

# An analysis method for tethered system and a study on their efficient deployment

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**Abstract** In recent years, systems with masses and extremely flexible components (hereinafter called “SMEF”) are often employed for satellites in order to realize various vast structures in orbit, and an efficient analysis method for such a system is required. In this paper, we propose an analysis method which can perform efficient and high speed calculation by describing the state transition of the SMEF in Linear Complementarity Problem by utilizing the SMEF characteristics. By using proposed calculation method, we conduct optimization analysis of deployment behaviors of tether satellites, as a practical example of SMEF, and make a proposal for efficient development of the tether satellites.

## テザー衛星に対する解析手法と効率的な展開における研究

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**摘要** 近年、大規模な宇宙構造物を実現するために、極めて柔軟かつ軽量な材料からなる柔軟部および質量部を有するシステム（以下、「超柔軟体 - 質量系(SMEF)」）が用いられることがあり、そのようなシステムの効率的な解析手法が求められている。本研究では、SMEFの特徴を利用し、SMEFの状態遷移を線形相補性問題に帰着させることで、効率的で高速計算が可能な解析手法を提案している。提案手法を用いて、SMEFの実用例として、テザー衛星の展開挙動の最適化解析を行い、テザー衛星の効率的な展開に対する一提案を行う。

### 1. Introduction

In recent years, systems with masses and extremely flexible components (hereinafter called “SMEF”) are often employed for satellites in order to realize various vast structures in orbit. For example, S310-36 <sup>(1)</sup> project demonstrated a deployment of a large antenna. As Figure 1 shows, the antenna had a triangle shape and had consisted of a mesh made by thin strings of less than 1[mm] diameter. It also had a mother satellite and three daughter satellites.

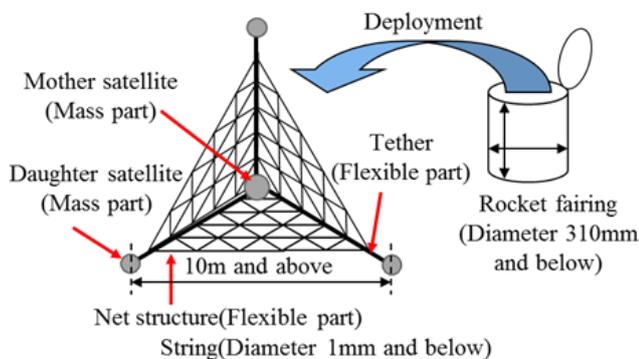


Fig. 1 Example of the SEFM (S310-36 project)

In such a system, mass and stiffness about the non-axial direction of the flexible components are very small, therefore the effect by them on the whole dynamics of the system can be ignored if there is no tensile force in the flexible components.

On the other hand, if tensile force exists in some flexible components, the whole dynamics of the system is subject to the influence of those flexible components. Therefore, state transition of the system arises depending on the configuration and careful treatments are required for the analysis of such a system, because number of the combination of state increases dramatically depending on the number of masses and flexible components, which results in the difficulty in the computation. So, the conventional method such as the finite element method has a problem because such a conventional method can not deal with aforementioned state transition.

Sugawara et al. found analogy between the state transition of the SMEF and contact problem and, proposed an effective analysis method for the SMEF <sup>(2)</sup> by the use of Pfeiffer's method <sup>(3)</sup>. Pfeiffer described the state transition of the contact problem in Linear Complementarity Problem (hereinafter called “LCP”) and solved the problem in an efficient manner <sup>(3)</sup>. In the same way, Sugawara described the state transition of the SMEF by LCP and carried out an analysis of linear motion of the SMEF. Numerical examples shows the validities of the proposed method and the method gives a moderate accuracy and low computational cost. Sugawara

also conducts comparative analyses of the proposed method and the conventional method, in one case, we succeeded in reducing the calculation time of 99.7% compared with the conventional method (Non-linear FEM). Fast calculation is realized. Experimental validation of the proposed method for linear motion was also performed, and good agreement was obtained <sup>(4)</sup>. Usefulness of the proposed method was confirmed by analysis and experiment

One of the advantages of the proposed method is fast calculation. Compared with the conventional method, it is feasible to perform iterative calculation because of its fast calculation. In other words, system optimization and parameter determination are possible by iterative calculation.

In this research, as an object of optimization, we use a tether satellite having a satellite with a thruster at the tip of a tether as shown in Fig. 2.

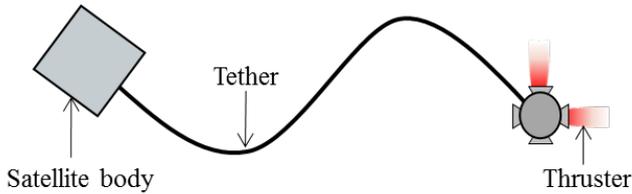


Fig. 2 The tethered satellite model

Tether satellite is a satellite that has a long and strong string called a tether, and its tether is used for various purposes: capturing asteroids, recovery of space debris, the function of cables in space elevators, propulsion and posture stabilization of satellites, and so on. In performing these missions, it is sometimes required to deploy and recover the tether part promptly, and the tether's behavior often involves undesirable rotational movements and vibrations. Therefore, suppressing these effects and realizing more efficient tether behavior is desired. Here, it is known that by preliminary numerical analysis, it is possible to passively change the behavior of the tether by attaching several masses to the intermediate positions of the tether. The mass attached to the tether is called the intermediate mass and Fig.3 shows the tether with the intermediate masses. These intermediate masses have the role of diverging the kinetic energy of the system. It is known that the behavior of the tether changes by changing the number, mass, position of the intermediate mass, and as a result, we expect that the behavior of the tether is changed.

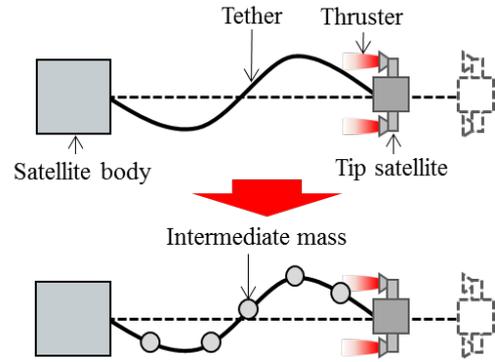


Fig. 3 Attaching the intermediate masses to the tether

Thus, there is a parameter dependence on the behavior of the tether, and if we can determine appropriate parameters, we think that passive and more efficient behavior (low consumption fuel) can be realized.

In this paper, by performing iterative calculation using fast calculation which is the feature of the proposed method, we confirm parameter dependence of the tether satellite and make a proposal on deployment of tethered satellites

## 2. Proposed method

The basic model of SMEF is shown in Fig.4. As Fig.4 shows, there are two state transitions in the dynamics of the SMEF. One is the transition from a state with tensile force to that without tensile force (F1), and the other is the transition between states before and after occurrence of impulsive tensile force in the string (F2).

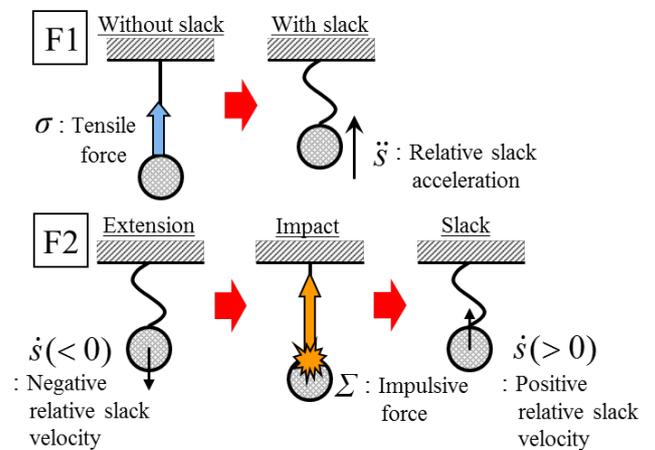


Fig. 4 Two state transitions in the SMEF

As shown in Fig.5, Sugawara found an analogy between the state transition of the SMEF and contact problem. Fig.5 shows an example of analogy in F1.

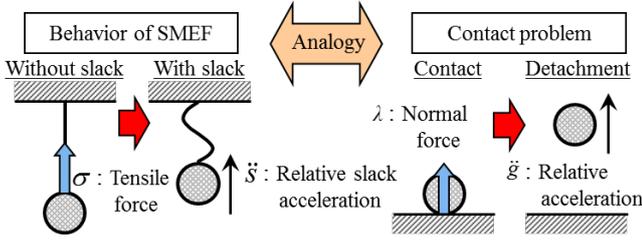


Fig. 5 An analogy between the SMEF and Contact problem

Pfeiffer et al. described the state transition of the contact problem in LCP and solved the problem in an efficient manner<sup>(3)</sup>. In the same way, Sugawara described the state transition of F1 and F2 by LCP.

As an example of SMEF formulation, an example of one-dimensional  $N$ -mass SMEF is shown in Fig.6.

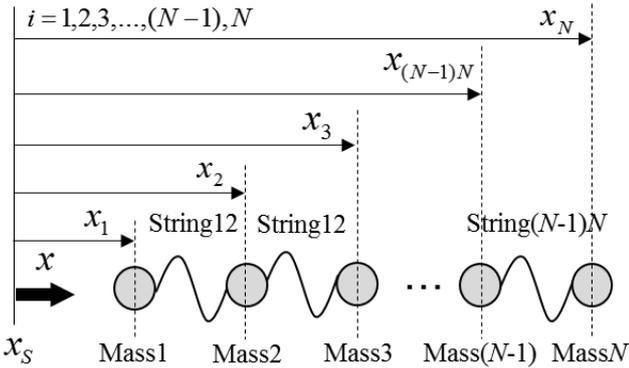


Fig. 6 One-dimensional multi-mass SMEF

Assuming that the number of masses is  $N$ , from the reference point  $x_s$ , we call Mass1, Mass2, ..., Mass $N$ . Generalized coordinates for each Mass are defined as shown in Fig.6. The string that is connected between Mass  $(i-1)$  and Mass  $i$  is called String  $(i-1)i$ . Focusing on Mass  $(i-1)$  and Mass  $i$ , relative slack coordinates and each parameter are defined as shown in Fig.7. String's mass and stiffness about the non-axial direction are very small, therefore the effect on the whole dynamics of the SMEF can be ignored if there is no tensile force in Strings. The length of String  $(i-1)i$  is  $l_{(i-1)i}$ . Mass is mass point and mass of Mass  $i$  is  $m_i$ .  $\epsilon_{(i-1)i}$  is the coefficient of momentum exchange of String  $(i-1)i$  after impulsive tensile force.  $\sigma_{(i-1)i}$  is tensile force of String  $(i-1)i$  and  $f_i$  is external force applied on Mass  $i$ .

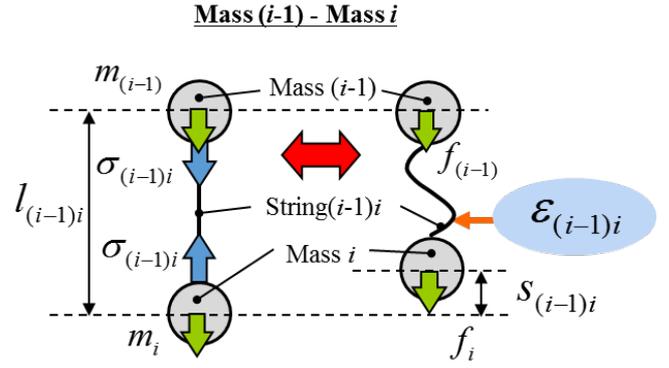


Fig. 7 One-dimensional multi-mass SMEF

The relative slack displacement  $s$ , velocity  $\dot{s}$  and acceleration  $\ddot{s}$  are given as

$$s = W^T q + c, \dot{s} = W^T \dot{q}, \ddot{s} = W^T \ddot{q} \quad (1)$$

where the generalized coordinates  $q$  is given by

$$q = [x_1 \ x_2 \ \dots \ x_{(N-1)} \ x_N]^T \quad (2)$$

and the relative slack displacement  $s$  is given by

$$s = [s_{12} \ s_{23} \ \dots \ s_{(N-1)N}]^T \quad (3)$$

$W$  is Jacobian matrix of this model and  $c$  is a vector function which depends on the length of Strings.  $(\cdot)$  denotes differentiation with respect to time  $t$ . The equation of motion of this model is as follows:

$$M\ddot{q} - h - W\sigma = 0 \quad (4)$$

where  $h$  is a vector that includes external force. As Fig.5 shows, in the state transition F1,  $\dot{s}$  and  $\sigma$  have complementarity as follows:

$$\dot{s} \geq 0, \sigma \geq 0, \dot{s} \cdot \sigma = 0 \quad (5)$$

and linear equation with respect to  $\ddot{s}$  and  $\sigma$  can be expressed by combination of Eq.(1) and (4) as

$$\ddot{s} = A\sigma + B \quad (6)$$

where  $A$  and  $B$  are the parameters which are determined from the system parameters. Eq.(5) and (6) are called LCP of acceleration level. In the state transition F2,  $\dot{s}$  and  $\Sigma$  show complementarity. F2 can also be described by LCP and called LCP of velocity level. Consequently, the state transition problems in the SMEF are also solvable by LCP of acceleration level and velocity level in efficient manner.

### 3. Tether satellite model

Fig.8 shows a tethered satellite as an analysis

model. As shown in Fig.8, the system is connected with a tip satellite and a tether on both sides, centering on the mother satellite.

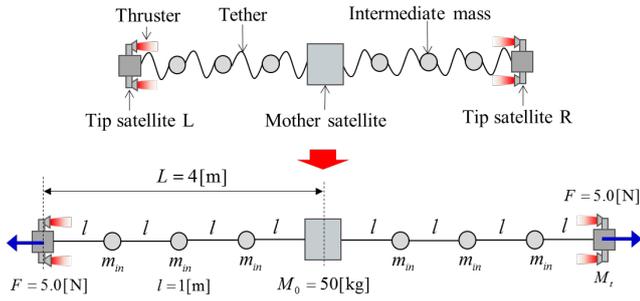


Fig.8 The analysis model

It is assumed that the satellites and the intermediate masses move only in the one-dimensional direction. Here, the total number of masses is 9, and the number of intermediate masses attached to one tether is 3. The mass of the mother satellite is  $M_0$ , the mass of the tip satellite is  $M_t$  and the intermediate mass is  $m_{in}$ . Length of tether is  $L$  and the length of Strings divided by the intermediate masses is  $l$ . A thruster is attached to each tip satellite, and the tether is deployed with a constant thrust force  $F$ . The reference point  $x_s$  is set on the initial position of the mother satellite and  $x_s = 0$ . However, note that the left direction of the figure is positive. The mass of the Strings and the rigidity of the bending deformation are assumed to be negligibly small, and the rigidity in the axial direction is sufficiently large so that the length does not change by deformation. That is, Strings do not stretch or shrink. Also, we do not think about the effects of heat, gravity and air resistance. No external force is applied to the intermediate masses. Note that collision between masses is not considered in this analysis.

#### 4. Numerical analysis

A numerical analysis is conducted by the proposed method. Analysis conditions are summarized in Table 1.

Table 1. Analysis conditions

Parameter	Value
$M_0$	50 [kg]
$M_t$	2.0 [kg]
$F$	5.0 [N]
$L$	4 [m]
$l$	1 [m]

As initial condition, each object has no initial velocity and the initial position is given by how slack String is in the initial state. In this analysis, the slack amount is set to 0.1 times the length of String. That is, the distance between the Masses in the initial state is 0.1 [m].

In this analysis, we compare the settling time by changing the intermediate mass  $m_{in}$  in case of a coefficient of momentum exchange  $\varepsilon$ . The settling time in this study is the time until the tether fully develops and the system completely stops.

As an analysis example, Fig.9 shows the analysis results when the intermediate mass  $m_{in}$  is 0.1 [kg] and the coefficient of momentum exchange  $\varepsilon$  is 0.8. Analysis was performed with the step time of 0.01 [s] and the integration time of 5 [s].

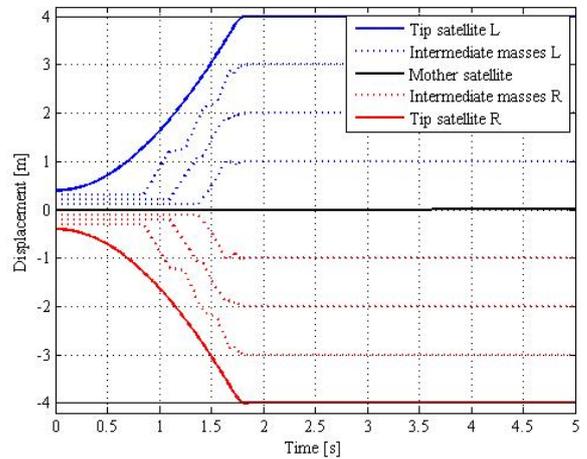


Fig.9 The analysis result

The tip masses at both ends of the tether satellite begin to expand due to the thrust and the intermediate masses begin to be pulled by the tip masses. Impulsive tensile forces also occur between the intermediate masses, and the kinetic energy of the entire system is attenuated by the coefficient of momentum exchange. Here, the settling time is 1.854 [s]. The settling time when no intermediate mass is arranged is 15.274 [s], and it can be seen that the time to completion of development can be reduced by about 89% by attaching 3 intermediate masses of 0.1 [kg] to the tether,

By attaching the intermediate mass to the tether as shown in Fig.9, the intermediate masses play the role of conserving and diverging the energy of the system, so the settling time can be shortened. However, simply reducing the intermediate mass cannot shorten the settling time. On the contrary, it is known that if  $m_{in}$  is large, the deployment speed

of the entire system will be delayed. In other words, we think that the intermediate mass has an optimum value. Here, by using high-speed calculation which is an advantage in the proposed method, if the intermediate mass when the settling time is the shortest (optimum mass) can be obtained by iterative calculation, we think that it can be one guideline in considering the structure of tethered satellites.

We perform repeated analysis by changing the intermediate mass  $m_{in}$  from 0.01 [kg] to 4.0 [kg] by 0.01 [kg]. The step time is 0.01 [s]. The analysis results of  $\varepsilon = 0.8$  are shown in Fig.10.

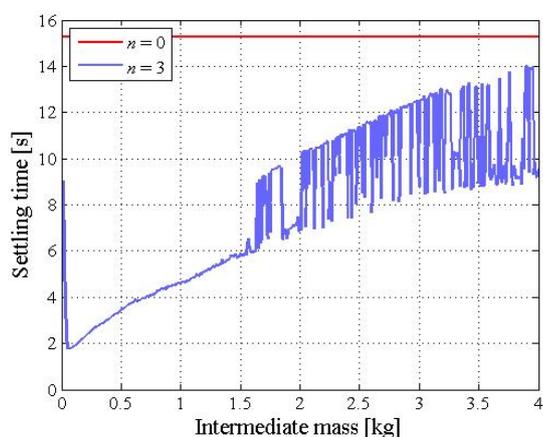


Fig.10 The analysis result of  $\varepsilon = 0.8$

The vertical axis represents the settling time, and the horizontal axis represents the intermediate mass.  $n$  is the number of the intermediate masses attached to one tether, and the line parallel to the horizontal axis shows the settling time when no intermediate mass is attached ( $n = 0$ ). In other words, the result below the parallel line indicates that the settling time could be shortened by attaching the intermediate masses to the tether. From Fig.10, it can be seen that there are the intermediate mass that the settling time is the smallest (optimum mass). We carried out the analyses in the case of other values of  $\varepsilon$ , and a similar tendency was found.

The discontinuity in the figure is due to the timing of impulsive tensile force. By arranging appropriate masses in the middle of the tether, it is possible to change the settling time passively. Thus, by the iterative calculation by the proposed method, it is possible to determine parameters that enable efficient deployment behavior

## 5. Conclusion

By using high speed calculation which is a feature of the proposed method used in this study, the iterative calculation is practical. It is possible to determine SMEF parameters such as a tether satellite in a round-robin manner by using iterative calculation. Although we focused on the mass of intermediate mass in this paper, as a future work, it is also necessary to investigate the influence such as the number of the intermediate masses and how to attach them to the tether.

## References

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