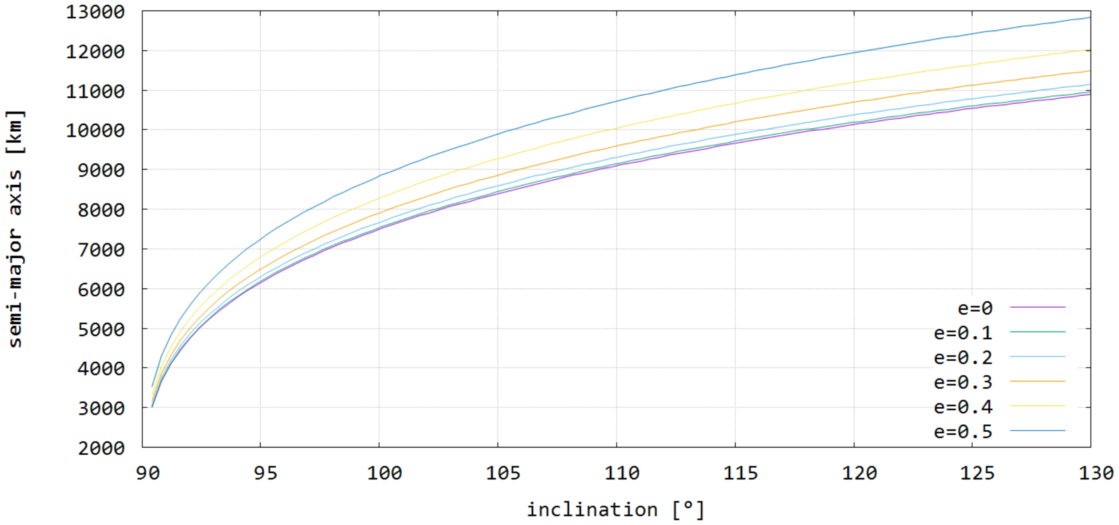


Figure 1: Semi-major axis of SSOs, plotted in reference to [1]

2. Model

In this study, planar circular restricted 4-body (Sun, Earth, Moon and Satellite) problem is assumed. In this system, the equation of motion of the satellite in the Hill's coordinate system (Figure 2) is written as

$$\begin{cases} \frac{d^2}{d\theta^2}x - 2\frac{n_{earth}}{n}\frac{d}{d\theta}y - 3\frac{n_{earth}^2}{n^2}x = \frac{1}{n^2}f_x \\ \frac{d^2}{d\theta^2}y + 2\frac{n_{earth}}{n}\frac{d}{d\theta}x = \frac{1}{n^2}f_y \\ \frac{d^2}{d\theta^2}z + \frac{n_{earth}^2}{n^2}z = \frac{1}{n^2}f_z \end{cases} \tag{1}$$

$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = -\frac{\mu_{earth}}{r^3}\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \frac{\mu_{moon}}{\{r^2 + R_{moon}^2 - 2R_{moon}(x\cos\theta + y\sin\theta)\}^{\frac{3}{2}}}\begin{pmatrix} x - R_{moon}\cos\theta \\ y - R_{moon}\sin\theta \\ z \end{pmatrix} \tag{2}$$

where θ is the phase of the moon, n_{earth} is the angular velocity of the earth about the sun, n is the apparent angular velocity of the moon: $n \equiv n_{earth} - n_{moon}$ where n_{moon} is the angular velocity of the moon, and μ_{earth} and μ_{moon} are the gravitational constants of the earth and the moon respectively.

3. Idea

In this system, since only the gravity of the sun, the earth and the moon is considered, the dominant force which can rotate the orbital plane is the tidal force of the sun. Considering orbits which face to the sun, orbits with high eccentricity whose semi-major axis tilts toward the sun from the z axis can be candidates (Figure 3). However, no solution was found by naïve numerical search we conducted preliminary.

Figure 4 is the result of a sample case in the preliminary search. The blue curve in the right graph is Ω of the osculating orbit and the green line is the reference value. Although the blue curve does not grow enough, from 20th day to 60th day, its growth rate is as fast as reference's. This domain corresponds to when the z component of the satellite's position has the larger value. We thought that this means the hypothesis about the candidates' shape is correct, although the effect of the tidal force is not enough. Therefore, to enlarge the effect of the tidal force, we attempted to make the shape of orbits more extreme, as shown in Figure 5, by the use of gravity assist by the moon. This shape is obtained by connecting two ellipses on the y axis of the Hill's coordinates.

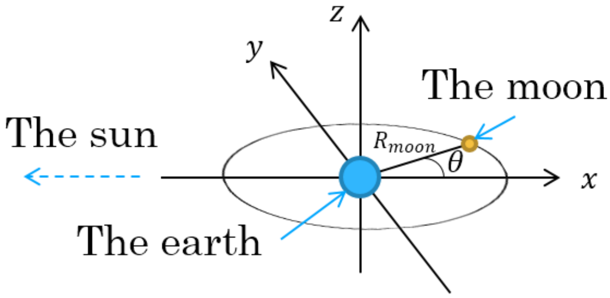


Figure 2: The Hill's coordinate system

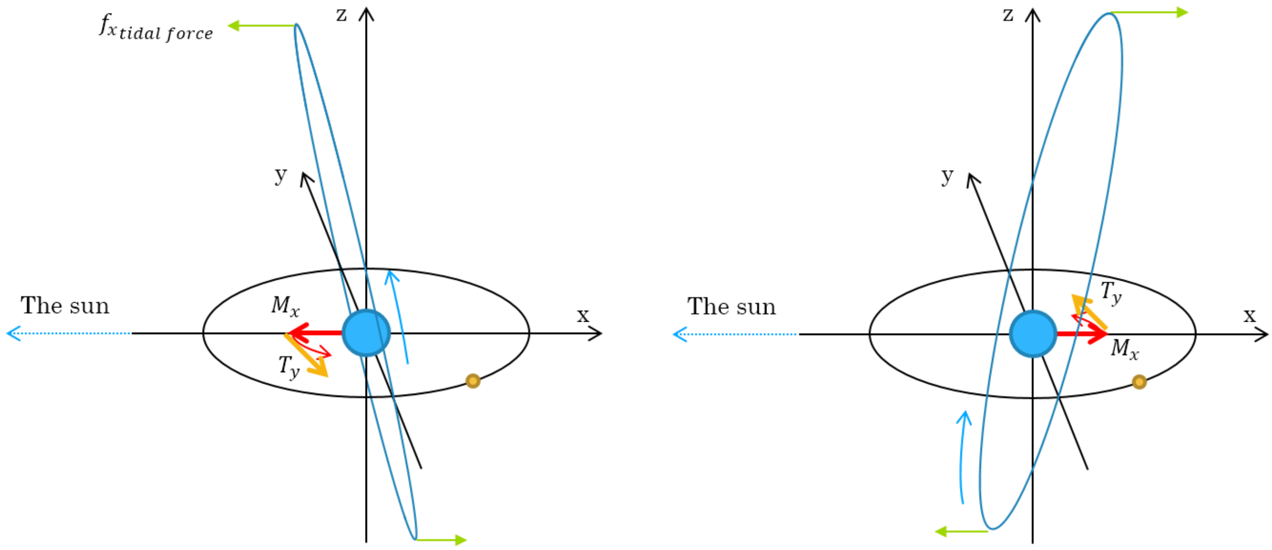


Figure 3: The tidal force on an orbit

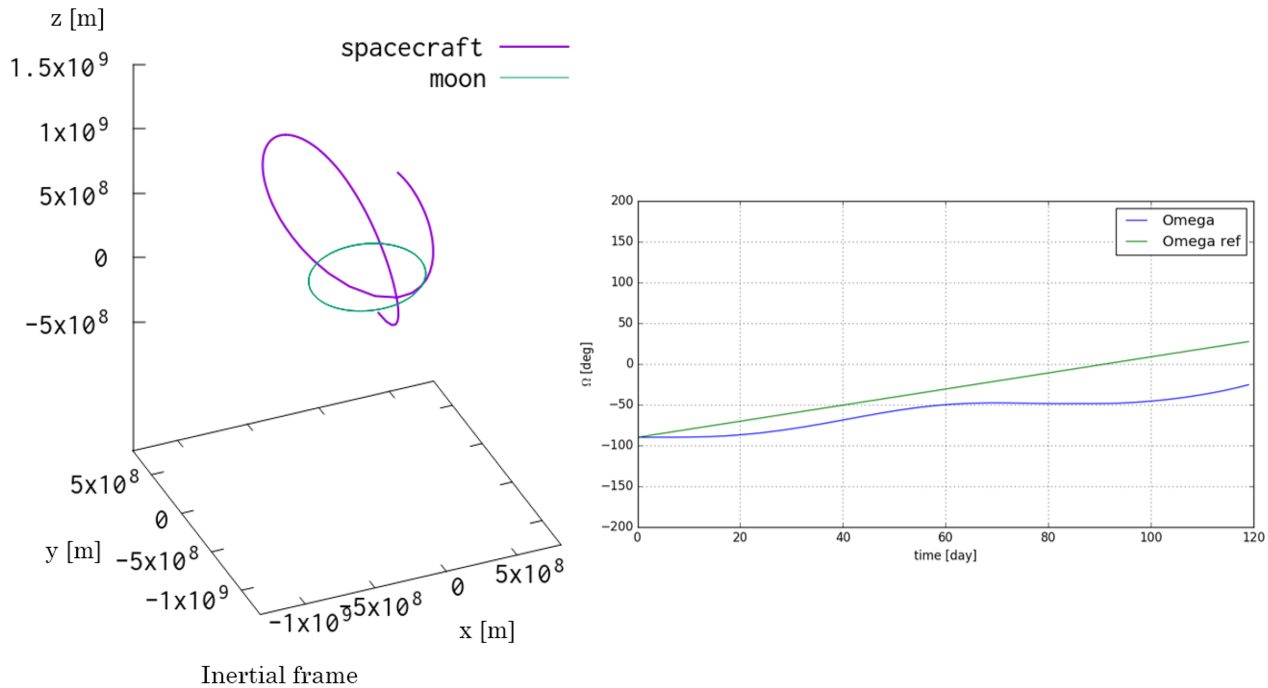


Figure 4: A sample case in the preliminary search

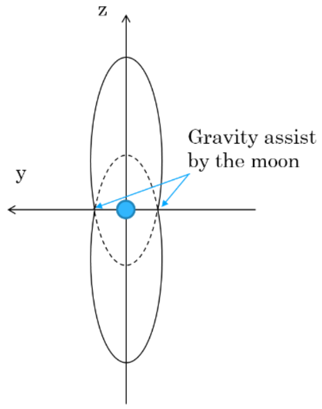


Figure 5: Candidate orbit

4. Symmetry

Before pursuing the solution, let us discuss the symmetry of the system.

In this system, there are two symmetry which can be used to make it easy to find closed orbits. One is that if a certain $\left(x(\theta), y(\theta), z(\theta)\right)$ is a solution of the system, the transformation $\left(x, y, z, cos\,\theta, sin\,\theta, \frac{d}{d\theta}\right) \rightarrow \left(x,\ -y,\ z, cos\,\theta, -sin\,\theta,\ -\frac{d}{d\theta}\right)$ also satisfies the system. This means that for each trajectory, there is an xz-plane symmetrical trajectory which propagates in the opposite direction (Figure 6).

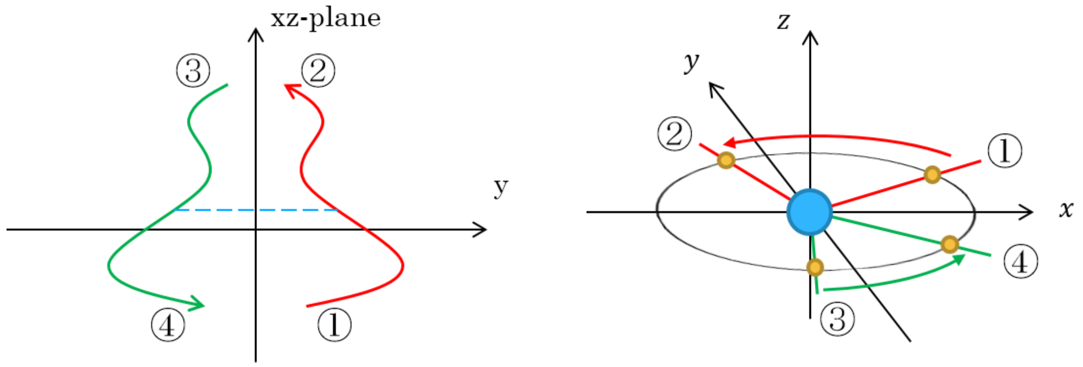


Figure 6: xz-plane symmetrical trajectory

Another symmetry is that the same thing can be said on the transformation $\left(x, y, z, \cos\theta, \sin\theta, \frac{d}{d\theta}\right) \rightarrow \left(-x,\ -y,\ -z,\ -\cos\theta, -\sin\theta, \frac{d}{d\theta}\right)$. This means that there is an origin symmetrical trajectory for each trajectory (Figure 7).

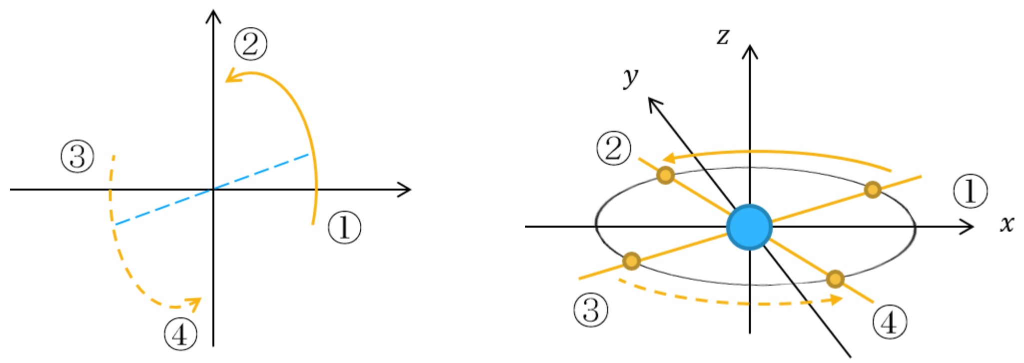


Figure 7: Origin symmetrical trajectory

With these symmetry, closed orbits can be obtained numerically. A trajectory which arrives at the xz-plane at the moment when the moon phase is $m\pi$ with 0 velocity in the direction of x and z, this trajectory can be connected to its xz-symmetrical trajectory (Figure 8). If the initial condition of the original trajectory is $(0, y_i, 0, \dot{x}_i, 0, \dot{z}_i)$, the final condition of the connected trajectory is $(0, -y_i, 0, -\dot{x}_i, 0, -\dot{z}_i)$, which is origin symmetrical, too. Since the phase of the moon is also xz-plane symmetrical, if it is on the y axis at the beginning, it is on the opposite point on the y axis at the end, which is also origin symmetrical. Thus, this pair of xz-plane symmetrical trajectories can be connected to its origin symmetrical trajectory (Figure 9). Therefore, we can reduce the number of parameters from 8 $(x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i, \theta_i, T)$ to 5 $(y_i, \dot{x}_i, \dot{z}_i, \theta_i, T)$.

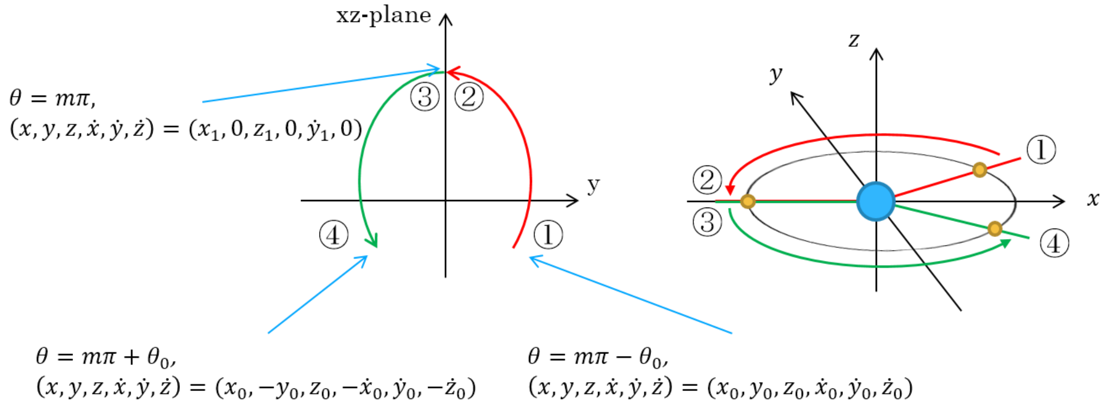


Figure 8: Connection of xz-plane symmetrical trajectories

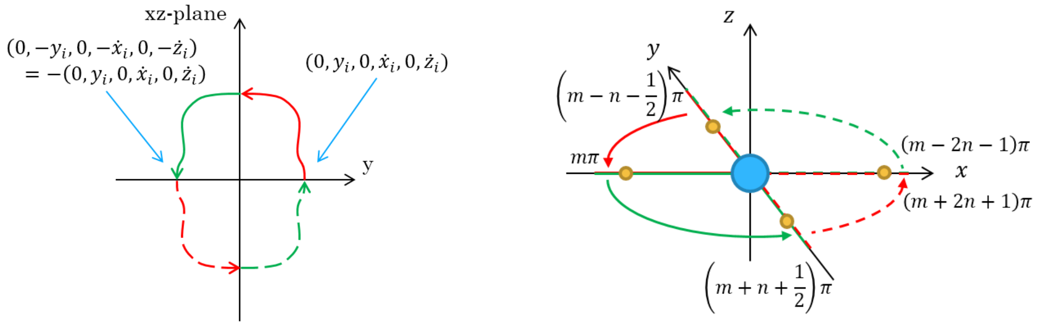


Figure 9: Closed orbit obtained by the symmetries

5. Gravity assist

Since the symmetry above requires the moon to be on the y axis when the orbit begins at a point on the y axis, we can achieve gravity assist by the moon by setting the initial position as more inside than the moon on the y axis (Figure 10).

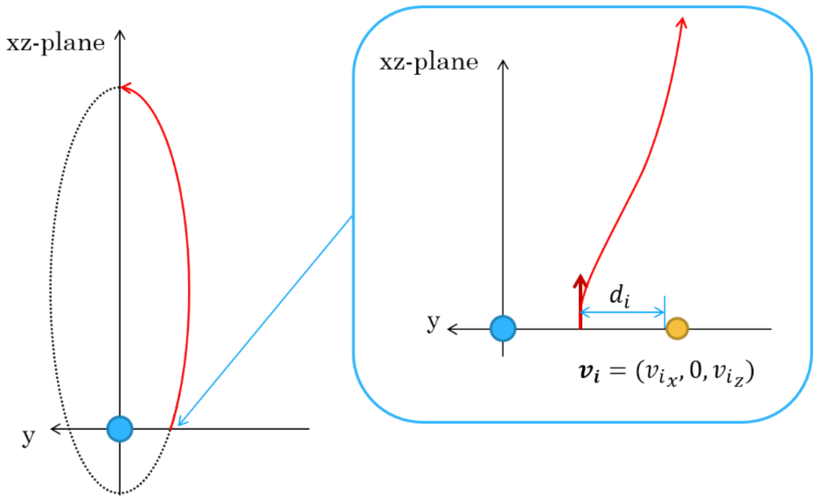
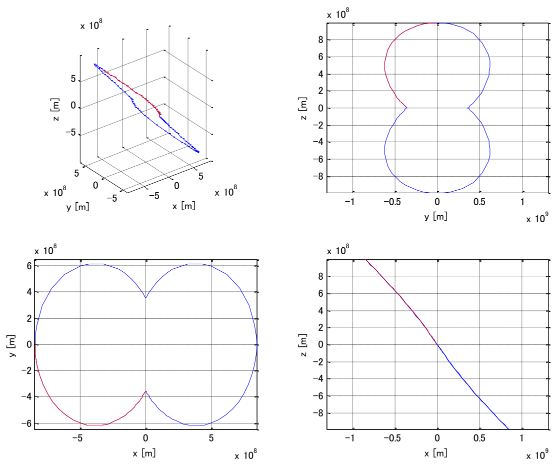


Figure 10: Gravity assist by the moon

6. Result of numerical search

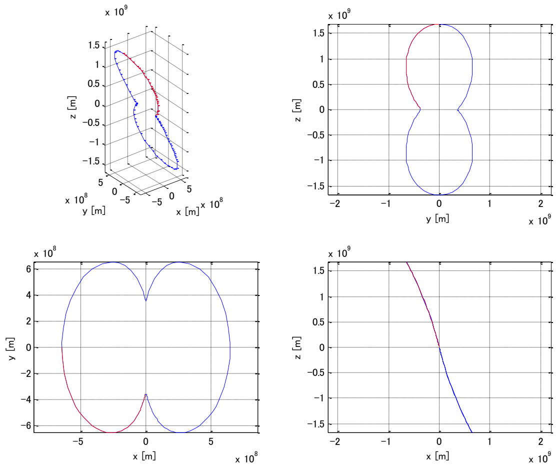
Restricting the orbital period up to one year, four result are obtained (Figure 11).

Note that the cases whose periods are 5, 7 and 9months are infeasible because in each case, d_i, the minimum distance from the surface of the moon (see Figure 10), is negative. This means the satellite collides with the moon. In these case, only the case whose period is 11months is feasible.



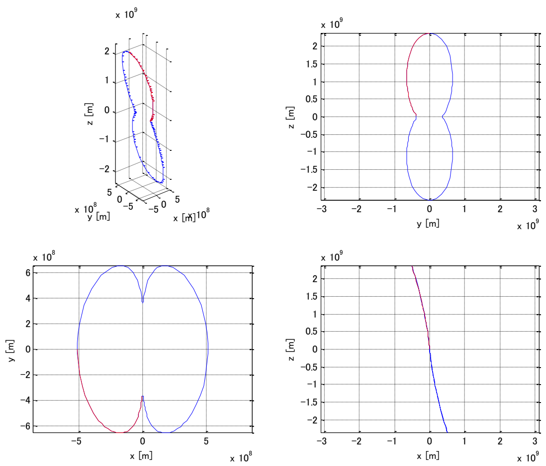
Period: 5 months

$d_i = -370$ km



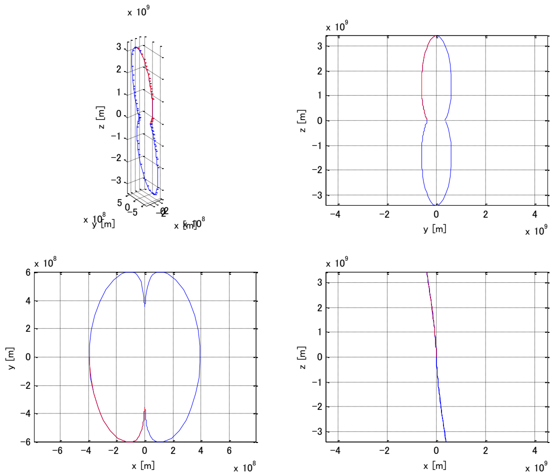
Period: 7 months

$d_i = -304$ km



Period: 9 months

$d_i = -133$ km



Period: 11 month

$d_i = 194$ km

Figure 11: Result of numerical search

7. Conclusion

Using the symmetry of the 4-body planar circular restricted problem, new SSOs were found. They have higher altitude by 5 orders of magnitude compared to the currently known SSOs. Although most of them are infeasible because of the collision with the moon, one solution which has 11months period is feasible.

8. References

[1] 半. 稔雄, ミッション解析と軌道設計の基礎, 2014.

[2] R. J. Boain, "A-B-Cs of Sun-Synchronous Orbit Mission Design," in *14 AAS/AIAA Space Flight Mechanics Conference*, Maui, Hawaii, 2004.