Magnetometers are commonly used attitude sensors for Earth orbiting spinning spacecraft. Specifically for spinning small satellites they are ideal sensors as they are cheap, small and lightweight. If the magnetometers are error free, attitude of the satellite can be estimated with a sufficient accuracy. However an in-orbit calibration algorithm is required for using magnetometers onboard the small satellites. The subsystems are closely located on these satellites and magnetometer errors vary due to the electromagnetic interference. In this study, two simple algorithms for real time estimation of magnetometer errors for spinning spacecraft are proposed. First two quasi-measurements for the bias estimation are built benefiting from the spin dynamics. A pseudo-linear Kalman filter (KF) with state-dependent measurement model and a linear KF with linearized measurement model are designed. The filters are tested for a hypothetical nanosatellite via numerical simulations. The results are discussed in detail and suggestions for implementation of the algorithms are given.

I. Introduction

Spin-stabilization is the simplest method of stabilizing the attitude of a spacecraft. In the early era of space exploration it was widely employed. Today, as the number of micro and nanosatellites increases, interest in spin-stabilization revives, mainly because of its simplicity that makes the method suitable for the small satellites\(^1\). The spin-stabilization is used for the nanosatellites either for the whole mission duration or temporarily to conduct an experiment, especially to de-orbit the spacecraft\(^2\).

Magnetometers are widely used onboard the small satellites as they are lightweight, inexpensive and reliable. However a real-time magnetometer calibration algorithm is required for correcting their measurements. Using a real-time algorithm is specifically advantageous as the subsystems are closely located on small satellites and the magnetometer errors may vary due to the electromagnetic interference\(^3\).

Magnetometer calibration for spinning spacecraft is usually not treated distinctly from the common calibration algorithms which are used also for three-axis stabilized spacecraft. For example for the ST-5 and THEMIS missions of NASA\(^4\), the Two-Step algorithm\(^5\) is used. The Two-Step method is a fast and robust algorithm for estimating the magnetometer errors but it is only applicable as a batch method, not in real-time. A real-time attitude independent magnetometer calibration approach is derived in Ref.\(^6\) and also used for a spinning spacecraft in Ref.\(^7\). This algorithm works well only with an Unscented Kalman Filter (UKF), which comes with a computational cost, and has convergence issues for spinning spacecraft\(^7\).

In Ref.\(^8\) magnetometers for a spinning projectile are calibrated in real-time with a sliding-window filter formulated in the EKF framework. Despite being used for a spinning projectile, the filter algorithm does not make use of any information that can be extracted from the spin motion. Besides, the measurements within an N-sized window should be kept in the filter memory. This increases the computational load.

A calibration method for magnetometers on a spinning spacecraft is discussed in Ref.\(^9\). It is one of the few studies that are intended solely for spinning spacecraft magnetometers and making advantage of the spin dynamics. As a drawback this method requires on-ground processing with an excessive amount of data collected from different periods. In fact, it is proposed for correcting the readings from scientific magnetometers rather than calibrating the attitude sensors.

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In this paper we propose two simple Kalman filter (KF) algorithms for real-time attitude-independent magnetometer bias estimation for spinning spacecraft. First we show that it is possible to benefit from the spin motion of the spacecraft to build two quasi-measurements for the bias estimation. This considerably improves the convergence characteristics of the filter. The second contribution is using a pseudo-linear KF algorithm to estimate the magnetometer bias without any need for attitude knowledge. The formulation for a linear KF that uses centered variables is also given. The performance of the KF algorithms is investigated using the simulated data for a nanosatellite.

II. Magnetometer Measurements for a Spinning Spacecraft

The measurement equation for three-axis magnetometer (TAM) at \( k^{th} \) time step is,

\[
B_k = A_k H + b_k + v_{b,k},
\]

where, \( A \) is the attitude matrix for transformation from the inertial frame to the spacecraft body frame, \( b \) is the bias vector and \( v_b \) is the Gaussian zero-mean measurement noise with covariance \( \sigma_b^2 \) as \( v_b \sim \mathcal{N}(0, \sigma_b^2) \).

Consider a spin-stabilized spacecraft rotating about body \( Z \) axis with an angular rate \( \omega \). The body \( Z \) axis is called as spin axis and the plane normal to this axis is referred as spin plane. The assumptions are:

- There is no dynamic error (e.g. coning) for the spacecraft or these errors have been already estimated and corrected.
- Each sensor of the TAM is aligned with the body axes.
- Bias is the only magnetometer error and there is no scaling, nonorthagonality or sensor misalignment for the measurements.

The measurements for the spin plane magnetometers have spin-modulated dynamics. In an ideal case, where there is no bias, the measurements in the spin plane oscillate about a zero-mean (Fig.1)\(^{9} \). However when the bias exists, the mean of oscillation for the spin plane measurements will shift depending on the magnitude of the bias in these measurement terms (Fig 2). Regarding this fact, averaging the measurements for the spin plane magnetometers within a window gives,

\[
\hat{b}_x = \frac{1}{N} \sum_{i=k-N+1}^{k} B_{x,i} ,
\]

\[
\hat{b}_y = \frac{1}{N} \sum_{i=k-N+1}^{k} B_{y,i} .
\]

Here \( N \) is the window size. \( \hat{b}_x \) and \( \hat{b}_y \) are noisy estimates for the corresponding bias terms. If the window size is large, the noise in the bias estimates, \( \hat{b}_x \), \( \hat{b}_y \) can be reduced and the estimation accuracy can be improved. But apparently this is not practical for real-time applications. Instead we can keep the window size small and use the averaged values as measurements for a KF algorithm:

\[
\bar{b}_x = \frac{1}{N} \sum_{i=k-N+1}^{k} B_{x,i} = b_x + v_{b_x},
\]

\[
\bar{b}_y = \frac{1}{N} \sum_{i=k-N+1}^{k} B_{y,i} = b_y + v_{b_y}.
\]

\[
v_{b_x} = \frac{1}{N} \sum_{i=k-N+1}^{k} v_{b_x} ,
\]

\[
v_{b_y} = \frac{1}{N} \sum_{i=k-N+1}^{k} v_{b_y} .
\]

Here \( v_{b_x} \) and \( v_{b_y} \) are Gaussian white noises as \( v_{b_x/y} \sim \mathcal{N}(0, \sigma_{b_x/y}^2 / N) \).

Thus far we have two measurements for estimating the bias terms for the magnetometers in the spin plane; we need a third measurement to estimate the bias for the magnetometer aligned in the spin direction. As clearly seen in Fig.1 and Fig.2, there is no spin modulation for this magnetometer measurements and averaging procedure is not
applicable to get $\hat{b}_y$ estimate (or noisy measurement). Instead, we use the fact that the magnitude of the magnetic field must be same in the inertial and spacecraft body frames in an ideal case. The procedure starts with eliminating the attitude dependence in Eq.1. We transpose the terms and compute the square:

$$|H_i|^2 = |B_i|^2 - 2B_i \cdot b_i + |b_i|^2 - 2(B_i - b_i) \cdot v_{B,i} + |v_{B,i}|^2.$$  \hfill (5)

![Fig. 1 Magnetometer measurements in ideal case.](image)

Next we define the measurement as the difference of the magnetic field magnitudes in two frames,

$$\beta_i = |B_i|^2 - |H_i|^2 = 2B_i \cdot b_i - |b_i|^2 + \eta_i.$$  \hfill (6)

Here,

$$\eta_i = 2(B_i - b_i) \cdot v_{B,i} + |v_{B,i}|^2,$$  \hfill (7)

is the measurement noise, which may be assumed as Gaussian noise as $\eta_i \sim \mathcal{N}(\mu_i, \Sigma_i)$ for especially a satellite in low-Earth orbit since $B_i \gg v_{B,i}$ \hfill (5). Regarding $R_i = \sigma_{B,i}^2$ here

$$\mu_i = -\text{tr}(R_i),$$  \hfill (8a)

$$\Sigma_i = 4(B_i - b_i)^T R_i (B_i - b_i) + 2\text{tr}(R_i^2).$$  \hfill (8b)

As a result we have three measurements to estimate all three magnetometer bias terms. Two of these equations are derived based on the information that is inherent to the spin motion of the spacecraft. The third equation is
obtained using the magnitude difference between the reference and measured magnetic field vectors. Using these measurements a KF can be designed to estimate the bias terms in real-time. How we treat the nonlinearity in the third measurement equation determines the type of the filter we use for bias estimation.

III. Kalman Filtering for Magnetometer Bias Estimation

The system equation for the constant magnetometer bias vector is,
\[ \dot{\mathbf{b}} = 0. \] (9)

The Eq.(6) is nonlinear in terms of the bias vector \( \mathbf{b} \). So a linear KF algorithm cannot be used unless this equation is further processed to save from nonlinearity. In this study we examine two KF algorithms that differ depending on the method for treating the nonlinearity.

A. Pseudo-Linear Kalman Filter for Magnetometer Bias Estimation

The first option is to leave Eq.(6) as it is and build a pseudo-linear measurement model for the KF as,

\[
\begin{bmatrix}
\dot{b}_x \\
\dot{b}_y \\
\dot{b}_z
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
2B_x - b_x & 2B_y - b_y & 2B_z - b_z
\end{bmatrix}
\begin{bmatrix}
b_x \\
b_y \\
b_z + \eta
\end{bmatrix}.
\] (10)

This measurement model is used together with the process model (Eq.9) in a pseudo-linear KF to estimate the magnetometer bias vector. The algorithm is an ordinary linear KF that operates on a linear model but the measurement matrix given in Eq.(10) is state-dependent.

Note that the measurement variance in Eq.(8b) is a function of the estimated bias terms and it can be calculated using the estimates from the previous step, \( \hat{\mathbf{b}}_{k-1} \).

B. Linear Kalman Filter for Magnetometer Bias Estimation

Another possibility for the KF measurement model is to use centering method\(^5,6\) to linearize the Eq.(6). In this case the method leads to
\[ \hat{\beta}_k = 2\bar{B}_k \cdot \mathbf{b} + \tilde{\eta}_k \] (11)
as the third measurement equation instead of Eq.(6). Then the measurement equation for the KF becomes,

\[
\begin{bmatrix}
\dot{b}_x \\
\dot{b}_y \\
\dot{b}_z
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
2\bar{B}_{x,k} - b_x & 2\bar{B}_{y,k} - b_y & 2\bar{B}_{z,k} - b_z
\end{bmatrix}
\begin{bmatrix}
b_x \\
b_y \\
b_z + \tilde{\eta}
\end{bmatrix}.
\] (12)

The centered variables are defined as,
\[ \bar{\beta}_k = \beta_k - \Sigma \frac{1}{\Sigma_k} \beta_k, \quad \bar{B}_k = B_k - \Sigma \frac{1}{\Sigma_k} B_k, \quad \tilde{\eta}_k = \eta_k - \Sigma \frac{1}{\Sigma_k} \eta_k \] (13)
where,
\[ \Sigma = \Sigma \frac{1}{\Sigma_k}. \] (14)

In fact, a linear KF running with Eq.(11) as a part of the measurement model is similar in essence with applying a sequential algorithm for bias estimation using the centering approach\(^6\). The given algorithm differs only as it applies centering within a moving window and adds Eq.(3) inherent to spinning spacecraft to the measurement model.

Both the pseudo-linear KF and the linear KF can be initialized using a batch of data. Spin period of the spacecraft is one of the factors determining the moving window and data size.
C. Simulations

The algorithms are tested for a hypothetical spinning nanosatellite whose orbit is assumed to be a low Earth orbit with a small eccentricity of $e = 6.4 \times 10^{-5}$, an inclination of $i = 74^\circ$ and approximate altitude of 612km. The satellite spins about body $Z$ axis with a spin rate of 7.5 rpm.

For the magnetometer measurements, the sensor noise is characterized by zero mean Gaussian white noise with a standard deviation of $\sigma_B = 300\, \text{nT}$. We assume that the magnetometers are free of errors other than the bias. The bias error is assumed to be constant and true values are given in Table 1. Simulations are run for 18000s, which is about 3 orbits for the satellite and the sampling time (both for filter propagation and magnetometer measurements) is $\Delta t = 1\, \text{s}$. The geomagnetic field is simulated using the International Geomagnetic Reference Field model.$^{10}$

The process noise covariance matrix for all bias terms is $Q_b = 10^{-8}\,(\text{nT})^2$. The filter initialized with bias estimates set to 0nT. The initial covariance matrix is diagonal and corresponding values are $P_{b,0} = 2 \times 10^7\,(\text{nT})^2$. The window size $N$ is selected as 8 samples which is exactly 1 spin period.

Table 1 summarizes the results for 100 runs executed with each KF algorithm. The results for a pseudo-linear KF that runs using only the scalar measurement (Eq.6) are also given. This helps understanding the advantages of including Eq.(3) in the measurement model. The mean the filter estimates at the final time are given together with their 3\(\sigma\) bounds. The pseudo-linear KF performs best and including the Eq.(3) in the measurement model improves the estimation accuracy. Although the mean values are close for the single measurement pseudo-linear KF and full three measurements pseudo-linear KF, 3\(\sigma\) bounds, which are smaller for the latter one, shows that including quasi-measurements increases the estimation accuracy.

<table>
<thead>
<tr>
<th>Bias Term</th>
<th>Truth</th>
<th>Single Measurement</th>
<th>Pseudo-Linear KF</th>
<th>Linear KF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_x$</td>
<td>5000 nT</td>
<td>5001.25 ± 18.06</td>
<td>5000.35 ± 15.51</td>
<td>4993.07 ± 17.34</td>
</tr>
<tr>
<td>$b_y$</td>
<td>3000 nT</td>
<td>3000.13 ± 16.54</td>
<td>3000.02 ± 15.21</td>
<td>2995.05 ± 16.62</td>
</tr>
<tr>
<td>$b_z$</td>
<td>4000 nT</td>
<td>3996.85 ± 14.40</td>
<td>3996.06 ± 11.64</td>
<td>3990.99 ± 13.38</td>
</tr>
</tbody>
</table>

Comparing the pseudo-linear KF and the linear KF, the pseudo-linear KF converges to the true values faster. Fig.3 and 4 show estimation results for a representative run. Using the nonlinear measurement (Eq.6) instead of its linear form (Eq.11) specifically quickens the convergence of $\hat{b}_z$ estimation, which is for the spin direction magnetometer. This is due to more accurate measurement modeling with the nonlinear equation. Moreover, the measurement noise in Eq.(11) is not uncorrelated although we ignore the correlation in practice.$^5$ In contrast the measurement variance for nonlinear equation (Eq.8b) can be calculated accurately using the estimates from the previous step, $\hat{b}_{z,-1}$.

Fig. 3 Pseudo-Linear KF estimates.
Varying the window size, \( N \), for mean calculation and centering requires retuning for the filters. Yet the pseudo-linear KF is more robust to such changes. The linear KF requires slightly more computation due to centering operation but this is negligible. Compared to more complex algorithms for real-time bias estimation such as the UKF both algorithm require very few computations.

IV. Conclusion

Two simple real-time attitude-independent magnetometer bias estimation algorithms were developed for spinning spacecraft. In addition to the well-known attitude-independent scalar measurement for magnetometer calibration, two auxiliary measurements were derived by making use of the spin dynamics. Simulations showed that using these auxiliary measurements increases the bias estimation accuracy as well as improving algorithm’s convergence speed. The pseudo-linear Kalman filter, which uses nonlinear form of the scalar measurement equation, is the most robust algorithm in terms of both overall accuracy and convergence properties. The other benefit of the algorithms is having very few computational requirements for real-time implementation.

Acknowledgments

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References