Earth Return Trajectory for Martian Moons eXplorer Combining Three and Two-Body Dynamics

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Abstract: The hybrid usage of chemical and electric propulsion has come into consideration in order to seek a fast low-energy escape from Mars for Martian Moons eXplorer using chemical propulsion in Mars escape phase and electric propulsion in interplanetary phase. In order to minimize the fuel consumption for chemical propulsion, escape trajectory's v-infinity is nearly equal to zero and gradually passes through the edge of the sphere of influence. Therefore, Mars escape phase should be considered as a three-body problem, though the tools for optimizing such a low-thrust trajectory have been much developed rather in the context of the two-body problem. In this paper, we will investigate the integration point from the three-body problem to two-body problem and reveal the effect of moving the point toward Mars.

火星衛星探査計画における三体問題と二体問題の結合による地球帰還軌道の検討

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摘要：火星衛星探査計画において復路の検討を進める中で，化学推進によって火星を離脱し，電気推進によって惑星間加速を行い地球に帰還する化電複合利用案が提案された．化学推進剤の使用量を抑えるため火星離脱軌道は無限遠速度がゼロに近く影響圏の境界を緩やかに通過することから三体問題として取り扱う必要がある．本研究では，三体問題と二体問題の軌道を繋ぎ合わせる結合点について考案し，結合点を火星に近付けた時の影響を探る．

I. Introduction

Martian Moons eXplorer, or MMX, is a mission proposed by ISAS/JAXA which aims to explore both Martian moons, Phobos and Deimos, in order to achieve the first sample return from one of the Martian moon. [1] For the Earth-Mars and Mars-Earth transfer, the usage of both chemical and electric propulsion was taken into a consideration. Using chemical propulsion for both legs of the trajectory gives us a fast transfer to collect and return samples from the Martian moon. However, the total mass of the spacecraft would increase due to its low specific impulse. Using electric propulsion for both legs could minimize the spacecraft mass, while it may increase the transfer time to the Mars as well as to the Earth. Then, the third option was considered in which chemical propulsion is used for Earth-Mars transfer and electric propulsion for Mars-Earth transfer. The first mission objective is to explore Martian moons, which chemical propulsion would be a better option for its fast encounter with Mars. The return leg of the journey could take time, which electric propulsion would be used to decrease the total spacecraft mass. However, the slow spiraling out from Mars and the window for the ballistic return to the Earth would impact the total observation time at Martian moons. To overcome this disadvantage, the hybrid usage of chemical and electric propulsion has been brought into consideration. In this configuration, the chemical propulsion is used to escape from Martian system quickly, while electric propulsion is considered in the interplanetary section of the trajectory for the efficient acceleration.

In the traditional two-body problem, we construct the return leg by patched conics method. The use of chemical propulsion must be minimized where the Mars departure trajectory may be a parabolic orbit with $v_\infty = 0$. After the escape from the Martian system, the low-thrust trajectory can be optimized in the Sun-Spacecraft two-body system to increase v-infinity relative to Mars by V-Infinity Leveraging Maneuver or VILM and return to the Earth on a ballistic orbit. However, by considering three-body problem, the spacecraft could escape from the Martian system with velocity less than a parabolic escape orbit's velocity. The invariant manifolds known as tubes, which are obtained from the unstable periodic orbits in the three-body problem, allow a spacecraft to escape from a planet. [2]

Considering the use of three-body problem's advantage, several options were considered for the constructing methods. [3,4,5] The practical option for optimizing trajectories may be given in the context of the three-body problem. However, as mentioned, the optimization tools for such a low-thrust trajectory have been more matured in the context of the two-body problem with reliability. Therefore, there's a need for trajectory design to somehow connect the three-body system with the two-body system. Using the transition point, where the sum of kinetic energy of the three-body trajectory in the Mars centered inertial frame and the potential energy of the Mars is equal to zero, may be one option. [6,7] Another option is to use a sphere of influence since after the escape from the Martian system, spacecraft would increase the energy to gain v-infinity.

In this paper, we will compare methods to construct the Earth return trajectory combining the Sun-Mars-Spacecraft three-body system and the Sun-Spacecraft two-body system. We will first show the fundamental setting of this problem. Then we will develop some methods followed by comparison with trajectories, all of which are constructed in the Sun-Mars-Spacecraft system.

II. Fundamental Settings

A. Dynamical Model

First let us consider the dynamics of the three-body system and the two-body systems. The Sun-Mars-Spacecraft (S-M-S/C) system may be regarded as a three-body system, in particular, the Circular Restricted Three-Body System (CR3BS). In this setting, the Sun and Mars have circular orbits about their common mass center. Phobos is assumed to have circular orbit about Mars on Mars's equatorial plane. Each of the spacecraft and Phobos is assumed to have a negligible mass. The Sun-Spacecraft and Mars-Spacecraft system is regarded as a two-body system, i.e., the restricted two-body problem. In the Sun-Spacecraft system, Mars and the Earth have respectively circular orbits about the Sun on the ecliptic plane with negligible masses.

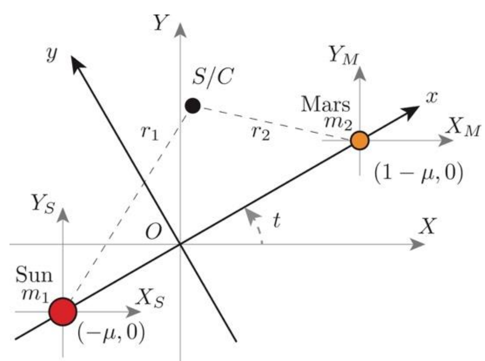

Fig. 1 Circular Restricted Three-Body Problem shown with four different coordinate systems.

B. CR3BP and Tube Dynamics

The spacecraft with a negligible mass is moving under the gravitational influence of the Sun and Mars with each having a mass of m_1 and m_2 respectively. For normalization, we take the unit of length by the distance between the Sun and Mars, and we choose the unit of time such that the angular velocity of the Sun and Mars is equal to 1. The unit of mass is taken by the sum of the Sun and Mars. Using the mass parameter $\mu = m_2/(m_1 + m_2)$, we can derive the equation of motion in the rotating frame as follows:

$$\ddot{x} - 2\dot{y} = -\frac{\partial \bar{U}}{\partial x},$$
$$\ddot{y} + 2\dot{x} = -\frac{\partial \bar{U}}{\partial y},$$
$$\ddot{z} = -\frac{\partial \bar{U}}{\partial z},$$

where

$$\bar{U}(x,y,z) = -\frac{1}{2}(x^2+y^2) - \frac{1-\mu}{r_1} - \frac{\mu}{r_2} - \frac{1}{2}\mu(1-\mu)$$

is the effective potential. The energy of the system, E is defined as a function of position and velocities by

$$E(x,y,z,\dot{x},\dot{y},\dot{z}) = \frac{1}{2}(\dot{x}^2+\dot{y}^2+\dot{z}^2) + \bar{U}(x,y,z),$$

which preserves constant along the solution curve. When the spacecraft is coasting, the energy of the system is fixed, and we could define the zero velocity curve. Note that the region surrounded by the zero velocity curve is called the forbidden region, in which region a spacecraft can not enter.

In the CR3BP, there are five equilibrium points known as the Lagrange points, three collinear equilibrium points (L_1, L_2, L_3) and two equilateral points (L_4, L_5). In particular, we focus on the L_1 and L_2, each of which has a local topology with the saddle×center structure.

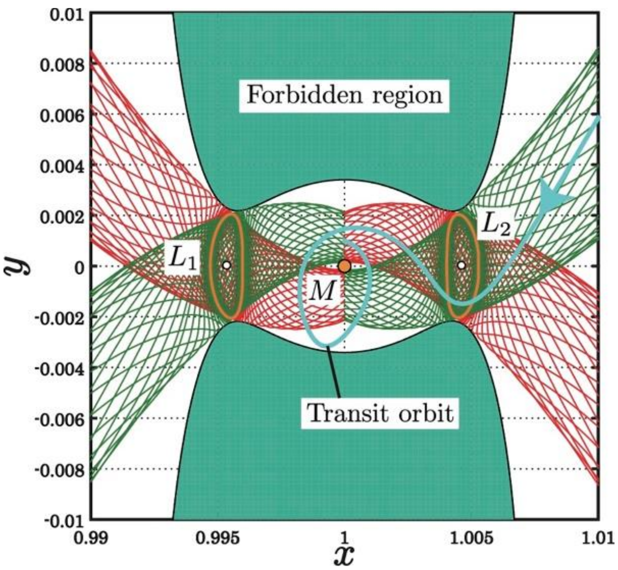

Fig. 2 Manifolds near Mars in CR3BP

There are unstable periodic orbits near the collinear Lagrange points, which associate with stable and unstable invariant manifolds called tubes. When the spacecraft is inside of an invariant manifold, the spacecraft moves through regions by passing through the collinear Lagrange points. If the spacecraft is outside of the tube, the spacecraft remains in the same region. In Fig. 2 we show the region near Mars. The orange orbits around two Lagrange points, L_1 and L_2 are the Lyapunov orbit. Associated with each Lyapunov orbit, there are two unstable manifolds that are repelling depicted in the color of red and two stable manifolds that are attracting in the color of green. A trajectory depicted in the color of blue, whose initial condition is located inside the green stable manifold, comes from the exterior region and passes through L_2, gate to get inside the Mars region. The trajectory locates inside the red unstable manifold derived from L_2, but it is located outside the green stable manifold derived from L_1, which traps the spacecraft in the Mars region.

III. Chemical-Electric Hybrid Propulsion Trajectory

A. Setting

The initial mass of the spacecraft is set as 600 kg. The specific impulse of chemical propulsion is 300 s and modeled as an impulsive maneuver. The specific impulse of electric propulsion is 3800 s and the maximum thrust is 40 mN. The maximum thrust of the electric propulsion is independent of the distance from the Sun.

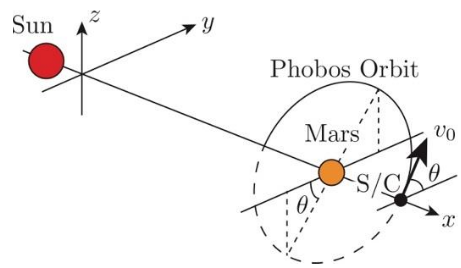

Fig. 3 Mars Escape Injection

For the Mars escape injection (MEI), we consider the time when orbital nodes of Phobos lie on the x-axis and the spacecraft is located at the ascending node. By the tangential thrust, the spacecraft escapes the Martian system from L_1.

B. Patched Three-and-Two-Body Method (Sphere of Influence)

The patched three-and-two-body method using the sphere of influence of Mars consists of two phases. The spacecraft would first be injected into a three-body trajectory after MEI and coast to the edge of Sphere of Influence (SOI), where the first integration point lies (coasting phase: green line). At the integration point, the trajectory may be considered as the Sun-Spacecraft two-body system. Beyond the integration point, the electric propulsion can be used to increase the v-infinity of the spacecraft and to reencounter with Mars for a gravity assist (thrust phase: black line). After getting the Mars gravity's assist, the spacecraft will be trapped on a Hohmann transfer orbit back to the Earth. Total time of flight is defined as the time from MEI to reencounter with Mars.

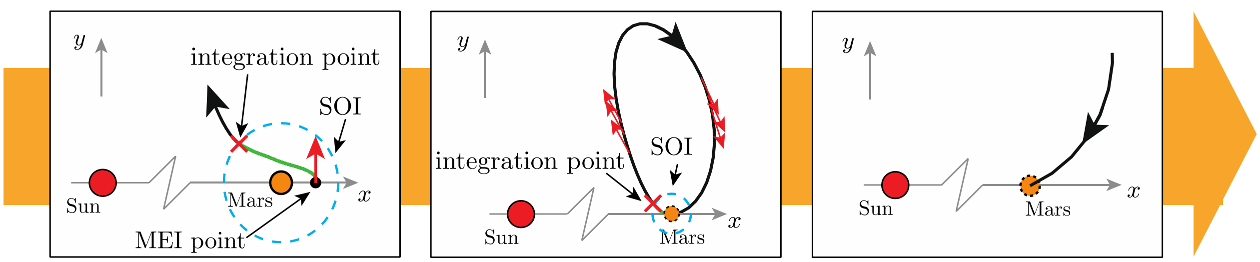

Fig. 4 Conceptual diagram of patched three-and-two-body method using the Sphere of Influence of Mars. Green line is the coasting phase in three-body problem. Black line represents the thrust phase is two-body problem. The three-body and two-body trajectories are integrated at the edge of the Sphere of Influence. Red arrows represent thrust.

In the thrust phase, the Sun-Spacecraft two-body problem is considered in the rotating frame. The differential equations of the trajectory are

$$\dot{x} = v_x,$$
$$\dot{y} = v_y,$$
$$\dot{z} = v_z,$$
$$\dot{v}_x = 2v_y + x - \frac{(1-\mu)(x+\mu)}{r_1^3} + \frac{T_x}{m},$$
$$\dot{v}_y = -2v_x + y - \frac{(1-\mu)y}{r_1^3} + \frac{T_y}{m},$$
$$\dot{v}_z = -\frac{(1-\mu)z}{r_1^3} + \frac{T_z}{m},$$
$$\dot{m} = -\frac{T}{I_{sp} g_0},$$

where

$$T = \sqrt{T_x^2 + T_y^2 + T_z^2}.$$

We impose the following constraints to be satisfied: The initial position and velocity are equal to those at the integration point. The terminal position is set to the position of the Mars and the terminal velocity to the v-infinity that allows the spacecraft to return to the Earth on the Hohmann transfer. Thrust magnitude is lower than the maximum thrust available. Total time of flight is lower than the given upper bound value.

The objective function is to maximize the final mass. The low-thrust trajectory problem is directly collocated with a nonlinear programming (DCNLP). The nonlinear programming (NLP) is solved by the sequential quadratic programming (SQP) method, which is implemented in MATLAB's toolbox of fmincon.

IV. Chemical-Electric Hybrid Propulsion Trajectory Optimal Solution

Two different points were used to compare the final mass, the sphere of influence and the transition point. These points are used to divide the three-body ballistic trajectory after the MEI and two-body low-thrust optimized trajectory to increase the v-infinity of the spacecraft. The optimized solution in all-three-body method is used to compare with a more realistic solution.

From Fig. 5, it apparently follows that using the edge of the sphere of influence as the transition from three-body to two-body problem has nearly the same mass as when the trajectory is optimized under three-body problem. The difference in the final mass is about 200g. Also, since the electric propulsion can be used earlier compared to the transition point, the final mass improves by more than 2kg especially in low ΔV_{MEI} cases.

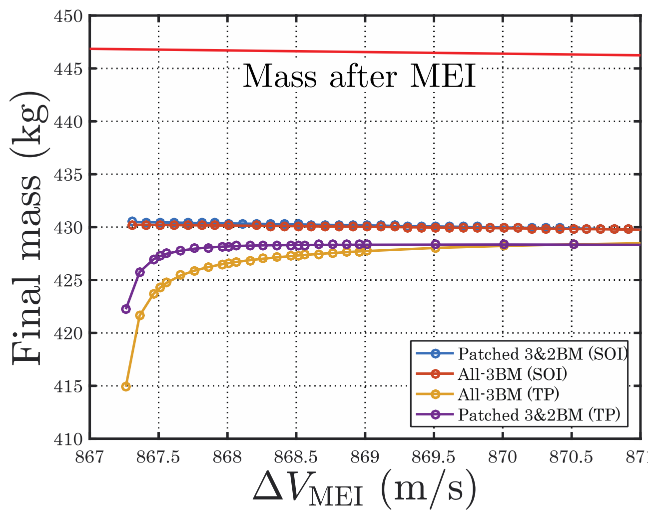

Fig. 5 Relation of ΔV_{MEI} and the final mass in four different methods are shown. Solutions of "All-3BP(TP)" and "Patched 3&2BM (TP)" are extracted from Horikawa et al. [7]

Considering the optimized trajectory obtained by the patched three-and-two-body method using the edge of sphere of influence, the thrust is used immediately. The energy seen from the three-body dynamics increases and the energy becomes greater than $E = -1.5$, where there would be no forbidden regions and spacecraft may be considered as a two-body problem. If we take the transition point, then the spacecraft may immediately pass through the transition point since the energy increases.

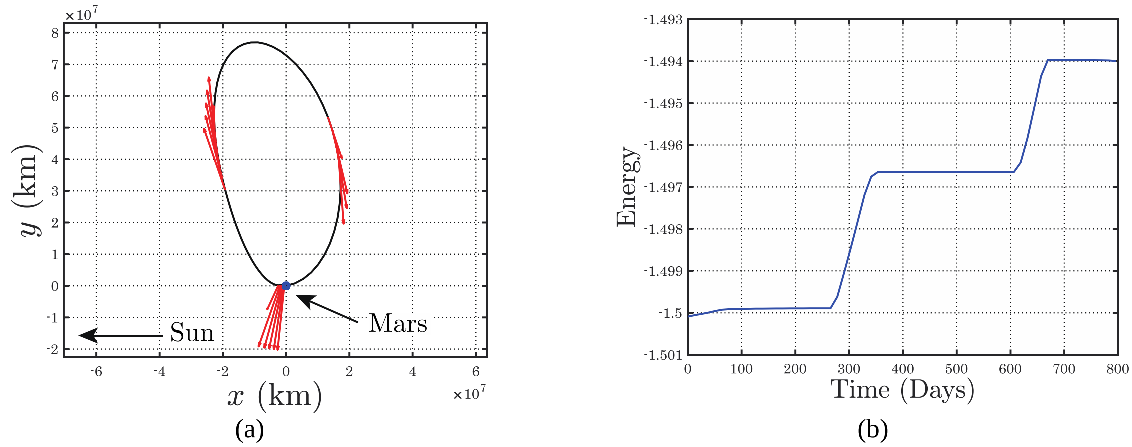

Fig. 6 (a) Optimized two-body trajectory in TOF of 1.2 Martian year, ΔV_{MEI}= 867.3 m/s, transition from the three-body system to the two-body system at the edge of sphere of influence. (b) The relation of time and expected three-body energy calculated along the trajectory.

V. Conclusion

In this study, we have focused on the construction of chemical-electric hybrid propulsion for the Earth return trajectory of Martian Moons eXplorer, in which we develop a method by combining the Sun-Mars-Spacecraft three-body dynamics and the Sun-Spacecraft two-body dynamics. We have compared the obtained trajectories by the patched three-and-two-body method using the transition point and SOI with the all-three-body method to show the accuracy of the simplified model.

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