

# Optimal Orbit Design for Multiple Space Debris Removal Utilizing Nodal Regression Induced by an Orbital Perturbation

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**Abstract:** This work deals with an orbit transfer problem aiming at multiple space debris removal. In order to find optimal solutions from a viewpoint of fuel consumption, this work derives an optimization method utilizing nodal regression induced by an orbital perturbation in Low Earth Orbit, J2 perturbation.

## 摂動による昇交点移動を用いたデブリ除去を目的とする マルチランデブー最適化に関する研究

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**概要** 複数のデブリ除去を目的とするランデブー問題を考える。燃料効率を考慮した複数のデブリ除去のためには、全てのデブリを除去するまでに必要な増速量が極力小さくなることが望ましいが、本研究では、地球低軌道上において特徴的な J2 摂動により引き起こされる昇交点移動を用いたランデブー最適化手法を導出する。

## 1 Introduction

Since the launch of Sputnik in 1957, the number of space debris consisting of rocket bodies and inactive satellites in Earth orbit have been increasing for about 60 years. Recently, Active Debris Removal (ADR) missions start to be discussed, in which various ways to capture and remove space debris using spacecraft are studied.

On the other hand, optimal orbit design to visit multiple space objects such as a planet, asteroid, and space debris with lower fuel consumption has been actively studied over the past several decades[1]-[19].

This study focuses on optimal trajectory with impulsive maneuver, and proposes an efficient orbit design utilizing nodal regressions induced by a perturbation force in Earth orbit, called a J2 perturbation. If amounts of semi-major axis between two orbits of debris to be transferred are different, the two orbital planes coincide with each other because of different rates of their nodal regressions. This study derives a

strategy of an orbit design introducing a relay orbit to effectively utilize the different nodal regressions, and a method to minimize velocity increments to transfer orbits for multiple debris using a Genetic Algorithm (GA).

This paper contains six sections. First, the current section briefly describes a background and objective of this study. The next section describes a nodal regression induced by a perturbation force called a J2 perturbation, as well as shows distributions of orbital planes expressed by two orbital elements, the inclination and the right ascension of ascending node (RAAN) for space debris in a priority list. After proposing an orbit design which includes a relay orbit to boost effectiveness of different nodal regressions between two debris' orbits in the third section, a GA-based optimization algorithm for the velocity increments is derived in the fourth section. The fifth section shows results from a series of numerical experiments, and discusses the results. Finally in the sixth section, concluding remarks are made.

## 2 Nodal Regression Induced by a J2 Perturbation

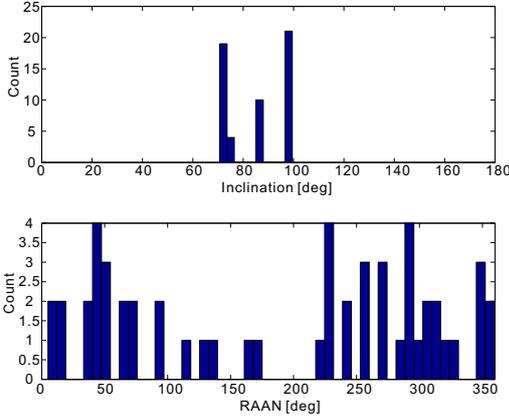


Fig. 1: Distributions of the inclination and the RAANs for catalogued debris in a priority list.

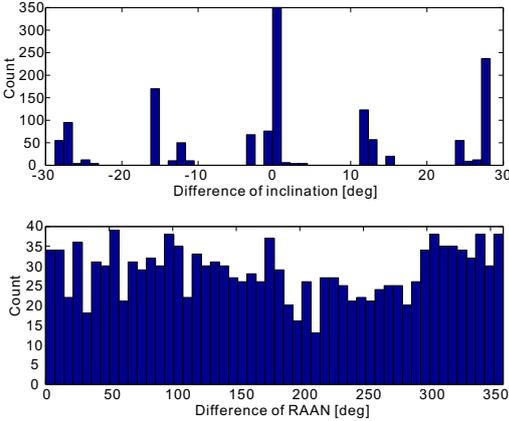


Fig. 2: Distributions of the differences of the two orbital elements for catalogued debris in a priority list.

According to a new priority list for catalogued space debris in Low Earth Orbit (LEO), there are 50 objects which should be preferentially removed considering risk and impacts of the fragmented debris on the space environment[20]. Focusing on the inclinations and the RAANs of their orbital elements, distributions of those two orbital elements are shown in Figure 1. The figures show that the RAANs of the critical debris are distributed in a relatively wide range, whereas their inclination angles are limited in three narrow ranges. Therefore, when a debris removal spacecraft transfers from one debris to another which are arbitrarily se-

lected in the priority list, it should overcome various amounts of difference for the RAANs, whereas that is not necessarily required for the inclination angle.

On the other hand, an object in LEO usually suffers from an orbital perturbation forces caused from a non-spherical part of Earth's gravitational attraction, or a so-called J2 perturbation. Such perturbation force results in time change of the RAAN,  $\dot{\Omega}$ , derived as the following equation[21]:

$$\dot{\Omega} = -\frac{3nR^2J_2}{2p^2} \cos i \quad (1)$$

where  $i$  is the inclination,  $p(=a(1-e^2))$  the semi-latus rectum, which can be computed from the semi-major axis  $a$  and the eccentricity  $e$ ,  $n$  the mean motion,  $R$  the Earth's radius, and  $J_2$  the first zonal harmonic of the Earth's gravitational potential.

As shown in the above equation, the nodal regression depends on three orbital elements,  $a$ ,  $e$ , and  $i$ . Thus, a relationship between two orbital planes for space debris dynamically changes with respect to time.

## 3 Impulsive Orbit Transfer Utilizing Nodal Regression

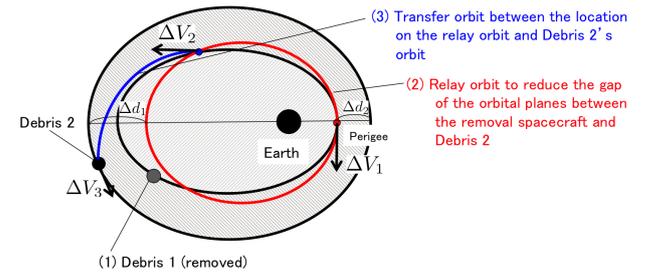


Fig. 3: An orbit transfer utilizing a relay orbit.

In the case of transferring between two orbits of debris which share the same inclination, that is possible as shown in the previous section, we can expect a low-thrust coplanar orbital transfer only by waiting for time to nearly coincide with the two RAANs. However, such an orbital transfer mission can be time-consuming. For example, in order to transfer between two orbital planes for three space debris in CASE 1, which will be shown

later in a later section, 278 to 4573 days are necessary only for waiting to coincide with the two orbital planes. In order to improve the effect of the difference of nodal regression between two orbits of a removal spacecraft and a target debris, this study proposes to utilize a relay orbit which accelerates reducing the angular difference between the two orbits. As briefly shown in Fig. 3, after removing a debris (Debris 1) on a primary orbit, the spacecraft is injected into a relay orbit at the perigee of the primary orbit, which could increase the difference of nodal regression rates between the relay orbit and a secondary orbit (an orbit of Debris 2). The spacecraft would keep on coasting on the relay orbit until the two orbital planes sufficiently get closer, and finally rendezvous with the debris (Debris 2) on the secondary orbit by a typical two-impulse maneuver for Lambert's problem.

Although the above orbital maneuver needs three impulses for a transition between two debris, which are one more than for a typical solution for Lambert's problem needs, it can decrease total velocity increments ( $\Delta V$ s) because large amount of velocity increments is often necessary for changing orbital planes. The most advantageous point of the proposed method is that it can decrease a mission duration at the same time.

#### 4 Genetic Algorithm-Based Optimization Method for Multiple Debris Removal

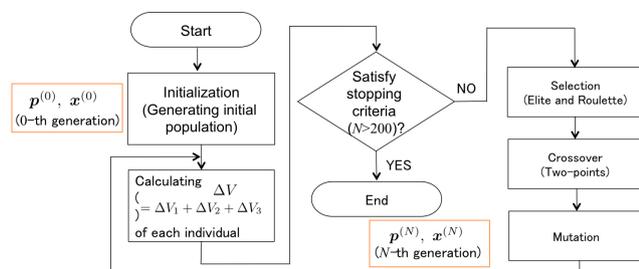


Fig. 4: A flowchart of the GA-based optimization for trajectory of multiple debris removal.

Aiming at an optimal solution from the strategy proposed in the previous section, a Genetic Algorithm-

(GA-) based method is derived. In this paper, considering single objective related to fuel consumption, a sum of velocity increments are defined as a fitness function of the GA. The optimized variables are defined as follows:

$$\begin{aligned} \mathbf{p} &= \{p_1, \dots, p_n\} \\ \mathbf{x} &= \{\Delta a_1, \Delta t_{c_1}, \Delta t_{t_1}, \dots, \Delta a_n, \Delta t_{c_n}, \Delta t_{t_n}\} \end{aligned} \quad (2)$$

where  $\mathbf{p}$  describes the order of capturing for  $n$  debris, and  $p_j$  ( $j = 1, \dots, n$ ) shows the identity number for the  $j$ -th debris.  $\Delta a_j$  describes the difference of semi-major axes between the  $(j - 1)$ -th debris' and the  $j$ -th relay orbits,  $\Delta t_{c_j}$  the sum of the coasting times on the  $(j - 1)$ -th debris' and the  $j$ -th relay orbits, and  $\Delta t_j$  the transfer time between the  $j$ -th relay and the  $j$ -th debris' orbits.

Fig. 4 shows a flowchart of the computational algorithm. Note here that a fitness function is defined as follows:

$$\Delta V = \sum_{j=1}^K \Delta V_1^j + \Delta V_2^j + \Delta V_3^j, \quad (4)$$

where  $\Delta V_1^j$  describes the Delta V required to inject into a relay orbit for transferring to the  $j$ -th ( $j = 1, \dots, K$ ) debris, and  $\Delta V_2^j, \Delta V_3^j$  the ones necessary for a typical maneuver to solve Lambert's problem.

#### 5 Numerical Experiments

In order to demonstrate an effectiveness of the method proposed in the previous sections, some numerical experiments based on orbital data of actual space debris are conducted.

Table 1 shows the orbital elements for an initial parking orbit of a removal spacecraft and three cases of target debris. The number of target debris in the experiments is three, and a starting time (or an epoch for the orbit propagations) was set on the 1st of July in 2020. In each case, the target debris were selected with respect to the difference of the orbital planes, which are determined by inclination and RAAN. Whereas the inclinations for the spacecraft and target debris are almost the same, various RAANs are assumed for the

three cases. For example, the maximum difference of the RAANs for Case 1 becomes about 40 degrees, and the one for Case 3 becomes as large as 302 degrees.

Table 2 shows the experimental conditions on the GA. Note here that the number of population was set to search for a solution space, which should be relatively large because of long coasting time. On the other hand, some constraints for the optimized variables and the maximum number of rotations to solve the Lambert's problem were set as shown in the same table considering appropriate values for an actual mission scenario.

Results of the optimization for all the cases are shown in Table 3. For comparison, results obtained from another experimental condition in which the fitness evaluations were conducted based on a typical two-impulse maneuver for two debris' orbits are also shown. As shown in these results, the Delta Vs obtained from the proposed method are about tenth to twentieth smaller than those based on a typical Lambert's solver, whereas the total time to rendezvous with all the debris becomes larger.

The effectiveness of the proposed orbital design utilizing the relay orbit is shown in the above results, however, global optimalities of the obtained results are not necessarily clear. Since a coasting time for a primary orbit and a relay orbit is randomly selected in GA, it is not guaranteed for an angular difference between the relay orbit and a secondary orbit to be minimum. Choosing appropriate GA parameters is an important issue to be discussed in future work. Also, considering that the relay orbit proposed in this study aims to decrease the mission duration  $\Delta t_{\text{total}}$ , a multi-objective optimization problem should be treated.

## 6 Conclusion

This paper proposes an optimal orbit design for multiple space debris removal utilizing a difference of the nodal regressions, which is induced by J2 perturbation occurring to the objects in Low Earth Orbit. Whereas the effectiveness of introducing a relay orbit to actively change the nodal regression is clear, discussions about the global optimality for the proposed strategy still

remain. Some issues on the optimality would be improved in future work.

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Table 1: The orbital elements of the removal spacecraft and target debris.

		$a$ [km]	$e$	$i$ [deg]	$\Omega$ [deg]	$\omega$ [deg]	$f$ [deg]
Parking orbit (Initial)		7176.1	0.0001	98.57	267.78	357.67	0.00
Case 1	1:Zenit-2	7192.6	0.0033	98.45	295.27	101.54	273.32
	2:Resurs-O2	7195.9	0.0017	98.53	297.26	50.00	292.36
	3:NOAA 17	7283.3	0.0021	98.44	307.57	44.95	269.45
Case 2	1:Zenit-2	7292.6	0.0033	98.45	295.27	101.54	273.32
	2:Resurs-O2	7195.9	0.0017	98.53	297.26	50.00	292.36
	3:SPOT 3	7202.9	0.0038	99.02	350.75	101.27	131.84
Case 3	1:Zenit-2	7192.6	0.0033	98.45	295.27	101.54	273.32
	2:SPOT 3	7202.9	0.0038	99.02	350.75	101.27	131.84
	3:Ariane 40	7253.2	0.0009	98.90	48.86	71.49	5.84

Table 2: Experimental conditions on the GA algorithm.

Number of population	4000
Number of generation	300
Crossover rate	0.8
Mutation rate	0.01
Constraints for the Optimized variables ( $j = 1, \dots, K$ )	$-300.0\text{km} \leq \Delta a_j \leq 300.0\text{km}$ $1.0\text{day} \leq \Delta t_{c_j} \leq 630.0\text{days}$ $0.0417\text{days} \leq \Delta t_{t_j} \leq 0.5\text{days}$ $N_{\max} = 50$

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Table 3: Results of the numerical experiments.

Case 1	
The proposed method: $\Delta V = 0.26\text{km/s}$ , $\Delta t_{\text{total}} = 1632.2\text{days}$ $\mathbf{p} = \{1, 3, 2\}$ $\mathbf{x} = \{-95.3, 415.4, 0.5,$ $\quad -60.0, 617.7, 0.3,$ $\quad 27.1, 597.9, 0.3\}$	GA with a typical Lambert's solver: $\Delta V = 4.27\text{km/s}$ , $\Delta t_{\text{total}} = 1313.5\text{days}$ $\mathbf{p} = \{1, 2, 3\}$ $\mathbf{x} = \{0.0, 630.0, 0.3,$ $\quad 0.0, 82.4, 0.3,$ $\quad 0.0, 600.4, 0.1\}$
Case 2	
The proposed method: $\Delta V = 0.89\text{km/s}$ , $\Delta t_{\text{total}} = 985.9\text{days}$ $\mathbf{p} = \{1, 2, 3\}$ $\mathbf{x} = \{-182.4, 240.3, 0.2,$ $\quad -90.6, 129.3, 0.5,$ $\quad -245.9, 615.2, 0.4\}$	GA with a typical Lambert's solver: $\Delta V = 10.80\text{km/s}$ , $\Delta t_{\text{total}} = 16.6\text{days}$ $\mathbf{p} = \{1, 2, 3\}$ $\mathbf{x} = \{0.0, 1.0, 0.4,$ $\quad 0.0, 13.3, 0.4,$ $\quad 0.0, 1.0, 0.5\}$
Case 3	
The proposed method: $\Delta V = 0.96\text{km/s}$ , $\Delta t_{\text{total}} = 1405.1\text{days}$ $\mathbf{p} = \{1, 2, 3\}$ $\mathbf{x} = \{-222.4, 193.4, 0.2,$ $\quad -250.6, 602.9, 0.4,$ $\quad -184.7, 607.8, 0.3\}$	GA with a typical Lambert's solver: $\Delta V = 17.94\text{km/s}$ , $\Delta t_{\text{total}} = 82.7\text{days}$ $\mathbf{p} = \{1, 2, 3\}$ $\mathbf{x} = \{0.0, 20.7, 0.2,$ $\quad 0.0, 15.8, 0.3,$ $\quad 0.0, 45.4, 0.2\}$

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